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# FELTER'S

# COMPLETE ARITHMETIC.

*COMPRISING THE NEW INTERMEDIATE ARITHMETIC AND PART OF THE ADVANCED ARITHMETIC.*

BY

**S. A. FELTER, A.M.,**
AUTHOR OF A SERIES OF ARITHMETICS,
AND
**S. A. FARRAND, Ph.D.,**
PRINCIPAL OF THE NEWARK ACADEMY.

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# PREFACE.

THIS book is designed to make the pupil quick and accurate in calculation, and to give him a knowledge of those principles and processes of arithmetic which are needed in the ordinary transactions of life, together with skill in their application.

To accomplish this, the DRILL CARD EXERCISES are arranged to furnish any desired amount of practice in computation; while the processes and analyses leading directly to the rule, together with the number, gradation, and character of the practical examples, give the knowledge of necessary principles, and skill in their use.

All unnecessary distinctions, processes, and classifications have been omitted, and those only retained which are required in the preparation for ordinary business.

Business-men generally and justly complain of the inability of pupils fresh from school to perform accurately the calculations required in ordinary transactions. This inaccuracy arises from want of suitable and sufficient practice. The Drill Card Exercises furnish an unlimited amount of graded work, from which the teacher, without labor, may select such examples as any pupil or class may require.

The combination of oral and written work will commend itself to the intelligent teacher.

The authors wish to acknowledge their indebtedness to Mr. S. W. Clark, Principal of Washington Street Public School, Newark, N.J., for valuable assistance in the preparation of the work; to the principals of other public schools of Newark, for important suggestions; and to Rev. M. R. Hooper of Yonkers, N.Y., for efficient aid while the work was passing through the press.

S. A F.
S. A. F.

JUNE, 1875.

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## SECTION I.

### DEFINITIONS, NOTATION AND NUMERATION.

In writing numbers we use ten characters, called *figures*. The first figure, 0, is called a *cipher*, or *nought*, and denotes *nothing*, or the absence of number.

Each of the other figures represents a number of *single things*, or units.

***1. A Unit**** is a single thing; as, 1 horse, 1 dog, 1 apple, 1 dozen, 1 fourth.

---

* Note for the Teacher. — Let this be fully explained, that the pupil may observe the distinction between a hundred as one unit, illustrated by a hundred-dollar-note, and as many units, illustrated by one hundred one-dollar-notes.

The teacher should question the class thoroughly on each subject: he should

***2. A Number*** is a unit, or collection of units; as, 2 cows, 1 house, 5 sheep, 1, 3, 4, 9.

***3. A Concrete Number*** is one which indicates some definite kind of thing; as, 1 bird, 7 sheep, 2 ounces.

***4. A Denominate Number*** is one in which the kind of thing named is a unit of weight, measure, or currency; as, 5 pounds, 2 yards, 6 dimes.

***5. An Abstract Number*** is one which does not indicate any definite kind of thing; 1, 5, 7, 6, 9, 14.

***6. Arithmetic*** treats of numbers, and the art of using them.

## NOTATION.

***7. Notation*** is a method of expressing numbers by words, letters, or figures. The methods most used are the **Roman**, which employs letters, and the **Arabic**, which employs figures.

***8. The Roman Notation*** uses the letters I, V, X, L, C, D, M. The numbers of sections, chapters, and

---

strive by illustrations and suggestions to develop in the mind of the pupil a clear conception of the idea contained in the definition, proposition, or problem, before requiring the one to be learned, or the other to be solved. The following notes and hints are merely suggestive, and are not intended to be either systematic or exhaustive. No book can take the place of a living teacher; neither can any teacher follow any prescribed method of instruction, for by so doing he sacrifices that individuality which is essential to success.

REVIEW QUESTIONS.* — What is a unit? (1.) What is a number? (2.) What is a concrete number? (3.) What is a denominate number? (4.) What is an abstract number? (5.) Of what does arithmetic treat? (6.)

* NOTE FOR THE TEACHER. — The numbers at the end of the questions refer to the paragraphs containing the answers.

N.B. — These review questions should by no means be omitted.

other divisions of books, are usually written by this method.

### TABLE.

| | | | | | |
|---|---|---|---|---|---|
| I. denotes | *one,* | 1. | C. denotes | *one hundred,* | 100. |
| V. " | *five,* | 5. | D. " | *five hundred,* | 500. |
| X. " | *teh,* | 10. | M. " | *one thousand,* | 1,000. |
| L. " | *fifty,* | 50. | $\overline{M}$. " | *one million,* | 1,000,000. |

All other numbers are expressed by the use of these letters repeated or combined; as,—

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| I. | one. | IX. | nine. | LXXX. | eighty | DCC. | seven hundred. |
| II. | two. | X. | ten. | XC. | ninety. | DCCC. | eight hundred. |
| III. | three. | XX. | twenty. | C. | one hundred. | DCCCC. | nine hundred. |
| IV. | four. | XXX. | thirty. | CC. | two hundred. | M. | one thousand. |
| V. | five. | XL. | forty. | CCC. | three hundred. | MD. | fifteen hundred. |
| VI. | six. | L. | fifty. | CCCC. | four hundred. | MM. | two thousand. |
| VII. | seven. | LX. | sixty. | D. | five hundred. | $\overline{X}$. | ten thousand. |
| VIII. | eight. | LXX. | seventy. | DC. | six hundred. | $\overline{M}$. | one million. |

OBSERVATIONS.—From the above it will be noticed, that

1. The value of a letter is repeated as many times as the letter itself is repeated; as, XXX. *thirty,* CC. *two hundred.*

2. If a letter of *less value* stands *before* a letter of *greater value,* the value of the less is *subtracted* from that of the greater; as, X. *ten,* IX. *nine,* L. *fifty,* XL. *forty.*

3. If a letter of *less value* stands *after* one of *greater value,* the value of the *less* is *added* to that of the greater; as, V. *five,* VI. *six,* M. *one thousand,* MD. *fifteen hundred.*

4. A dash (—) placed over a letter, or a combination of letters, repeats the value a thousand times; as X. *ten,* $\overline{X}$. *ten thousand,* XII. *twelve,* $\overline{XII}$. *twelve thousand.*

**NOTE FOR THE TEACHER.**—Slate and blackboard drill in writing and reading numbers in Roman notation should be furnished by the teacher.

**REVIEW QUESTIONS.**—What is the Roman notation? (7.) Why so called? What is its use? How many letters does it use? (8.) Name them. Which letter has the greatest value? Which the least? Which letter stands for fifty? Which for ten? Which for one hundred? Which for five hundred? What effect has the repetition of a letter?

## ARABIC NOTATION.

***9. The Arabic Notation*** uses ten characters, or figures; as,

| | | | |
|---|---|---|---|
| 0, | *cipher.* | 2, *two.* | 6, *six.* |
| | *nought.* | 3, *three.* | 7, *seven.* |
| | *zero.* | 4, *four.* | 8, *eight.* |
| 1, | *one.* | 5, *five.* | 9, *nine.* |

1, 2, 3, 4, 5, 6, 7, 8, and 9 are often called *digits.*

When we count ten things, as we have no figure to represent the number *ten,* we unite the ten into a group,* and call it *ten,* or, 1 *ten,* and represent it by writing the 1 in the second place, thus, 10; the cipher showing, that, while we have 1 *ten,* we have no additional units.

When we count twelve things, we have one ten and two single things, represented thus; 12.

When we count ten *tens,* as we have no figure to represent the number, we unite the *tens* into a group, and call it 1 hundred, and write the 1 in the third place; thus, 100; the ciphers showing that we have no additional *tens* or *units.*

***10. The Simple Value*** of a figure is the one expressed by it when standing alone, or in the unit's place.

***11. The Local Value*** of a figure is the one it acquires by standing in a particular place in a number; as, 3 in 30, where it represents three *tens,* or thirty; 3 in 364, where it represents three *hundreds;* &c.

***12. Numeration*** is the art of reading numbers expressed by figures.

---

* The different orders of units can be very clearly illustrated by tying ten sticks into a bundle to represent 1 *ten,* then a package of ten such bundles to represent 1 *hundred,* &c.

### NUMERATION TABLE.

| 6th Period.* | | | 5th Period. | | | 4th Period. | | | 3d Period. | | | 2d Period. | | | 1st Period. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Quadrillions. | | | Trillions. | | | Billions. | | | Millions. | | | Thousands. | | | Units. | | |
| Hundreds. | Tens. | Units. | Hundreds. | Tens. | Units. | Hundreds. | Tens. | Units. | Hundreds. | Tens. | Units. | Hundreds. | Tens. | Units. | Hundreds. | Tens. | Units. |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Every place in a number not occupied by digits must be filled with a cipher.

The periods of a number are often separated by commas.

### EXERCISES.

1. Write 4 hundreds and 2 units, commencing at the left hand, and read the number.

Write 3 tens and 7 units in the same manner, and read it.

Write 5 hundreds, 3 tens, and 2 units in the same manner.

Write 4 tens, and 6 units.

Write 5 hundreds and 4 units.

Write 7 hundreds and 2 tens.

2. Write 5 hundreds, 7 tens, and 2 units, and read the number.

Write 3 tens and 1 unit.

Write 5 hundreds and 6 units.

| Units' Period. | | |
|---|---|---|
| H. | T. | U. |
| 4 | 0 | 2 |
| | 3 | 7 |
| 5 | 3 | 2 |
| | 4 | 6 |
| 5 | 0 | 4 |
| 7 | 2 | 0 |

---

* From quadrillions, the table may be extended to quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, &c.

3. Write 6 units.
9 tens and 4 units.
6 hundreds and 7 units.
5 hundreds and 6 tens.
9 hundreds and 5 tens.

4. Write 7 hundreds and 6 tens.
7 tens and 3 units.
5 hundreds and 8 tens.
6 hundreds and 5 units.
3 tens and 7 units.

5. Write 8 tens and 3 units.
7 hundreds and 6 tens.
3 tens and 4 units.
8 tens and 3 units.
4 hundreds.

6. Write 3 tens and 8 units.
6 tens and 3 units.
5 hundreds and 4 tens.
9 hundreds and 3 tens.
7 tens and 8 units.

7. Write 2 in each period, commencing at the left hand, and read the number.

Write 22 in each period, commencing at the left hand, and read the number.

Write 222 in each period, commencing at the left hand, and read the number.

**Note for the Teacher.** — The pupil should be required to prepare the following exercises thus arranged, on slate or paper, to be read as a class-exercise. The mechanical execution of the work should be carefully inspected.

MODEL OPERATION.

| Tr. | | | Bill. | | | Mill. | | | Th. | | | U. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| H. | T. | U. | H. | T. | U. | H. | T. | U. | H. | T. | U. | H. | T. | U. |
| | | 2 | 0 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 2 |
| | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 2 | 0 | 2 | 2 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

To be read. — Two *trillions*, two *billions*, two *millions*, two *thousand* and two.

Twenty-two *trillions*, twenty-two *billions*, twenty-two *millions*, twenty-two *thousand* and twenty-two.

Two hundred and twenty-two *trillions*, two hundred and twenty-two *billions*, two hundred and twenty-two *millions*, two hundred and twenty-two *thousand*, two hundred and twenty-two.

8. Vary example 7, by writing 3 wherever 2 was used; vary again by using 4, and thus by using each of the digits in succession. Read the numbers.

9. Write in each period the numbers from 11 to 20, *and read.*

10. Let each pupil dictate an example, to be written and read by the class.

The teacher should require the pupils, in the same manner, to write and read, as a class-exercise, series of numbers; as, from 1 to 20, from 100 to 130, &c., until promptness and accuracy are secured.

The exercise may be varied by dictating from the drill-card, Tables I. and II. (see page 17), as well as reading from the same. The teacher should write in figures, on the blackboard, numbers, to be copied in words by the pupils on their slates. The pupils' work should be criticised in spelling, &c.: numbers should also be copied in Roman notation, to be read as a class-exercise.

***13. Observe,*** that, in the Arabic notation,—

1. Ten units of any order make one unit of the next higher order.

2. A digit in any place has a value ten times as large as that of the same digit one place to the right: hence,

3. The value of any figure is increased tenfold by removing it one place to the left. It is decreased tenfold by removing it one place to the right; thus tens will become UNITS; UNITS will become *tenths; tenths* become *hundredths*, &c.

## REDUCTION.

Extend this table to millions, and present it as an exercise:—

10 *thousandths* = 1 *hundredth.*
10 *hundredths* = 1 *tenth.*
10 *tenths* = 1 UNIT.
10 UNITS = 1 ten, &c.

### ANALYSIS OF NUMBERS.

It is required to analyze the number 3,917,893.

ANALYSIS. — 9 tens are equal to 90 units; 8 hundreds are equal to 80 tens, or 800 units; in the thousands' period 7 thousands are equal to 70 hundreds, or 700 tens, or 7,000 units; 1 ten of thousands is equal to 10 thousands, or 100 hundreds, or 1,000 tens, or 10,000 units: 9 hundreds of thousands are equal to 90 tens of thousands, or 900 thousands, or 9,000 hundreds, or 90,000 tens, &c.

After the model, analyze the following numbers: —

1. One million three hundred and eighty-five thousand six hundred and seventy-four.
2. 19 millions 713 thousand 486.
3. 186 millions 247 thousand 391.
4. 421 millions 223 thousand 86.
5. 5 trillions 876 millions 876 thousand.
6. 189 quadrillions 317 thousand 346.
7. 896 millions 348 thousand.
8. 876 millions 384 thousand.

**14.** ***A Rule*** is a concise direction for solving problems.

**15.** RULE FOR NOTATION. — *Begin at the left hand, and write in the place of each successive order the figure belonging to that order, filling with ciphers the places otherwise vacant.*

*Indicate the place of units by a decimal point whenever an order is written which has a lower value than units.*

**16.** RULE FOR NUMERATION. — *Begin at units' place, and separate the number into periods of three figures each.*

*Beginning at the highest period, read each separately, giving the name of each period except that of units.*

## DECIMAL SCALE. — U. S. CURRENCY.

U. S. Currency.

| Dollars. | Cents. | Mills. |
|---|---|---|
| 3 | 42 | 5 |

| Units. | 10ths. | 100ths. | 1000ths. |
|---|---|---|---|
| 3. | 4 | 2 | 5 |

Decimal Scale.

**1.** It will be seen by the accompanying scale that the United States currency is based upon the decimal scale of notation; the "dollars" corresponding to the *units*, the "cents" to the *tenths* and *hundredths*, and the "mills" to the *thousandths.*

**2.** Teachers who may desire can easily dictate numbers like the following: —

| | |
|---|---|
| 1. Write 3 and 4 *tenths.* | 2. Write 3 dollars and 40 cents. |
| 1 and 42 *hundredths.* | 1 dollar and 42 cents. |
| 16 and 327 *thousandths.* | 16 dollars, 32 cents, and 7 mills. |
| 5 and 41 *thousandths.* | 5 dollars, 4 cents, and 1 mill. |

For exercise in reading decimals, or United States Currency, see Drill Card, Table III. (page 17).

## ANALYSIS OF SECTION I.*

### I. Number.

1. CONCRETE.
2. DENOMINATE.
3. ABSTRACT.

### II. Notation.

1. ROMAN.
   *Letters.*
2. ARABIC.
   *Figures.*
   Simple value.
   Local value.

### III. Numeration.

ORDERS.
- (*a.*) *Ratio of increase.*
- (*b.*) *Names.*

---

* The "analysis" at the close of each section should form the outline for a topical review, which may be either oral or written, or both.

# SECTION II.

## ADDITION.

In this picture some boys and girls have been gathering peaches.

Arthur has 1 peach in each hand.

Ezra has 2 peaches in one hand, and 1 in the other.

Sophie has 3 peaches in a little basket, and 1 in her hand.

Selina has 4 peaches in her lap, and 1 in her hand.

How many has each?

If Sophie gives her peaches to Selina, how many will she have?

If all put their peaches into Selina's lap, how many will she have?

This putting of things together to find the entire number is called, in arithmetic, *addition;* and the number found is called the *sum*, or *amount:* hence,

**17. *Addition*** is the process of finding the sum of two or more numbers.

**18. *The Sum*** of two or more numbers is the number which contains all their units.

**19. *The Sign of Addition*** is a vertical cross; thus, +. It is called *plus*, and, when placed between two numbers, it denotes that they are to be added together; as, $4 + 6$ equals 10.

**20. *The Sign of Equality*** is two short horizontal parallel lines; thus, =. When placed between two numbers or quantities, it denotes that they are equal; as, $4 + 6 = 3 + 7$.

**21. *The Dollar Sign*** ($) is placed before dollars, and the decimal point after them; thus, $86.42, to be read eighty-six dollars and forty-two cents.

**22. *An Axiom*** is a self-evident truth. The following is used in addition:—

AXIOM.—*Things can be added to like things only.*

When it is required to find how many peaches Selina and Ezra have together, the question, in arithmetic, is called a *problem;* and the explanation of the act, or process of putting them together is called a *solution*, or an *analysis.* A problem having but one such question is called a *simple problem;* but one that has two or more questions, requiring an answer to each, is called a *complex problem.* Each question in a complex problem indicates an *analytical step:* hence,

---

REVIEW.—What is the Arabic notation? (9.) How many figures, or characters, are used? What is the simple value of a figure? (10.) What is the local value? (11.) What is numeration? (12.) Repeat the numeration table by periods from right to left. What is the use of the cipher?

**23.** ***A Problem,*** in arithmetic, is a question which requires solution.

**24.** ***A Simple Problem*** contains but one elementary question.

**25.** ***A Complex Problem*** contains more than one elementary question.

**26.** ***An Analytical Step*** is one of the steps of the process of solving a complex problem.

**27.** ***Order of Solving Problems:—***

1. State the problem.
2. Give necessary definitions.
3. Analyze.
4. Give conclusions.
5. Prove.

**28.** ***Analysis of Addition.****

PROBLEM.—How many are 389 + 486 + 394 + 437?

1. STATEMENT.—We are to find the sum of 389, 486, 394, and 437.

2. DEFINITION.—The sum of two or more numbers is the number which contains all their units.

PROCESS.

```
 389
 486
 394
 437
----
1706
```

3. ANALYSIS.—(1.) For convenience, the numbers are so written that units shall stand in the column of units, tens in the column of tens, &c.

(2.) For convenience, add the column of the lowest order; thus, 7, 11, 17, 26: 26 units = 2 tens and 6 units. Write the units in the column of units, and add the tens with the column of tens; thus,

(3.) 2, 5, 14, 22, 30: 30 tens = 3 hundreds. Write 0 in the column of tens, and add the hundreds with the column of hundreds; thus,

(4.) 3, 7, 10, 14, 17: 17 hundreds = 1 thousand and 7 hundreds.

---

* The analysis of addition may be omitted until a final review of the book.

Write the hundreds in the column of hundreds, and the 1 thousand in the place of thousands.

4. CONCLUSION. — Therefore the sum of 389, 486, 394, and 437, is 1 thousand 7 hundred and 6.

Hence we have the

5. RULE. — I. *Write the numbers so that units shall stand in the column of units, tens in the column of tens, &c.*

II. *Beginning with the lowest order, add each column separately: if the sum be less than ten, place it under the column added.*

III. *If the sum of any column be ten or more, write the units' figure of that order under the column added, and add the others with those of the next column.*

6. PROOF. — *Repetition is the only practical proof of addition. Add the numbers from the top downward, and, if the sum agrees with that previously obtained, it is presumed to be right.*

### *29. Blackboard Exercises.*

The teacher should require the class, department, or entire school, to spend at least five minutes daily for ONE YEAR, in concert drill on the following column exercises in addition. In order to prevent habits of hesitation and inaccuracy, each column exercise should be THOROUGHLY MASTERED before attempting the next.

---

**REVIEW.** — What should be done with the vacant places of a period? (12.) What with vacant periods? How are the periods of a number usually separated? What is a rule? (14.) What is the rule for notation? (15.) Repeat the rule for numeration. (16.) What is addition? (17.) What is the sum of two or more numbers? (18.) Give the description and use of the sign of addition? (19.) Of the sign of equality? (20.) Of the dollar sign? (21.) What is an axiom? (22.) Repeat the axiom used in addition. What is a problem? (23.) What is a simple problem? (24.) Give an example. What is a complex problem? (25.) What is an analytical step? (26.) Give the order of solving problems. (27.)

The following table gives the key to the secret of rapid addition. It contains all the different changes in the combination of the digits by addition.

### TABLE.

| (1.) | (2.) | (3.) | (4.) | (5.) | (6.) | (7.) | (8.) | (9.) | (10.) |
|---|---|---|---|---|---|---|---|---|---|
| &c. | &c. | &c. | &c. | &c. | &c. | &c. | &c. | &c. | &c. |
| 2 | 2 | 1 | 3 | 1 | 4 | 4 | 1 | 5 | 3 |
| 2 | 2 | 2 | 3 | 2 | 4 | 4 | 4 | 5 | 2 |
| 2 | 2 | 1 | 3 | 1 | 4 | 4 | 3 | 5 | 1 |
| 2 | 2 | 1 | 3 | 2 | 4 | 4 | 1 | 5 | 2 |
| 2 | 2 | 2 | 3 | 3 | 4 | 4 | 2 | 5 | 4 |
| 2 | 2 | 2 | 3 | 2 | 4 | 4 | 1 | 5 | 3 |
| 2 | 2 | 1 | 3 | 1 | 4 | 4 | 4 | 5 | 1 |
| 2 | 2 | 2 | 3 | 3 | 4 | 4 | 2 | 5 | 2 |
| 2 | 2 | 1 | 3 | 1 | 4 | 4 | 3 | 5 | 3 |
| 2 | 2 | 1 | 3 | 2 | 4 | 4 | 4 | 5 | 1 |
| 2 | 2 | 2 | 3 | 3 | 4 | 4 | 3 | 5 | 4 |
| 2 | 1 | 1 | 3 | 1 | 4 | 1 | 1 | 5 | 5 |
| — | — | — | — | — | — | — | — | — | — |

| (11.) | (12.) | (13.) | (14.) | (15.) | (16.) | (17.) | (18.) | (19.) | (20.) |
|---|---|---|---|---|---|---|---|---|---|
| &c. | &c. | &c. | &c. | &c. | &c. | &c. | &c. | &c. | &c. |
| 6 | 6 | 3 | 7 | 3 | 8 | 8 | 1 | 9 | 3 |
| 6 | 6 | 4 | 7 | 4 | 8 | 8 | 7 | 9 | 1 |
| 6 | 6 | 3 | 7 | 3 | 8 | 8 | 8 | 9 | 4 |
| 6 | 6 | 6 | 7 | 1 | 8 | 8 | 3 | 9 | 2 |
| 6 | 6 | 4 | 7 | 2 | 8 | 8 | 4 | 9 | 3 |
| 6 | 6 | 6 | 7 | 3 | 8 | 8 | 3 | 9 | 1 |
| 6 | 6 | 5 | 7 | 6 | 8 | 8 | 1 | 9 | 4 |
| 6 | 6 | 3 | 7 | 7 | 8 | 8 | 7 | 9 | 3 |
| 6 | 6 | 6 | 7 | 1 | 8 | 8 | 8 | 9 | 6 |
| 6 | 6 | 3 | 7 | 3 | 8 | 8 | 7 | 9 | 7 |
| 6 | 6 | 5 | 7 | 1 | 8 | 8 | 3 | 9 | 1 |
| 6 | 1 | 1 | 7 | 7 | 8 | 1 | 1 | 9 | 9 |
| — | — | — | — | — | — | — | — | — | — |

## METHOD OF RECITATION.

Copy the exercise on the blackboard, and let a pupil, or class in concert, commence at the bottom, and add, repeating only the result of each addition: thus, (Exercise 1) 2, 4, 6, 8, 10, 12, 14, &c., to 50, or more, as the teacher may judge best. When the first exercise is THOROUGHLY mastered, proceed with the second in the same manner, MASTERING each exercise successively, until all are finished. The exercises in this lesson need not be completed before the pupil proceeds; but they should form a part of each recitation until finished.

## DRILL CARD EXERCISES.

N.B. — The problems in the following card exercises are graded by digits and steps, in the same way as those in the Primary Arithmetic. Such exercises should be selected as are best suited to the necessities of the class, as few if any classes will need them all.

The problems may be assigned for home work, seat or rapid class drill; but in no case should the pupils be allowed the answers while preparing lessons.

The *answers*, *explanations*, and *methods* are published in a KEY for the use of teachers only, and may be obtained through the mail by addressing the *publishers*.

| TABLE I., *Units' Period.* | TABLE I., *Thousands.* | TABLE II., *Units.* |
|---|---|---|
| Exercise I. — 3 numbers. | Exer. VIII. — 3 numbers. | Exer. XV. — 2 numbers. |
| II. — 4 " | IX. — 4 " | XVI. — 3 " |
| III. — 5 " | X. — 5 " | XVII. — 4 " |
| IV. — 6 " | XI. — 6 " | XVIII. — 5 " |
| V. — 7 " | XII. — 7 " | XIX. — 6 " |
| VI. — 8 " | XIII. — 8 " | XX. — 7 " |
| VII — 9 " | XIV. — 9 " | XXI. — 8 " |
| | | XXII. — 9 " |

| TABLE II., *Thousands.** | TABLE II., *Billions.* | *U.S. Currency*, TABLE III. |
|---|---|---|
| Exer. XXIII. — 2 numbers. | Exer. XXXI. — 10 numbers. | Exer. I — 3 numbers. |
| XXIV. — 3 " | | II. — 4 " |
| XXV. — 4 " | | III. — 5 " |
| XXVI — 5 " | | IV. — 6 " |
| XXVII. — 6 " | | V. — 7 " |
| XXVIII. — 7 " | | VI. — 8 " |
| XXIX. — 8 " | | VII. — 9 " |
| XXX. — 9 " | | VIII. — 10 " |

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* Includes units' and thousands' period.

## DIRECTIONS FOR USING FELTER'S DRILL CARDS.

### EXERCISE I.

**PREPARATION: 1ST SERIES OF TWENTY PROBLEMS.** 1st Example. — Copy three numbers from the card on a slate or paper, commencing with the number opposite the letter A (Table I., Units' Period).

2d Example. — Copy three numbers, commencing with the number opposite the letter B, as before.

3d Example. — Copy three numbers, commencing with the letter C.

4th Example. — Copy three numbers, commencing with the number opposite the letter D: thus,

| (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|
| A. 022 | B. 202 | C. 200 | D. 220 |
| 202 | 200 | 220 | 222 |
| 200 | 220 | 222 | 110 |
| Ans. 424 | Ans. 622 | Ans. 642 | Ans. 552 |

In the same manner, commence with the number opposite each letter in the column between the tables. If there are not numbers enough in the column of figures to finish the example, commence again at the top, and take a sufficient number.

### EXERCISE II.

**SECOND SERIES: FOUR NUMBERS.** 1st Example. — Copy (Table I., Units' Period) four numbers, commencing with the number opposite the letter A.

2d Example. — Copy four numbers, commencing with the letter B.

3d Example. — Copy as before, &c.

### EXERCISE III.

**THIRD SERIES.** — Five numbers, copy as before.

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**REMARKS.** — Pupils should not be allowed to copy results from each other. The careful teacher can easily detect and prevent it. No work should be accepted, unless neatly and carefully done. Once or twice a week, the lesson should be copied from dictation. A committee of one or two pupils may be appointed to examine the slates, and report on the neatness and accuracy of the work, and thereby save the teacher's time.

**ANSWERS.** — The answers in the key will be found to correspond with the letters opposite the first number in each problem.

**RECITATION.** — 1. Let the teacher mention the letter opposite the first number of each example; at which, let the class rise.

2. Let the teacher name one of the class to read his answer.
3. Let the pupils who have the same result sit.
4. If any remain standing, let the teacher name another pupil, as before.
5. Let the teacher read the correct answer; at which, let those who are correct rise, those who are wrong remain seated. Draw a line distinctly across the incorrect answers.
6. At the close, let the teacher make a record of the recitation. Those who are wrong correct their work at their seats.

## FELTER'S
# ARITHMETICAL DRILL CARDS.

| TABLE I. | | TABLE II. | | TABLE III. |
|---|---|---|---|---|
| Thou. Units. | | B. Mill. Thou. Units. | | Dollars. Cts. m. |
| 579,438 | | 9,381,908,191 | | 5,319.43 4 |
| 242,022 | A | 8,448,646,446 | A | 3,402 cts. |
| 442,202 | B | 6,688,286,466 | B | 1,811 m. |
| 442,200 | C | 868,424,246 | C | 60 dol. |
| 240,220 | D | 606,486,644 | D | 403 cts. |
| 402,222 | E | 428,664,246 | E | 10 dol. |
| 331,110 | F | 75,351,535 | F | 121 m. |
| 303,211 | G | 73,735,353 | G | 8 cts. |
| 311,121 | H | 53,533,353 | H | 195 dol. |
| 342,221 | I | 53,131,533 | I | 2,000 m. |
| 221,121 | K | 33,713,555 | K | 715 dol. |
| 550,112 | L | 29,036,632 | L | 86 cts. |
| 404,201 | M | 19,241,747 | M | 7 dol. |
| 040,121 | N | 9,866,474 | N | 9 m. |
| 443,121 | O | 9,673,342 | O | 8,189 cts. |
| 552,232 | P | 9,477,773 | P | 908 m. |
| 243,110 | Q | 8,347,544 | Q | 4,003 cts. |
| 152,231 | R | 887,744 | R | 500 dol. |
| 435,112 | S | 347,654 | S | 6,001 cts. |
| 543,121 | T | 281,263 | T | 2,100 m. |
| 402,301 | U | 187,602 | U | 5,000 cts. |

N.B. — A **Key** containing the answers to the problems on the above card may be obtained by *mail*, by addressing the publishers.

## EXERCISES IN NOTATION AND ADDITION.

The following problems should be prepared from the book WITHOUT ANSWERS, on slates, or paper, by the pupils at their seats. They may be recited as follows: —

Those who are prepared on the first problem rise. One pupil is required to read *his* answer; and those whose answers agree with it then sit down. If any remain standing, they, also, are required to read, after which the teacher announces the correct result from the key. Those who are wrong draw a line across the incorrect answer, and report at the close of the recitation. The other problems may be recited in the same way. The recitation of ten problems should only take from *five to seven* minutes.

1. Add,

1 thousand,
1 thousand, 280,
10 thousand, 50,
50 thousand, 200,
529 thousand, 625.

MODEL PROCESS.

1000
1280
10050
50200
529625
———
592155

2. Add,

80 thousand, 7,
40 thousand, 40,
70 thousand, 20,
50 thousand, 5,
90 thousand, 90,
4 thousand, 4.

3. Add,

300 thousand, 6,
400 thousand, 60,
900 thousand, 600,
11 thousand, 11,
14 thousand, 9,
700 thousand, 683.

4. Add,

500 thousand, 32,
600 thousand, 438,
200 thousand, 29,
900 thousand, 30,
800 thousand, 284,
20 thousand, 384.

5. Add,

538 thousand, 224,
829 thousand, 720,
80 thousand, 9,
400 thousand, 560,
940 thousand, 12,
82 thousand, 36.

---

REVIEW. — What is a unit? (1.) A number? (2.) A concrete number? (3.) A denominate number? (4.) An abstract number? (5.) Of what does arithmetic treat? (6.)

6. Add,

750 thousand, 20,
870 thousand, 496,
100 thousand, 20,
99 thousand, 50,
30 thousand, 98,
700 thousand, 561.

7. Add,

350 thousand, 880,
60 thousand, 56,
499 thousand,
22 thousand, 85,
456 thousand, 790,
600 thousand, 70.

8. Add,

3 millions, 387 thousand, 24,
4 millions, 46 thousand, 4,
9 millions, 8 thousand, 6,
20 millions, 7 thousand, 720,
10 millions, 900 thousand,
5 millions, 248 thousand, 6.

9. Add,

14 millions, 14 thousand, 14,
3 millions, 760 thousand, 480,
8 millions, 20 thousand, 2,
50 millions, 640 thousand,
22 millions, 321 thousand, 56,
8 millions, 777 thousand, 988.

10. Add,

25 millions, 927 thousand,
14 millions, 26 thousand, 327,
32 millions, 591 thousand, 33,
40 millions, 800 thousand,
8 millions, 700 thousand, 7,
11 millions, 256 thousand, 277.

11. Add,

65 millions, 999 thousand, 80,
8 millions, 200 thousand,
38 millions, 45 thousand, 240,
17 millions, 590 thousand,
25 millions, 25 thousand, 25,
20 millions, 444 thousand, 960.

12. Add,

400 millions, 900 thousand,
100 millions, 450,
700 millions, 7 thousand, 9,
900 millions, 489 thousand,
300 millions, 7 thousand, 60,
800 millions, 700 thousand, 19.

13. Add,

1100 millions, 920,
1800 millions,
300 millions, 4 thousand, 50,
500 millions, 200 thousand,
800 millions, 800 thousand,
100 millions, 567

14. Add,

400 millions, 9 thousand, 30,
900 millions, 99,
600 millions, 24 thousand,
1200 millions, 884,
300 millions, 350 thousand,
700 millions, 256.

15. Add,

1200 millions, 93 thousand,
1800 millions, 27,
200 millions, 426 thousand, 200
500 millions, 956 thousand,
900 millions,
300 millions, 50.

16. Add,
5 hundred, 20,
99,
3 thousand, 96,
9 hundred,
26 thousand, 780,
1 million, 480.

17. Add,
36,
9 thousand, 900,
228 thousand,
4 millions, 26 thousand, 580,
500 millions, 4 thousand, 10,
4 hundred, 65.

18. Add,
1100 millions,
98,
24 thousand, 420,
7 millions, 20 thousand,
58,
200 millions, 4 thousand, 50.

19. Add,
10 thousand, 10,
11 millions, 726 thousand,
5 hundred, 90,
78,
300 millions, 989,
66 thousand, 251.

20. Add,
4 hundred, 50,
720 thousand, 787,
500 millions, 66 thousand,
69,
400 millions, 228 thousand, 90,
26 thousand, 10.

21. Add,
6 millions, 84 thousand, 291,
900 millions,
29,
650,
3 thousand, 80,
400 millions, 86.

22. Add,
790,
800 millions, 596 thousand,
500 thousand, 970,
88,
3 millions, 700,
26 thousand, 777.

23. Add,
900 thousand,
47,
19 millions, 19 thousand,
7 thousand, 47,
600 millions, 59,
9 hundred, 60.

NOTE TO TEACHER. — Examples *ad libitum* may be dictated from the drill cards, answers to which will be found in the key.

| 24. Add, | PROCESS. |
|---|---|
| 3 dollars, 20 cents, 1 mill, | $3.201 |
| 23 dollars, 40 cents, 2 mills, | 23.402 |
| 63 cents, 2 mills, | .632 |
| 4 mills, | .004 |
| 4 cents, 4 mills. | .044 |
| | $27.283 |

25. Add,
5 dollars, 33 cents, 4 mills,
2 dollars, 3 cents, 1 mill,
4 dollars, 5 cents, 3 mills,
6 dollars, 1 cent, 2 mills,
8 dollars, 5 cents, 1 mill.

26. Add,
4 dollars, 18 cents, 3 mills,
6 dollars, 5 cents, 2 mills,
46 dollars, 14 cents, 3 mills,
14 dollars, 21 cents, 1 mill,
142 dollars, 16 cents, 3 mills.

27. Add,
9 dollars, 49 cents, 7 mills,
5 dollars, 16 cents, 2 mills,
37 dollars, 22 cents, 4 mills,
1 dollar, 1 cent, 5 mills,
2 dollars, 0 cents, 4 mills.

28. Add,
13 dollars, 80 cents, 2 mills,
16 dollars, 3 cents, 3 mills,
15 dollars, 0 cents, 0 mills,
0 dollars, 0 cents, 8 mills,
0 dollars, 10 cents, 0 mills,

29. Add,
85 dollars, 85 cents, 3 mills,
1 dollar, 1 cent, 1 mill,
34 dollars, 14 cents, 0 mills,
58 dollars, 13 cents,
43 dollars.

30. Add,
58 dollars, 16 cents, 2 mills,
41 dollars, 38 cents, 4 mills,
4 cents,
4 dollars,
43 dollars, 23 cents.

31. Add,
48 cents,
5 mills,
10 dollars,
6 cents,
50 dollars.

32. Add,
32 dollars,
46 cents,
5 mills,
3 dollars,
2 cents.

**Note to Teacher.** — For additional examples, dictate from drill cards, Table III., in the same manner as in addition of simple numbers.

## ORAL PROBLEMS CONTAINING BUT ONE ELEMENTARY QUESTION.

### SIMPLE NUMBERS.

Mary has 30 books, and Margaret has 40: how many have both?

**Formula.** — 30 books + 40 books = 70 books.

**Analysis.** — If Mary has 30 books, and Margaret has 40, they both have the sum of 30 books and 40 books, which is 70 books.

1. A man bought a cow for $30, and a horse for $80: what was the cost of both?

2. A farmer sold 60 sheep, and kept 40: how many had he at first?

3. Susan is 20 years older than John, and John is 40 years old: how old is Susan?

4. Some silk cost 60 cents, some calico 80 cents: what is the cost of both?

5. Edward has 60 nuts, which is 50 less than the number Charles has: how many nuts has Charles?

At the option of the teacher, additional oral examples may be formed by substituting the following numbers in the corresponding examples; as, 40 and 85 in the place of 30 and 80 in the first example; then, again, 26 and 82 in the place of 30 and 80.

1. 40 and 85, 26 and 82, 96 and 47.
2. 50 and 67, 93 and 72, 47 and 96.
3. 53 and 70, 67 and 21, 87 and 37.
4. 63 and 80, 57 and 62, 29 and 67.
5. 49 and 60, 23 and 72, 67 and 95.

## UNITED STATES CURRENCY.

The examples of United States currency in the fundamental rules may be omitted until a review, if thought best by the teacher. We think, however, with the proper explanation, and sufficient card drill work, no difficulty will be experienced by the pupils.

6. A boy bought a knife for 30 cents, and a book for 80: how many cents did both cost?

FORMULA. — 30 cents + 80 cents = $1.10.

ANALYSIS. — They both cost the sum of 30 cents and 80 cents, which is (110 cents) 1 dollar and 10 cents.

7. A geography costs 80 cents more than an arithmetic: the arithmetic costs 60 cents. What is the cost of the geography? How many dollars and cents?

8. A pair of skates is worth 90 cents, which is ten cents less than the cost of a new sled: what is the cost of the sled? How many dollars and cents?

9. Henry sold some chestnuts for 80 cents, and some filberts for 90 cents: how much did he receive? How many dollars and cents?

10. If a pig cost 3 dollars, and a pair of chickens 90 cents, what is the cost of both? How many cents?

If desired, additional examples may be formed by substituting the following numbers in place of the numbers in the corresponding examples:—

6. 60 and 75, 83 and 72, 96 and 84.
7. 76 and 50, 64 and 63, 87 and 96.
8. 87 and 20, 63 and 22, 47 and 16.
9. 40 and 64, 78 and 31, 95 and 18.
10. 4 and 36, 3 and 94, 5 and 50.

## WRITTEN PROBLEMS.

33. Henry had 15 marbles, James 36, and John 49: how many marbles had all?

**Formula.** — 15 marbles + 36 marbles + 49 marbles = 100 marbles.

**Analysis.** — All must have had the sum of 15 marbles, 36 marbles, and 49 marbles, which is 100 marbles.

PROCESS.
15 marbles.
36 "
49 "
—
100 "

34. James gave 125 cents for a geography, 98 cents for a reader, and 75 cents for an arithmetic: what did all cost him?

The answers should be given in dollars and cents.

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**Review.** — What is notation? (7.) Give the method of Roman notation. (7.) What is the difference between the Roman and Arabic notation? (8, 9.) What is the difference between the simple and the local value of a figure? (10, 11.)

35. Mary gave 137 cents for some calico, 39 cents for some braid, and 230 cents for some silk: how much did she pay for all?

36. A builder paid $610 for a city lot, and built upon it a house which cost him 2,085 dollars: how much did the house and lot cost him?

37. In January a laborer deposited in the savings bank 12 dollars, in February 30 dollars, in March $13, in April $21, in May 17 dollars, and in June $16: how much did he deposit in the six months?

38. A fruit-grower sold 123 barrels of apples to one man, 31 barrels of pears to another, 210 barrels of apples to another, and 30 barrels of pears to a fourth: how many barrels of fruit did he sell?

39. James paid 12 cents for a slate, 97 cents for a geography, and 113 cents for a reader: how much did he pay out?

40. Long Island is divided into three counties. King's County contains 72 square miles; Queen's County, 410 square miles; and Suffolk County, 1,200 square miles: how many square miles in the island?

## UNITED STATES CURRENCY.

Additional work in United States currency may be given by substituting the following numbers in place of those in the corresponding examples above, the analyses and methods of recitation to be the same as in simple numbers.

35. $2.14, 86 cents, and $1.89, $1.67, $1.24, and $2.14.
36. $412.30 and $2,564.89, $512.67 and $1,234.73.
37. $13.13, $28, $15.17, $25, $31.17, and $3.427.
*39.* *25 cents, $1.19,* and $1.125, 17 cents, 69 cents, and 99 cents.

41. The mason work of a new schoolhouse cost $1,220; the carpenter work, 1,037 dollars; and the painting and glazing, $427: how much did the schoolhouse cost?

42. A farmer sold his milk for one month for 387 dollars, the next month for $320, and the third month for $129: how much did he receive for the three months?

43. I had 103 sheep, my father gave me 444, I bought 321 more: how many sheep have I in all?

44. If I have 413 peach-trees in one orchard, 34 in another, and 212 in another, how many have I in all?

45. If I have $141 in one bank, and $636 in another, how many dollars have I in both?

46. If I sell 3 horses for $430, 9 cows for $334, and 55 sheep for $223, for how many dollars do I sell them all?

47. A drover sold 123 sheep, and kept 197: how many had he at first?

48. A farmer paid 5,750 dollars for his farm, and $375 for his tools: what was the cost of the whole?

## UNITED STATES CURRENCY.

Form examples by substituting the following numbers in the corresponding examples which precede.

41. $1,371.20, $903.64, and $572.893.
42. $267.30, $241.37, $176.03.
45. $243.80 and $593.80, $972.46 and $913.80.
46. $398.67, $256.40, and $189.36.
48. $2,673.20 and $974.30, $1,867.60 and $213.
49. $9,167.60 and $8,913.23, $6,934 and $941.30.

---

**Review.**—Repeat the numeration table. (12.) How do the places of a number increase or decrease? (13.) In which direction do they increase? What is a rule? (14.) Repeat the rule for notation. (15). Repeat the rule for numeration. (16.)

49. The purser of a steamer received $9,346 from the passengers for fare, and $31,426 for carrying goods: how many dollars did he receive in all?

50. A man travelled 476 miles by railroad, 390 miles by steamboat, and 120 by stage: how many miles in all did he travel?

51. In one school are 516 pupils; in another, 314; and in another, 215: how many pupils in all?

52. A farmer sold 3 loads of hay. There were 1,789 pounds in the first load, 2,976 pounds in the second, and in the third 2,967 pounds: how many pounds in the three loads?

53. If I buy 3 horses for $430, 8 cows for $334, and 55 sheep for $321: how much will they cost me?

54. In the year 1869, I bought 1,472 sheep; in the year 1870, 1,943 sheep; in the year 1871, 1,473 sheep: how many did I buy in all?

55. In 1873, I sold 1,870 sheep; in 1874, 1,674 sheep; in 1875, 1,864 sheep: how many did I sell in all?

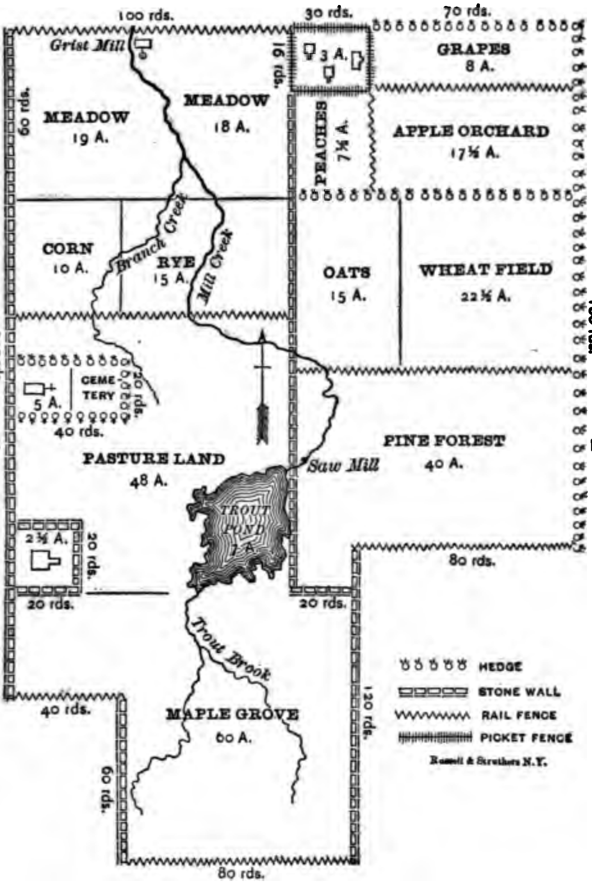
56. I bought 4,134 sheep, and sold 3,172; I then bought 2,134 more, and sold 1,413: how many sheep did I sell in all?

57. I bought 913 sheep, and sold 613; I then bought 217, and sold 188; after which I bought 691: how many sheep did I buy in all?

58. I had two fields of melons. In the first I had 347, from which 189 were stolen; in the second I had 439, from which 159 were stolen: how many melons were stolen from the fields?

59. A man by his will gave his wife $1,234, to his daughter 124 dollars, to his son $937: what was the amount of his bequests?

**60.** If I owe A 847 cents, B 5,674 cents, C 3,418 ents, how many cents do I owe them all?



The above is the map of the Maple Grove Farm, where I live. Can u tell me how many acres are contained in each of the following rtions:—

61. How many acres in the pine-forest and the maple-grove?

62. How many acres in the maple-grove and pasture-land?

63. How many acres in the pasture-land, including the church-grounds, schoolhouse-yard, and the trout-pond? How many acres of apples and peaches on the farm? How many acres of wheat, oats, corn, and rye?

64. How many acres of meadow on both sides of Mill Creek?

65. How many acres in the farm west of the stone wall extending through the centre of the farm, not including the maple-grove? How many acres in the farm east of the same wall?*

66. How many acres in the meadow, corn, and rye fields?

67. How many acres in the church and schoolhouse grounds?

68. What is the length of the east line? The length of the south line?

69. How many rods of hedge fence, not including the fence around the church-grounds? How many rods of hedge fence on the entire farm?

70. How many rods of wall on the west line? How many rods including the fence around the schoolhouse-grounds?

71. How many rods of rail fence on the south line" On both north and south line?

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* The teacher will find little difficulty in adding the fractions, if first illustrated by halves of an apple.

REVIEW.—What is addition? (17.) What is the sum of two or more numbers? (18.) Describe and give the use of the sign plus. (19.) Of the sign c. equality. (20.) Of the dollar sign. (21.)

2. How many rods of rail fence around the pine-st? How many around the maple-grove and pine-st?

3. I sold from my farm $124.30 worth of grapes, $434.72 worth of apples: how much did I receive oth?

4. I received $53.70 for 105 bushels of oats, and 20 for 112 bushels of corn: how much did I re-e? How many bushels did I sell?

5. I sold at one time 78 bushels of wheat for $93.20; nother time, 43 bushels for $75.23; and at another ushels for $125: how many bushels did I sell? w much did I receive for it?

6. I cut from the meadow west of Mill Creek 64 of hay; from the east of the creek, 51 tons: how y tons were cut from the entire meadow? The hay the west side was worth $150.50; on the east side, 0: how much was my hay worth?

7. The first week in April, there were sawed at the , 126,474 feet of maple lumber, and 623,478 feet of lumber: what number of feet were sawed during week?

8. There are 40 acres in the pine-forest: how many s in four such fields of equal size?

9. How many acres of land in the entire Maple ve Farm?

0. There are three barrels of eggs. The first con-s 675; the second, 1,276; the third, 1,575: how y eggs do these barrels contain?

1. A farmer sold three car-loads of potatoes. The contained 174 barrels; the second, 259 barrels; the 1, 199: *how many barrels* did the cars contain?

82. Gave 86 cents for a kite, 91 cents for a ball and bat, 138 cents for a wagon: what was given for all?

83. In an orchard there are 137 cherry-trees, 634 apple-trees, 72 pear-trees, 65 plum-trees: how many trees in the orchard?

84. The owner sold the cherries for $86, the apples for 387 dollars, the pears for $76, and the plums for 197 dollars : how many dollars did he receive for the fruit?

85. The hind-quarters of an ox weighed 375 pounds each; the two fore-quarters weighed 621 pounds; the hide and tallow, 112 pounds: what was the entire weight?

86. A merchant bought three pieces of cloth. The first measured 37 yards; the second, 41 yards; the third, 29 yards: how many yards in all the pieces?

87. He sold the first piece for 29 dollars, the second for 37 dollars, the third for $19: how many dollars did he receive for all the pieces?

88. Henry and James were playing marbles. Henry won in the first game 43, in the second, 57, and in the third, 156: how many did he have when he finished?

89. A father divided his money among his four sons, giving each 2,087 dollars: how much was he worth?

90. By selling a farm for $1,384, I lost $976: what was the cost of the farm?

91. Of three flocks of blackbirds, the first contained 3,467; the second, 4,171; the third, 3,741. A boy fired into the flocks, and frightened away 3,414: how many did the flocks contain before they were fired into?

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REVIEW. — What is an axiom? (22.) Repeat the one used in addition. Define a problem. (23.) What is the difference between a simple and a complex problem? (24, 25.)

92. A man deposited $341 in a certain bank on Monday, and drew out $314; on Tuesday he deposited $693, and drew out $574; on Wednesday he deposited $674, and drew out $513: how many dollars did he deposit in the three days?

93. How many dollars did he draw out in the three days?

94. If Henry can count 5,389 in one hour, how many can he count in nine hours?

95. I paid $1,655 for flour, $1,473 for pork, $1,342 for beef, $187 for sugar, $134 for cheese: how much did I pay out in all?

96. I paid away $396 in gold, $574 in silver, and $413 in currency, and had $413 left: how many dollars had I at first?

97. I bought 730 bushels of potatoes, of which I sold 396 bushels; I then bought 1,341 bushels, of which I sold 387 bushels: how many bushels did I buy in all?

98. My father gave me $347, and I earned $395; he then gave me an amount equal to the sum I earned; afterward I earned $345. How much did my father give me? How much did I earn? How much did I have in all?

99. My father had $2,380. He gave me $278, and my sister $467; he then bought me a farm for $1,008, and my sister a horse, for which he paid $347. How much cash did he give us? How much property?

100. With my money I bought a cow for $45, five

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**Review.**—Define an analytical step. (26.) Give the order of solving a problem. (27.) Give the analysis of an example in addition. (28.)

sheep for $33, and had $200 left: how much did my father give me?

101. By selling a farm for $6,479, the owner lost $2,734: what did it cost?

102. A man sold 327 bushels of corn for 296 dollars, 486 bushels for $329, and 320 for $211; this amount lacked $437 of what it cost him: what did it cost?

103. A lady went shopping, and bought some calico for 96 cents, some lace for 70 cents, some pins for 20 cents, and had 380 cents left: how much had she when she started?

104. Two boys started from the schoolhouse; one went north 76 rods, and the other went south 125 rods: how far were they then apart?

105. John and Henry started from the same place. John ran north 25 rods the first minute, and 27 rods the next minute; Henry ran south 23 rods the first minute, and 24 rods the next minute. How far did John run north? How far did Henry run south? How far were they apart at the end of the second minute?

106. John stood in the schoolhouse door, and shot an arrow from his cross-bow 20 yards up the road; a little boy from the schoolhouse brought it to him: how far did the boy run to bring it to John?

107. He then shot two arrows; one up the road 17 yards, and the other down the road 22 yards: how far must the little boy go to bring them to John?

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**Review.**—What is the difference between a unit and a number? (1, 2.) Between a concrete and an abstract number? (3, 5.) Between a concrete and a denominate number? (3, 4.) What is the use of arithmetic? (6.) What is the difference between the Arabic and the Roman notation? (8, 9.) Between the simple and the local value of a figure? (10, 11.)

108. Henry lends $913 to Thomas, $473 to Samuel, $576 to Theodore, and has $576 left: how many dollars had he at first?

109. A man was 37 years old when his son was born: how old will he be when his son is 59 years old?

110. Marcus had $347, and Oren had $576 more than Marcus: how many dollars had Oren?

111. Lillie had 17 roses; Laura has 11 more than Lillie, Charles has 18 dahlias more than Laura has roses, and Jennie has 14 dahlias more than Charles: how many dahlias has Jennie?

112. Henry's purse contains 329 cents; Edward's contains 43 more than Henry's, and Henry's contains as many cents as Sarah's, lacking 94 cents: how many cents does Sarah's purse contain?

113. Mary bought a pencil for which she gave 95 cents; Maude bought one for which she gave 13 cents more than Mary; and Maude's pencil cost as much as Jane's, lacking 23 cents: what was the cost of Jane's pencil?

### UNITED STATES CURRENCY.

114. Add $3.14, $2.17, $9.84, and $1.37.
115. Add $3.14, $0.39, $0.67, and $1.67.
116. Add $14.132, $31.47, $56.213, and $21.
117. Add $46.141, $2.13, $6.17, $0.234, and $31.64.
118. Add $21, $0.21; $0.021, $0.002, and $0.02.
119. Add $30, 50 cents, 40 dollars, and 6 mills.
120. Add 30 cents, 2 dollars and 2 cents, 2 cents and 2 mills.
121. Add 50 cents, 50 dollars, 3 dollars and 16 cents.

122. Add $1.14, 5 dollars and 5 cents, and 13 cents and 2 mills.

123. Add 49 cents, 4 cents and 9 mills, and 9 mills.

124. Add 6 dollars and 6 mills, 4 cents, 2 mills.

125. $43 + $25 + $1.30 + $21.303 + $0.004 = how much?

126. $37 + $41.30 + $13.13 + $2.46 + $0.13 + $1,918 = how much?

127. Add 5 dollars, 50 cents, 5 cents, and 5 mills.

128. Add 30 dollars, 30 dollars and 30 cents, 30 cents and 3 mills.

129. Add $50, 25 cents, 1 dime, 1 cent, and 1 mill.

130. Add 1 eagle, 1 dollar, 1 dime, 1 cent, and 1 mill.

131. What is the cost of the following items: 9 wagons for 312 dollars and 6 cents, 4 horses for 987 dollars, 1 wheelbarrow for 3 dollars, 9 ploughs for $35 and 4 cents, 2 pigs for $2.25.

132. What is the cost of the following items: 1 gallon of molasses for 38 cents, 1 pound of cheese for 13 cents, 3 pounds of butter for 1 dollar and 30 cents, 41 pounds of coffee for 25 dollars, 19 gallons of vinegar for 4 dollars and 30 cents.

133. A farmer bought a cow for which he paid 30 dollars, a horse for which he paid $104.31, a wagon for which he paid 85 dollars: how much did the whole cost?

134. Mr. D. Spring sold farmer Mahan 1 caddy of tea for $7.38, 9 yards of cloth for $27.16, a plough for

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REVIEW. — Give the rule for notation. (15.) For numeration. (16.) Give rule for addition. (28.) How is addition proved? Place on the blackboard, and show the use of, the sign of addition. (19.) The sign of equality. (20.) The dollar sign. (21).

9 dollars and 16 cents, a harness for 21 dollars, 16 cents and 5 mills: what was the amount of the bill?

135. A man is indebted to A $3.41, to B $6.14, to C $21.39, and to D 13 dollars and 5 cents: how much does he owe them?

136. Paid $4.37 for a hat, $13.13 for a coat, $5.62 for pair of boots, and 98 cents for an umbrella: what was paid for the whole?

137. One day a toll-gate keeper received $17.16; the next two days, $13.27 each: how much did he receive for the three days?

138. A furniture-dealer sold a washstand for $6.50, a bureau for 11 dollars and 13 cents, and 3 rocking-chairs for $4.37 each: for how much did he sell them all?

139. A man received $39.40. He paid $1.342 for flour, $3.171 for sugar, $0.124 for meal, and $0.137 for salt: how much did he pay out in all?

140. A man bought a village lot for 325 dollars; he sold it so as to gain $123.416: for how much did he sell it?

141. I bought a hat for $3.28, a coat for 11 dollars, and a pair of boots for 13 dollars: how much did I pay for them?

142. I bought 340 bushels of potatoes for $75, 68 bushels of corn for $39.20, and 112 bushels of wheat for $137.243: how much did all cost? How many bushels did I buy?

143. Bought 16 lbs. of sugar for 187 cents, 17 lbs. of coffee for 273 cents, and 34 lbs. of rice for 357 cents: how many cents did I give for all? How many dollars and cents? How many pounds did I buy?

144. A boy bought 13 oranges for 67 cents, 24 lemons

for 87 cents, 19 apples for 16 cents: how many cents did all cost? What number of oranges, lemons, and apples had he, counted together? How many oranges and lemons, counted together?

145. Thomas W. Scudder bought a city lot for $125. He built a house worth $538.76, he spent $38.26 for fencing, he then sold it for $189.72 more than the whole cost: for how much did he sell it?

146. 43 + 971 + 1,893 + 12 + 4 + 8 = how many?
147. $27 + $36.48 + $3.946 + $4.21 = how much?
148. $0.27 + $3.648 + $39.46 + $6.421 = how much?
149. $0.027 + $0.364 + $3946 + $0.042 = how much?
150. $0.002 + $3648 + $3946 + $421 = how much?

## ANALYSIS OF SECTION II.

### Addition.

1. **Illustration.**
2. **Definition.**
3. **Signs.**
4. **Axiom.**
5. **Problem.**
    - (*a.*) ***Abstract Problem.***
        - **Process.**
        - **Analysis.**
        - **Rule.**
    - (*b.*) ***Concrete Problem.***
        - **Simple.**
        - **Complex.**
        - **Formula.**
        - **Analytical Step.**
        - **Analysis.**

## SECTION III.

### SUBTRACTION.

In this picture the boys have a skating-party. There were at first 15 boys; but 6 of them, becoming tired of the fun, have started for home: how many remain?

Of those who remain, 2 have no skates: how many have skates?

Of those who have skates, 1 is just putting on his skates: how many have their skates on?

Of the boys who remain, all but 2 are standing: how many are on their feet?

Six of the boys started for home, leaving 9 on the ice: how many more stay than go?

This taking of one number from another, or taking away a part to find how many are left, is called *subtraction*, and is just the opposite of putting things together, or addition.

***31. Subtraction*** is the process of finding the difference between two numbers.

***32. The Subtrahend*** is the number or part taken away.

***33. The Minuend*** is the number from which the *subtrahend* is taken.

***34. The Remainder, or Difference,*** is the part of the minuend left after the subtrahend is taken away.

***35. The Sign of Subtraction*** is a short horizontal line; thus, —. It is called *minus*, meaning *less*; and, when placed between two numbers, it denotes that the second is to be taken from the first; as, $6 - 4 = 2$.

***36.*** The axioms used in subtraction are, —

1. *Things can be subtracted from like things only.*

2. *If two numbers are equally increased or diminished, their difference will not be changed.*

***37.*** ANALYSIS OF SUBTRACTION.

1. PROBLEM. — Find the difference between 3027 and 1538.

PROCESS.

| *Subtraction.* | | *Proof.* | |
|---|---|---|---|
| 3027 | minuend. | 1538 | subtrahend. |
| 1538 | subtrahend. | 1489 | difference. |
| 1489 | difference | 3027 | sum = minuend. |

**2.** ANALYSIS. — (1.) For convenience, write the subtrahend under the minuend, placing units under units, tens under tens, &c.

(2.) For convenience, begin to subtract at units; thus,

(3.) 8 units cannot be taken from 7 units; but one of the 2 tens can be changed to units (1 ten = 10 units), and 10 units and 7 units are 17 units. 8 units from 17 units leave 9 units, to be written in the place of units.

(4.) As one of the 2 tens has been used, only 1 ten remains, from *which 8 tens cannot* be taken. Take 1 from the next significant figure of the minuend (which, in this case, is 3 thousands), 1 thousand = 10 hundreds; of the 10 hundreds take 1 hundred = 10 tens: 10 tens and 1 ten are 11 tens; 3 tens from 11 tens leave 8 tens, to be written in the place of tens.

(5.) 5 hundreds from 9 hundreds leave 4 hundreds.

(6.) 1 thousand from 2 thousands leaves 1 thousand.

**3.** Conclusion. — Therefore, 1538 subtracted from 3027 leaves 1489 as a remainder.

**4.** Rule. — I. *Write the numbers so that the units of the subtrahend shall stand under the units of the minuend, the tens under the tens, &c.*

II. *Begin at the right hand, and, if it can be done, subtract each figure of the subtrahend from the figure above it, and write the result underneath.*

III. *If any figure of the subtrahend is greater than the corresponding figure of the minuend, add ten to the figure of the minuend before subtracting. In this case, make the next left-hand figure in the minuend* ONE *less, (or the next left-hand figure in the subtrahend* ONE *greater.)*

**5.** Proof. *Add the difference to the subtrahend; and, if the sum equals the minuend, the work is correct.*

***38.*** Some prefer the following analysis: —

$$\begin{array}{r}3897\\2938\\\hline 959\end{array}$$

Analysis. — (1.) As 8 units cannot be taken from 7 units, add 10 units to the 7, making 17 units: 8 units from 17 units leave 9 units, to be written in the place of units.

(2.) Since 10 units have been added to the minuend, add their equal, 1 ten (Axiom 2, **36**), to the subtrahend. 1 ten and 3 tens are 4 tens: 4 tens from 9 tens leave 5 tens, to be written in the place of tens.

(3.) As 9 hundreds cannot be taken from 8 hundreds, add 10 hundreds to the 8 hundreds, making 18 hundreds: 9 hundreds from 18 hundreds leave 9 hundreds, to be written in the place of hundreds.

(4.) Since 10 hundreds have been added to the minuend, add their equal, 1 thousand, to the subtrahend: 2 thousands and 1 thousand are 3 thousand; 3 thousands from 3 thousands leave nothing.

## DRILL CARD EXERCISES.

TABLE I., *Thousands.*
Exercise I. — 579438 minuend.
II. — 885973 "
III. — 868943 "
IV. — 998998 "
V. — 687789 "
VI. — 889787 "

TABLE II., *Thousands.*
Exer. VII. — 908191 minuend.
VIII. — 809081 "
IX. — 908080 "
X. — 911111 "
XI. — 922222 "
XII. — 933333 "

TABLE II., *Thousands.*
Exer. XIII. — 944444 minuend.
XIV. — 955555 "
XV. — 966666 "
XVI. — 977777 "
XVII. — 988888 "
XVIII. — 999999 "

TABLE II., *Thousands.*
Exer. XIX. — 1000000 minuend.
XX. — 1020401 "
XXI. — 2413621 "
XXII. — 1010201 "

TABLE II., *Billions.*
XXIII. — 9381908191 "
XXIV. — 9503403916 "

TABLE II., *Billions.*
Exer. XXV. — Find difference of A and B, B and C, C and D, &c.

TABLE I., *Thousands.*
Exer. XXVI. — Find difference of A and B, B and C, C and D, &c.

### SUBTRACTION. — UNITED STATES CURRENCY. — TABLE III.

Exer. I. — $973.864 minuend.
II. — $734.001 "
III. — $846 minuend.
IV. — $800 "
V. — Find difference of A and B, B and C, C and D, &c.
VI. — Find difference of A and C, B and D, C and E, &c.

## DIRECTIONS FOR USE OF DRILL CARDS.

EXERCISE 1. — FIRST SERIES OF TWENTY PROBLEMS (TABLE I., UNITS' AND THOUSANDS' PERIOD). Copy and subtract the number opposite each letter from 579438; thus,

| Example (1). | Example (2). | Example (3). |
|---|---|---|
| 579438 | 579438 | 579438 |
| A. 242022 | B. 442202 | C. 442200 |
| Ans. 337416 | Ans. 137236 | Ans. 137238 |

N.B. — See Exercises above for *minuends.* For answers, see *Key.*

# FELTER'S
# ARITHMETICAL DRILL CARDS.

| TABLE I. | | TABLE II. | | TABLE III. |
|---|---|---|---|---|
| Thou. Units. | | B. Mill. Thou. Units. | | Dollars. Cts. m. |
| 579,438 | | 9,381,908,191 | | 5,319.43 4 |
| 242,022 | A | 8,448,646,446 | A | 3,402 cts. |
| 442,202 | B | 6,688,286,466 | B | 1,811 m. |
| 442,200 | C | 868,424,246 | C | 60 dol. |
| 240,220 | D | 606,486,644 | D | 403 cts. |
| 402,222 | E | 428,664,246 | E | 10 dol. |
| 331,110 | F | 75,351,535 | F | 121 m. |
| 303,211 | G | 73,735,353 | G | 8 cts. |
| 311,121 | H | 53,533,353 | H | 195 dol. |
| 342,221 | I | 53,131,533 | I | 2,000 m. |
| 221,121 | K | 33,713,555 | K | 715 dol. |
| 550,112 | L | 29,036,632 | L | 86 cts. |
| 404,201 | M | 19,241,747 | M | 7 dol. |
| 040,121 | N | 9,866,474 | N | 9 m. |
| 443,121 | O | 9,673,342 | O | 8,189 cts. |
| 552,232 | P | 9,477,773 | P | 908 m. |
| 243,110 | Q | 8,347,544 | Q | 4,003 cts. |
| 152,231 | R | 887,744 | R | 500 dol. |
| 435,112 | S | 347,654 | S | 6,001 cts. |
| 543,121 | T | 281,263 | T | 2,100 m. |
| 402,301 | U | 187,602 | U | 5,000 cts. |

**N.B.—A Key containing the answers to the problems on the above card may be obtained by *mail*, by addressing the publishers.**

### ORAL PROBLEMS CONTAINING BUT ONE ELEMENTARY QUESTION.

1. Henry had 50 apples, he gave 30 of them to Charles: how many had he left?

**FORMULA.** — 50 apples − 30 apples = 20 apples.

**ANALYSIS.** — Henry must have had left as many apples as the difference between 50 apples and 30 apples, or 20 apples.

2. James is 80 years old, his son is 50: how much older is James than his son?

3. One bag contains 80 pounds of wheat, another bag contains 60 pounds: how many more pounds does one bag contain than the other?

4. There are 109 scholars in a school; 81 of them are girls: how many boys in the school?

5. A farm is worth $896, and a house is worth $194: how much more is the farm worth than the house?

6. A man travelled 890 miles; in returning, he travelled the first day 60 miles: how far had he yet to travel?

7. Mary went shopping with 120 cents, and, when she returned, she had only 80 cents left; how many cents had she spent?

8. A cistern which holds 110 gallons was full in the morning, but at night there were but 30 gallons remaining: how many gallons had leaked out?

Substitute the following numbers in the above examples: —

Example 2. 93 and 42, 76 and 54, 84 and 65.
" 3. 74 and 33, 120 and 60, 121 and 75.
" 4. 108 and 76, 97 and 55, 101 and 47.
" 5. $483 and $62, $372 and $49, $100 and $76.
" 6. 89 and 43, 78 and 69, 146 and 97.
" 7. 133 and 28, 67 and 34, 158 and 73.
" 8. 150 and 73, 100 and 84, 150 and 34.

### UNITED STATES CURRENCY.

1. Charles had $2; he gave 97 cents for a knife: how many cents had he left?

**Analysis.** — $2 equal 200 cents: he must have had left the difference between 200 cents and 97 cents, or 103 cents = $1.03.

2. Susan had 200 cents; she spent 46 cents: how many cents had she remaining? How many dollars and cents?

3. A boy had 100 dollars; he gave away 97 cents: how much money had he left? How many dollars and cents?

4. A man had 10 dollars; he spent 6 cents: how much had he left?

5. A little girl had $1.50; she bought a book for 80 cents: how much money had she left?

6. A woman went shopping with $3.30 in her purse; on her return she had only 80 cents left: how much did she spend?

7. A man went to market with a 5-dollar bill; on his return he had a 2-dollar bill, and 54 cents in change: how much had he spent?

8. A boy bought a sled for $1.45; he sold it so as to lose 86 cents: for how much did he sell it?

Substitute the following numbers in the above examples:—

Example 1. $3 and 55, $2 and 37, $2.50 and 99.
" 2. 108 and 54, 79 and 39, 185 and 77.
" 3. 100 and 67, 100 and 100, 10 and 10.
" 4. 40 and 40, 20 and 20, 5 and 5.
" 5. $2.20 and 30, $6 and 60, $1.10 and 39.
" 6. 4 dollars and 40 cents, $3.56 and 97 cents.
" 7. 10-dollar bill and 3-dollar bill, 60 cents.
" 8. $2.30 and 75, $1.67 and 58, $3.80 and 94.

## WRITTEN EXERCISES.

1. A man who had a farm of 623 acres gave his son 186 acres: how much land had he left?

2. In a certain school-district there are 389 children of school age; of these only 297 attend school: how many do not attend school?

3. A grocer receiving an order for 386 barrels of beef has only 275 barrels on hand: how many more barrels will he require to fill the order?

4. In a certain town, 435 men were drafted for the army, but 167 were rejected by the examining surgeon: how many passed examination?

5. A market-gardener in one year received $3,730 for fruits and vegetables, and his expenses were 2,134 dollars: how much were his profits?

6. I have $374 in the savings bank: if I draw out 89 dollars, how much money shall I have left on deposit?

7. A merchant sold for $1,034 goods which cost him 989 dollars: how much did he gain by the transaction?

8. Henry has 197 marbles less than John, and John has 200: how many has Henry?

9. The sum of two numbers is 25, one of the numbers is 17: what is the other?

10. The difference of two numbers is 34, the greater is 97: what is the less?

11. A farmer has 7,012 sheep, and as many lambs, lacking 749: how many lambs has he?

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**Review.**—What is a unit? (1.) Name a number. (2.) When is a concrete number called denominate? (4.) What is the difference between a concrete and an abstract number? (3, 5.) Of what does arithmetic treat? (6.)

12. Joseph has 513 cents, and Jerrie as many, lacking 213: how many cents has Jerrie?

13. The sum of two numbers is 13,416, and one of the numbers is 4,137: what is the other?

14. The sum of two numbers is 5,416, the greater of the numbers is 4,134: what is the less?

15. The sum of two numbers is 13,416, the less is 3,147: what is the greater?

16. Our national independence was declared in the year 1776: how many years from that time to the close of the war in 1815?

17. The war of the Revolution broke out in the year 1775: how many years to the breaking-out of the war with Mexico, in 1846?

18. America was discovered in the year 1492: how many years from that period to the landing of the pilgrims, in 1620?

19. The mariner's compass was invented in Naples, 190 years before America was discovered, in 1492: in what year was the compass invented?

20. Bonaparte was declared emperor in the year 1804, at which time he was 35 years of age: in what year was he born?

21. Sir Isaac Newton was born in 1642, and died in 1727: how old was he when he died?

22. Gunpowder was invented by Schwartz, in the year 1330: how long was this before the invention of printing, which took place in the year 1441?

23. What number is that which, taken from 2579, will leave 1397?

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**REVIEW.—How many methods of notation in common use? (7.) Name the letters used in Roman notation, and give the value of each. (8.) When is a figure said to have a simple value? (10.) When a local value? (11.) Recite the numeration-table by periods. (12.) In *what ratio* do the orders in a number increase? (13.)**

24. If the remainder is 15, the subtrahend 29, what is the minuend?

25. Supposing a man to have been 80 years old in 1854, in what year was he born?

26. There are two numbers whose difference is 876, the greater number is 1673: what is the less?

27. What number is that, which, taken from 3,795, will leave 473?

28. How long time will elapse between the years 1874 and 1892?

29. What number is that which, if added to 789, will make 1637.

30. One of two numbers is 387, the other is 673: what is their sum?

31. One of two numbers is 387, the other is 673: what is their difference?

32. The minuend is 3,917, the subtrahend is 413: what is the remainder?

33. The subtrahend is 4,167, the remainder is 413: what is the minuend?

34. 38 thousand 24 *less* 3 thousand 40 are how many?
35. 3 thousand 47 *less* 583 equals what?
36. 46 thousand and 6 *less* 3,413 equals what?
37. 25 thousand *less* 3 thousand and 3 = what?
38. 163 thousand *less* 46 thousand 213 = what?
39. 327 thousand 147 *less* 25 thousand 25 = what?
40. 439 thousand less 4,712 equals what?

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**REVIEW.—What is a rule? (14.) Recite the rule for notation. (15.) For numeration. (16.) Define addition. (17.) What is meant by the sum of two or more numbers. (18.) Illustrate the sign of addition. (19.) Of equality. (20) Illustrate the use of the dollar-sign. (21.) What is an axiom? (22.) Give the axioms used in addition; in subtraction. (22, 36.) Define a problem. (23.) A simple problem. (24.) A complex problem. (25.)**

### UNITED STATES CURRENCY.

41. From $896 take $41.30.
42. From $100 take $0.57.
43. From $389 take $0.643.
44. From $52.60 take $1.234.
45. From $327 take $21.34.
46. From $345 take $62.13.
47. From 95 dollars take 95 cents.
48. From 100 dollars take 100 cents.
49. From 10 dollars take 10 cents.
50. From 10 cents take 10 mills.
51. From 1 cent take 1 mill.
52. From $3.20 take 1 cent and 1 mill.
53. From $2.413 take 20 cents and 2 mills.
54. From $6.134 take $3.37.
55. From 58 dollars take $1.334.
56. From 97 dollars take $21.213.
57. From 12 dollars take $3.713.

58. A man purchased a yoke of oxen for $78, a cow for 26 dollars and 50 cents: how much more did he give for the oxen than for the cow?

59. A man buys a horse for $97.50, and gives a hundred-dollar-bill: how much change should he receive?

60. How much must be added to $60.039 to make the sum 1 thousand and 5 dollars?

61. How much must be added to 38 dollars 79 cents to make the sum 100 dollars and 3 mills?

62. A man bought a pair of oxen for $100, and sold them for $75.375: did he make, or lose? and how much?

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**Review.**—Define an analytical step. (26.) Give the analysis of an example in addition. (28.) How is addition proved? What is the answer called? (18.)

63. A man starts on a journey with a hundred dollars; he spends 87 dollars, 87 cents and 5 mills: how much has he left?

64. How much must be added to $40.173 to make 100 dollars?

### PROBLEMS CONTAINING NOT MORE THAN TWO ELEMENTARY QUESTIONS.

PROBLEM. — Sold a cow for $20, a calf for $12; received in pay $19; how much was still due?

FORMULA. — 1. $20 + $12 = $32.
2. $32 — $19 = $13. Ans.

ANALYTICAL STEPS. — 1. *Find the amount for which the cow and calf were sold.*
2. *Find the amount due.*

The analytical steps should be given in the analysis of every problem.

65. A farmer bought a horse for $87, and a wagon for $50; he exchanged both for two yoke of oxen worth $200: how much did he gain?

66. A man bought a yoke of oxen for $97; their keeping cost $13; he sold them for $125: did he gain, or lose? and how much?

67. A box of raisins was bought for $7, a firkin of butter for $28; both were sold for $40: how much was gained?

68. A jeweller bought a watch for $86, a chain for $36; he sold both for $175: how much did he gain by the bargain?

69. Henry and George started from the same place, and travelled in the same direction. Henry travelled 160 miles, and George 78: how far apart were they?

70. Two boats sail from the same place, and in the

same direction. One sails at the rate of 190 miles a day; the other, at the rate of 97 miles: how far apart will they be at the end of the day?

71. A father is 90 years of age, and is 37 years older than his eldest son: how old is his son?

72. James had 40 cents; he paid 12 cents for a whistle, and 25 cents for a knife: how many had he left?

73. Arthur had 97 nuts in a basket; he put 12 of them in one pocket, and 13 in the other: how many were left in the basket?

74. A schoolboy paid 56 cents for an atlas, and 30 cents for an arithmetic; he gave the shopkeeper a dollar-bill: how much change should he receive?

75. A drover bought 190 sheep of three farmers; he bought 27 of the first, 28 of the second: how many did he buy of the third?

76. Lillie had 9 oranges, her mother gave her 11 more, and her father enough to make 40: how many did her father give her?

77. A man had 88 dollars, and gave 28 dollars for a cow, and 14 dollars for a calf: how much money had he left?

78. A man bought one barrel of pork for $18, and another for $17; he sold both for $46: how much did he gain?

79. George spent 19 cents for candy, and 17 cents for fruit: how much more will he have to spend to make 50 cents.

80. Two men bought a horse; the one paying 110 dollars, and the other 25 dollars less: how much did both pay?

81. A man sold a firkin of butter for 17 dollars, a cheese for $19; he received in payment 4 barrels of flour worth $36: how much remains due?

82. Solomon gave 13 cents for some oranges, 27 cents for some cake, and had 47 cents remaining: how much had he at first?

83. Stoddard at a game of marbles won 13, and then lost 25, after which he had 17 remaining: how many had he at first?

84. A farmer bought a horse for $67, and exchanged it for a yoke of oxen, which he sold for $87: how much did he gain by the transaction?

85. A box of raisins was bought for 5 dollars, a firkin of butter for $18; both were sold for 20 dollars: how much was lost?

86. A drover bought some sheep for $40, some cattle for $130; he sold them all for $200: what was his gain?

87. A jeweller bought a watch for $83, and a chain for 15 dollars; he sold them both for $105: what was his gain?

88. A merchant bought a hogshead of molasses for $53; he paid $5 cartage: for how much must he sell it to gain $23?

89. I bought a horse for 90 dollars, a carriage for $75; I sold them both at a loss of $37: how much did I receive for them?

***39. A Parenthesis*** () denotes that all the numbers enclosed are to be taken as a whole, and that the same process is to apply to them all. Thus $14 - (3 + 4)$ signifies that the sum of 3 and 4 is to be taken from 14: $(13 - 4) - (5 + 2)$ signifies that the sum of 5 and 2 is to be taken from the difference of 13 and 4.

## COMPLEX PROBLEMS REQUIRING NOT MORE THAN TWO ANALYTICAL STEPS.

**Problem.**—Of a salary of \$2,447, a man spent \$650.40 for rent, and \$834.17 for family expenses: how much remained?

PROCESSES.

(*a.*) \$2,447 — \$650.40 — \$834.17 = \$962.43.
(*b.*) \$2,447 — (\$650.40 + \$834.17) = \$962.43.

**Anal. Steps.**—(*a.*) 1. *Find what was left after paying rent; then,*
2. *Find what was left after paying family expenses.*

(*b.*) 1. *Find what was paid for rent and expenses.*
2. *Find what was left of the salary.*

**Note for Teacher.**—The attention of the pupil should be called to the two methods of solving problems of this class.

90. A boy had 378 cents; he gave 127 cents for a book, and 40 cents for a slate: how many cents had he remaining?

91. A drover bought a drove of cattle for \$6,843; he paid at one time \$462, at another time \$413: how much did he still owe?

92. A boy had 100 cents; he bought an arithmetic costing 63 cents, and a slate which cost 30 cents, and gave the remainder for a sponge: what did the sponge cost him?

93. I lent a man \$13.74; he paid me at one time \$3.87, at another \$4.76: how much does he still owe me?

94. A merchant bought 946 yards of calico; he sold

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**Review.**—Define subtraction. (31.) Subtrahend. (32.) Minuend. (33.) Remainder. (34.) Difference. (34.). Describe the sign of subtraction. (35.)

137 yards to one person, and 673 to another: how many yards had he remaining?

95. A boy gave some nuts to his little brother and two sisters. To his brother he gave 125; to his elder sister, 13 less than to his brother; and to his younger, 24 less than to the elder: how many nuts did the younger receive?

96. A man had 4,832 sheep; on Monday he sold 1,199, on Tuesday he sold 2,259 more: how many had he then remaining?

97. A man had $2,986 in cash. He bought a building-lot, for which he paid $341; a house, for which he paid $1,418: how much cash had he left?

98. A boy went to a store with a ten-dollar bill; he bought an algebra, a history, and a chemistry for $3.83; on his way home he met a boy with a little carriage; he immediately gave for it all the money he had left, excepting $2: how much did the carriage cost him?

99. A gentleman, having $7,564, deposited $4,319 in a bank, and spent $86 of the remainder: how much then remained?

100. A boy had 374 cents; he gave 37 cents for a dove, 90 cents for a rabbit, and 114 cents for a pair of pigeons: how many cents had he left?

FORMULA.—$374-(37+90+114)=\$1.33$. Ans.

ANAL. STEPS.—1. *Find the total amount expended.*
2. *Find the amount remaining.*

101. Suppose a gentleman has an income of $1,597: if he pays $150 for taxes, and for his other expenses $873, how much of his income does he save?

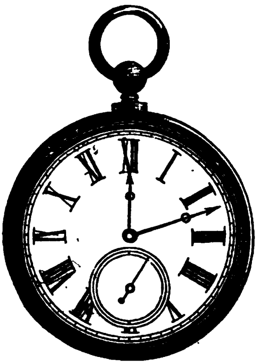
102. A farmer, having 554 sheep, sold 168, and then bought 896 more: how many had he then?

103. In a certain army there were 3,512 men; in a battle, 513 of them were killed, 627 were wounded, and 97 of them deserted: how many remained?

104. A farmer, having 625 bushels of grain, sold to A 97 bushels, to B 127, to C 197, and gave 110 bushels to the poor: how many bushels had he remaining?

105. A man travelled by railroad 372 miles, and started to return on foot. On the first day, he travelled 35 miles; on the second day, 43; on the third, 44; on the fourth day, he took the cars and came home: how many miles did he travel the last day?

106. Since there are 60 minutes in an hour, how many minutes are there between 2 o'clock and 4 o'clock?



107. How many minutes between 11 minutes before 8 o'clock and 10 o'clock?

108. How many minutes between 25 minutes before 8 o'clock and 13 minutes before 12 o'clock?

109. How many minutes between 28 minutes after 12, M., and 26 minutes after 1, P.M.?

110. How many minutes between 47 minutes before 11, A.M., and 32 minutes after 2 o'clock, P.M.?

111. How many minutes between 39 minutes after 4 o'clock, A.M., and 42 minutes before 4 o'clock, P.M.?

112. How long a time will elapse between 3 minutes before 9 o'clock, A.M., and 3 minutes after 3 o'clock, P.M.?

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**REVIEW.**—Name the axioms used in subtraction. (36.) Analyze by the first method an example in subtraction. (37.) By the second method. (38.) Illustrate the use of the parenthesis. (39.)

### UNITED STATES CURRENCY.

113. If a man's income is $175 per month, and he pays $25.38 for rent, $44 for provisions, and $18.25 for other expenses, how much will he have left?

114. A man gave his note for $32.45; he paid at one time $7.80, and at another time $4.84: how much remained unpaid?

115. A man paid $140.36 for a horse, and 165 dollars for a carriage; he afterwards sold them for $300: did he gain, or lose, and how much?

116. A flour-merchant, having 700 barrels of flour worth $3,500, sold 278 barrels to one man for $1,078, and 142 barrels to another for $542: how many barrels had he left? How much had the remainder cost him?

117. Three men bought a farm for 9,840 dollars. The first paid $2,672.31; the second paid $3,089.40; and the third, the remainder: how much did the third pay?

118. A man bought a house for $1,500, and, having expended $315.46 for repairs, sold it for $2,000: how much was his gain?

119. A man deposited 5,000 dollars in a bank. He drew at one time, $23.50; at another, $12.40; and, at another, 737 dollars: how much had he remaining in the bank?

120. A farmer bought a piece of land for $122.50; he bought a second piece for $153.42, lacking $63.40: how much more did he pay for the first than for the second?

121. A drover bought a drove of cattle of one farmer for $861.11, a drove of a second farmer for $987.63, lacking $189.90: how much more did he pay for the first than for the second?

122. A boy had 33 cents given him to buy a slate and sponge. He paid 9 cents for his sponge; and his slate cost 30 cents: how much did he owe for his slate?

123. If it costs a man $2 per day to support his family, and he receives $5 per day for his work, how much can he save in one week?

124. A merchant bought 500 barrels of flour for $3,500; he sold 250 barrels for $2,000: how many barrels had he on hand?

125. Since there are 30 days in June, how many days will elapse between the 7th of June and the 23d of the same month?

126. Since there are 31 days in May, how many days will elapse between the 9th of May and the 28th of the same month?

127. Having $4,937.60 in my possession, I wish to know how much I must add to this amount to be able to purchase a farm valued at $6,741, and yet save $576.34 for other purposes.

128. A boy bought a sled for $1.48, and gave 87 cents for having it repaired; he sold it for $1.49: did he gain, or lose, by the transaction? and how much?

129. One man travels 67 miles in a day, another man follows at the rate of 42 miles in a day: if they both start from the same place at the same time, how far apart will they be at the close of the day?

130. Bought 14 yards of cloth for $13.27, 5 yards of calico for $1.36, 12 pounds of sugar for $1.972, 4 pounds of tea for $1.75: what is the amount of the bill? Paid 8 dollars, 75 cents, and 3 mills: how much is still owed?

131. Bought 5 yards of ribbon for 79 cents, 6 yards of tape for 13 cents, 41 yards of calico for 5 dollars,

23 yards of cloth for 18 dollars and 23 cents: what is the amount of the bill? Paid two 20-dollar bills: how much change should be received in return?

132. Sold 5 bushels of corn for 3 dollars 87 cents, 6 bushels of rye for 7 dollars 14 cents, 9 bushels of potatoes for 3 dollars 17 cents, 11 bushels oats for 5 dollars 5 cents, 37 bushels of wheat for 35 dollars: what should be received on the above bill? How much does it lack of 100 dollars?

## ANALYSIS OF SECTION III.

### Subtraction.

1 ILLUSTRATION.
2. DEFINITIONS.
3. SIGN.
4. AXIOMS.
5. ABSTRACT PROBLEMS.
    (a.) *Process.*
    (b.) *Analysis.*
    (c.) *Rule.*
6. CONCRETE PROBLEMS.
    (a.) *Formula.*
    (b.) *Analytical Steps.*
    (c.) *Analysis.*

## TEST QUESTIONS FOR EXAMINATION.

**1.** What is a unit? Name a unit. Is a book a unit? A horse? A house?

**2.** What is a number? Write a number on the blackboard. Show a number of books. Can there be numbers of different kinds? Give an example.

**3.** What is a concrete number? Write a concrete number on the blackboard. Name five concrete numbers of different kinds. Name five concrete numbers of the same kind.

**4.** When are concrete numbers called denominate? Is 5 ounces a denominate number? 3 horses? 5 dimes? 6 cows? 7 yards?

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NOTE. — It is important that one day in the week should be spent in a review of the principles over which the class have passed during the other four days. The answers to these questions may be written on slate or paper in a topical form, to be read as a class-exercise, subject to criticism by the class or teacher.

**5.** What is an abstract number? Name an abstract number. What is the difference between a concrete and an abstract number? Name three abstract numbers. Can one abstract number be greater than another? Write an abstract number on the blackboard. Write a concrete number on the blackboard. What is the difference between a number and a unit? Is a unit always a number? Is a number always a unit? Give examples on the blackboard.

**6.** Of what does arithmetic treat?

**7.** What is notation? Illustrate the meaning of notation. Give an example of each method. Which is the most convenient? Why? Which is chiefly used in arithmetic? Why?

**8.** What is the Roman notation? Why called Roman? What characters are used in the Roman notation? How many? What is the value of each? Which has the greatest value? Which the least? What is the effect of repeating a letter? What is the effect of placing a letter at the left of one of greater value? What of placing a letter at the right of one of greater value? When a dash is placed over a letter, or a combination of letters, how is the value affected? Write forty-eight on the blackboard in Roman characters. Change it to forty-eight thousand. To forty thousand and eight. Ten thousand fifty-eight. Forty-five thousand and three. Forty-seven thousand and one.

**9.** What is the Arabic notation? Why so called? What characters are used in the Arabic notation? How many? By what name are nine of them sometimes called? How many single things does each represent? Which character has no value? How many names has it? Which is the most common? What do figures represent? Is a figure, or a combination of figures, a number? Why? What is the difference between a picture and a thing? When you look at figures, what do you think about?

**10.** What is the simple value of a figure?

**11.** What is the local value? Has every digit a simple value? When is the simple value the same as the local value? When are they different? In the number 395, how many values has the figure five? How many values has the figure three. Does its local value change its simple value? Does it increase it, or decrease it?

**12.** What is numeration? What is a period? How many figures in a period? What is the name of the first period? Of the second period? Of the third period? How many places in a period? Name the periods from right to left. From left to right. Write on the blackboard a number containing four periods. Read it. At which hand do you commence to write a number? Repeat the numeration-table by periods, from right to left. From left to right. What should be done with a vacant place in the period of a number? Write a number on the blackboard, each period of which has a vacant place. What should be done with vacant periods of a number? Write a number on the blackboard having a vacant period. What should be done with the vacant left-hand places of the left-hand period? Tell how a number is written. Tell how a number is read. How are periods usually separated in writing numbers?

**13.** A digit in units' place has how many times the value of the same digit in *tenths'* place? A digit in tens' place has how many times the value of the same digit in units' place?

**14.** What is a rule? Which is the more important, — a rule, or an analysis? Can a rule be made from an analysis? Why are rules used in arithmetic? [Let the teacher explain to the pupil that rules are deduced from analyses, and that an analysis is far more important than a rule.]

**15.** Recite the rule for notation.

**16.** Recite the rule for numeration.

**17.** What is addition? What is the first process in arithmetic? *Notation*. What is the next? *Numeration*. What is the next? *Addition*.

**18.** What is the sum of two or more numbers? When two numbers are added together, what is the result called? What is the sum of 6 and 8? What is the 14 called? Of what numbers is the 14 composed? Can 14 be separated into more than two parts? Which is the greater, — 14, or the sum of all the parts of 14? What is putting-together two or more numbers into one sum called?

**19.** What is the sign of addition? What is a sign? Illustrate the use of the sign of addition.

**20.** What is the sign of equality? Illustrate its use.

**21.** What is the dollar-sign? Illustrate its use.

**22.** What is an axiom? What is meant by "a self-evident truth"? Can an axiom be proved? Can it be illustrated? Do axioms need to be proved? Illustrate at the blackboard the axiom used in addition.

**23.** What is a problem? What is meant by the solution of a problem?

**24.** What are simple problems? How many questions in a simple problem? How many answers can be given? What is the meaning of "complex"?

**25.** What is a complex problem? Can a complex problem have more than one elementary question? Can it have more than one answer?

**26.** What is an analytical step? How many steps can there be in a simple problem? How many in a complex problem?

**27.** What is the order of solving problems?

**28.** Give a complete analysis of a problem in addition, following the given order.

**31.** What is subtraction? What is the process of taking 7 from 14 called? Why?

**32.** What is the subtrahend? Which is the greater, — the subtrahend, or the minuend? How should the subtrahend be written? Why?

**33.** What is the minuend? Is the minuend the greater, or the smaller number?

**34.** What is the remainder? Which is the greater, — the remainder, or the minuend? Can the remainder be less than the subtrahend? Can it be greater? What two numbers added together will give the minuend? What is the difference of two numbers? How does the difference differ from the remainder? Are they always alike? If the difference and minuend are given, how can the subtrahend be found? When the remainder and the subtrahend are given, how can the minuend be found?

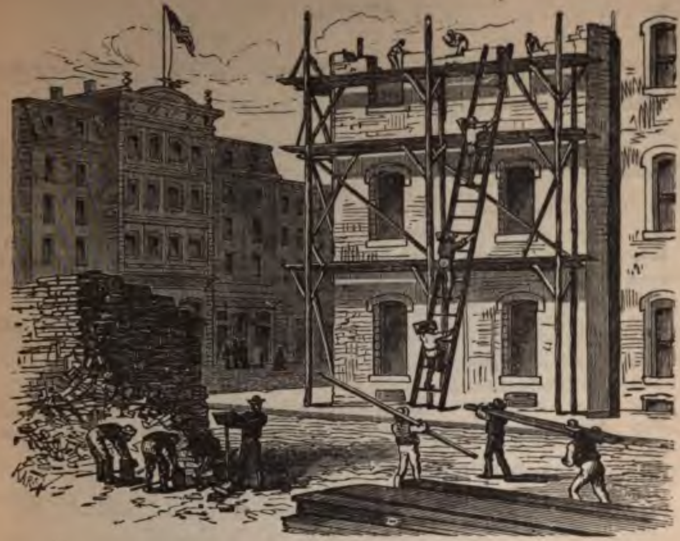
**35.** Illustrate the use of the sign in subtraction.

**36.** What are the axioms used in subtraction? Illustrate the first on the blackboard. The second

**37.** Give a complete analysis of an example in subtraction by first method.

**38.** Analyze an example by the second method.

**39.** Illustrate the use of the parenthesis in arithmetic.



## SECTION IV.

### MULTIPLICATION.

In the above picture some men are engaged in building a house. Three of the men are busy taking bricks from a pile, 3 others are carrying lumber, 3 are each carrying a hod of bricks up the ladder, while 3 bricklayers are placing the bricks in a wall.

How many men are at work?

How many groups of 3 men each are there?

How many are 3 men and 3 men and 3 men and 3 men? How many 3's of men are there?

Each of the 3 men climbing the ladder has 10 bricks in his hod: how many are 10 bricks + 10 bricks + 10 bricks? How many are 3 tens counted together?

In the building with a flagstaff are some windows. If you count them one way, you have 3 rows of windows, and 4 windows in each row, or 4 windows + 4 windows + 4 windows; but, if you count them the other way, you have 4 rows of windows, and 3 windows in each row, or 3 windows + 3 windows + 3 windows + 3 windows: how many windows in the building.?

When equal numbers are to be counted together, there is a shorter process than by addition. This process is called *multiplication*; thus, 3 times 4 windows = 12 windows, or 4 times 3 windows = 12 windows. Any one of the equal numbers to be united is called the *multiplicand*; the number of the equal numbers is called the *multiplier*; the result is called the *product.*

**40.** ***Multiplication*** is the process of taking a number a given number of times.

**41.** ***The Multiplicand*** is the number taken, or multiplied.

**42.** ***The Multiplier*** is the number by which we multiply, and shows how many times the multiplicand is taken.

**43.** ***The Product*** is the result of the multiplication.

**44.** ***The Factors*** of the *product* are the multiplicand and multiplier.

**45.** ***The Sign of Multiplication*** is an oblique cross, thus, ×; and, when placed between numbers, it denotes that they are to be multiplied together; as, $4 \times 8 = 32$.

**46.** ***The Axiom*** used in multiplication is,—

*The product is the same whichever factor is taken for the multiplier.*

**47.** ***Principles:***—

1. *The multiplicand may be abstract or concrete.*
2. *The multiplier is always abstract.*
3. *The product is like the multiplicand.*

## MULTIPLICATION TABLE.

| | | | |
|---|---|---|---|
| 2 × 1 = 2 | 3 × 1 = 3 | 4 × 1 = 4 | 5 × 1 = 5 |
| 2 × 2 = 4 | 3 × 2 = 6 | 4 × 2 = 8 | 5 × 2 = 10 |
| 2 × 3 = 6 | 3 × 3 = 9 | 4 × 3 = 12 | 5 × 3 = 15 |
| 2 × 4 = 8 | 3 × 4 = 12 | 4 × 4 = 16 | 5 × 4 = 20 |
| 2 × 5 = 10 | 3 × 5 = 15 | 4 × 5 = 20 | 5 × 5 = 25 |
| 2 × 6 = 12 | 3 × 6 = 18 | 4 × 6 = 24 | 5 × 6 = 30 |
| 2 × 7 = 14 | 3 × 7 = 21 | 4 × 7 = 28 | 5 × 7 = 35 |
| 2 × 8 = 16 | 3 × 8 = 24 | 4 × 8 = 32 | 5 × 8 = 40 |
| 2 × 9 = 18 | 3 × 9 = 27 | 4 × 9 = 36 | 5 × 9 = 45 |
| 2 × 10 = 20 | 3 × 10 = 30 | 4 × 10 = 40 | 5 × 10 = 50 |
| 2 × 11 = 22 | 3 × 11 = 33 | 4 × 11 = 44 | 5 × 11 = 55 |
| 2 × 12 = 24 | 3 × 12 = 36 | 4 × 12 = 48 | 5 × 12 = 60 |
| 6 × 1 = 6 | 7 × 1 = 7 | 8 × 1 = 8 | 9 × 1 = 9 |
| 6 × 2 = 12 | 7 × 2 = 14 | 8 × 2 = 16 | 9 × 2 = 18 |
| 6 × 3 = 18 | 7 × 3 = 21 | 8 × 3 = 24 | 9 × 3 = 27 |
| 6 × 4 = 24 | 7 × 4 = 28 | 8 × 4 = 32 | 9 × 4 = 36 |
| 6 × 5 = 30 | 7 × 5 = 35 | 8 × 5 = 40 | 9 × 5 = 45 |
| 6 × 6 = 36 | 7 × 6 = 42 | 8 × 6 = 48 | 9 × 6 = 54 |
| 6 × 7 = 42 | 7 × 7 = 49 | 8 × 7 = 56 | 9 × 7 = 63 |
| 6 × 8 = 48 | 7 × 8 = 56 | 8 × 8 = 64 | 9 × 8 = 72 |
| 6 × 9 = 54 | 7 × 9 = 63 | 8 × 9 = 72 | 9 × 9 = 81 |
| 6 × 10 = 60 | 7 × 10 = 70 | 8 × 10 = 80 | 9 × 10 = 90 |
| 6 × 11 = 66 | 7 × 11 = 77 | 8 × 11 = 88 | 9 × 11 = 99 |
| 6 × 12 = 72 | 7 × 12 = 84 | 8 × 12 = 96 | 9 × 12 = 108 |
| 10 × 1 = 10 | 11 × 1 = 11 | 12 × 1 = 12 | 13 × 1 = 13 |
| 10 × 2 = 20 | 11 × 2 = 22 | 12 × 2 = 24 | 13 × 2 = 26 |
| 10 × 3 = 30 | 11 × 3 = 33 | 12 × 3 = 36 | 13 × 3 = 39 |
| 10 × 4 = 40 | 11 × 4 = 44 | 12 × 4 = 48 | 13 × 4 = 52 |
| 10 × 5 = 50 | 11 × 5 = 55 | 12 × 5 = 60 | 13 × 5 = 65 |
| 10 × 6 = 60 | 11 × 6 = 66 | 12 × 6 = 72 | 13 × 6 = 78 |
| 10 × 7 = 70 | 11 × 7 = 77 | 12 × 7 = 84 | 13 × 7 = 91 |
| 10 × 8 = 80 | 11 × 8 = 88 | 12 × 8 = 96 | 13 × 8 = 104 |
| 10 × 9 = 90 | 11 × 9 = 99 | 12 × 9 = 108 | 13 × 9 = 117 |
| 10 × 10 = 100 | 11 × 10 = 110 | 12 × 10 = 120 | 13 × 10 = 130 |
| 10 × 11 = 110 | 11 × 11 = 121 | 12 × 11 = 132 | 13 × 11 = 143 |
| 10 × 12 = 120 | 11 × 12 = 132 | 12 × 12 = 144 | 13 × 12 = 156 |

## DRILL CARD EXERCISE.

N.B.—Assign to the class such portions only of the following graded drill exercises as will make the pupils prompt and accurate in their calculations *without answers.* The key is for the use of teachers only.

TABLE I., *Thousands.*

Exercise I.—2 multiplier.
II.—3 "
III.—4 "
IV.—5 "
V.—6 "
VI.—7 "
VII.—8 "
VIII.—9 "

TABLE II., *Billions.*

Ex. IX.—2 multiplier.
X.—3 "
XI.—4 "
XII.—5 "
XIII.—6 "
XIV.—7 "
XV.—8 "
XVI.—9 "

TABLE II., *Billions.*

Ex. XVII.—11 multiplier.
XVIII.—12 "
XIX.—22 "
XX.—33 "

TABLE II., *Thousands.*

Ex. XXI.—100 to 119 multipliers.
XXII.—220 " 239 "
XXIII.—340 " 359 "
XXIV.—460 " 479 "

Ex. XXV.—5091 multiplier.
XXVI.—90090 "
XXVII.—50902 "
XXVIII.—903004 "

### UNITED STATES CURRENCY.—TABLE III.

Ex. I.—2 multiplier. Ex. V.—6 multiplier. Ex. IX.—20 multiplier.
II.—3 " VI.—7 " X.—37 "
III.—4 " VII.—8 " XI.—105 "
IV.—5 " VIII.—9 " XII.—900 "

### DIRECTIONS.

Exercise I. 1st Example.—Copy (Table I., Thousands' Periods), and multiply A by 2, multiply B by 2; multiply C by 2, &c.; thus:

| | | |
|---|---|---|
| A. [illegible] | B. 442202 | C. 442200 |
| 2 | 2 | 2 |
| Ans. [illegible] | 884404 | 884400 |

Ex. II. Multiply A by 3, B by 3; C by 3, &c.
Ex. III. Multiply A by 4, B by 4; C by 4, &c.

## FELTER'S
## ARITHMETICAL DRILL CARDS.

| TABLE I. | | TABLE II. | | TABLE III. |
|---|---|---|---|---|
| Thou. Units. | | B. Mill. Thou. Units. | | Dollars. Cts. m. |
| 579,438 | | 9,381,908,191 | | 5,319.43 4 |
| 242,022 | A | 8,448,646,446 | A | 3,402 cts. |
| 442,202 | B | 6,688,286,466 | B | 1,811 m. |
| 442,200 | C | 868,424,246 | C | 60 dol. |
| 240,220 | D | 606,486,644 | D | 403 cts. |
| 402,222 | E | 428,664,246 | E | 10 dol. |
| 331,110 | F | 75,351,535 | F | 121 m. |
| 303,211 | G | 73,735,353 | G | 8 cts. |
| 311,121 | H | 53,533,353 | H | 195 dol. |
| 342,221 | I | 53,131,533 | I | 2,000 m. |
| 221,121 | K | 33,713,555 | K | 715 dol. |
| 550,112 | L | 29,036,632 | L | 86 cts. |
| 404,201 | M | 19,241,747 | M | 7 dol. |
| 040,121 | N | 9,866,474 | N | 9 m. |
| 443,121 | O | 9,673,342 | O | 8,189 cts. |
| 552,232 | P | 9,477,773 | P | 908 m. |
| 243,110 | Q | 8,347,544 | Q | 4,003 cts. |
| 152,231 | R | 887,744 | R | 500 dol. |
| 435,112 | S | 347,654 | S | 6,001 cts. |
| 543,121 | T | 281,263 | T | 2,100 m. |
| 402,301 | U | 187,602 | U | 5,000 cts. |

**N.B.—A Key containing the answers to the problems on the above card may be obtained by *mail*, by addressing the publishers.**

## 48. ANALYSIS OF MULTIPLICATION.

1. PROBLEM. — How many are 24 times 873?

PROCESS.

$$\begin{array}{rl} \left.\begin{array}{rl} 873 & \text{multiplicand,} \\ 24 & \text{multiplier,} \end{array}\right\} & \text{factors.} \\ \hline 3492 & \text{partial product, 4 times multiplicand.} \\ 1746\phantom{0} & \text{partial product, 20 `` \quad ``} \\ \hline 20952 & \text{whole product, 24 `` \quad ``} \end{array}$$

2. ANALYSIS. — (1.) For convenience, write the units' figure of the multiplier under the lowest order of the multiplicand.*

(2.) For convenience, multiply first by the units' figure.

(3.) 4 times 3 units are 12 units, equal to 1 ten and 2 units: write the units in the place of units, and reserve the tens to be added to the product of the tens.

(4.) 4 times 7 tens are 28 tens, plus the ten reserved, are 29 tens, equal to 2 hundreds and 9 tens: write the tens in the place of tens, and reserve the hundreds to be added to the product of hundreds.

Continue thus until each figure of the multiplicand has been multiplied by the units' figure of the multiplier.

(5.) Next multiply by the tens' figure. Because it is in tens' place, it has a value tenfold greater than if in units' place, or 20. 20 times 3 are 60 units = 6 tens, which must be written in the tens' place.

(6.) 20 times 7 tens are 140 tens, or 14 hundreds, equal to, &c.

(7.) Add the partial products: hence

3. CONCLUSION. — 24 times 873 are 20952.

4. RULE. — I. *Write the multiplier under the multiplicand.*

II. *Beginning with the unit figure, multiply each figure of the multiplicand by each figure of the multiplier successively, observing to write the first figure of each*

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* It should be remembered that the product is not changed by changing the order of the factors (46): hence, if more convenient, the multiplicand may be written under the multiplier. This analysis may be omitted until a final review of the work.

*partial product directly under its multiplier, writing down and carrying as in addition.*

III. *Add the partial products; and their sum will be the product sought.*

5. PROOFS. — 1. *Repeat the work.*
2. *Change the factors.*
3. *Divide the product by either factor.*

### ORAL AND WRITTEN PROBLEMS CONTAINING BUT ONE ELEMENTARY QUESTION.

**In the following problems the first question in each one is usually oral, and should be recited as such with the appropriate analysis. Those questions containing numbers too large to be thus performed constitute the written problems. The advantages of this arrangement must be obvious to all.**

**The letter *W* will be used to separate the oral from the written questions, and the answers of the written only will be given at the end of the book.**

1. (*a.*) What will 80 pounds of rice cost at 5 cents a pound? (*b.*) What will 4,572 pounds cost?

**ANALYSIS. — (*a.*) Since 1 pound of rice costs 5 cents, 80 pounds will cost 80 times 5 cents, which is 400 cents, or $4.**

**FORMULA. — (*b.*) 5 cts. × 4,572 = 22,860 cts. = $228.60.**

2. What will 50 spelling-books cost at 7 cents apiece? *W.* What will 325 books cost?

3. At 12 cents a pound, what will 40 pounds of sugar cost? *W.* 580 pounds?

4. At 3 cents apiece, what will 90 oranges cost? *W.* 97 oranges?

5. At 11 cents apiece, what will 40 pine-apples cost? *W.* 270 pine-apples?

6. What will 8 yards of cloth cost at 21 cents a yard? *W.* At 31 cents? At 91 cents?

7. If a quantity of provisions will last 2 men 8 days, how long will it last 1 man? *W.* If it will last 2 men 47 days, how long will it last one man?

8. If 2 men can do a piece of work in 31 days, how long will it take 1 man? *W.* If 9 men can do it in 149 days, how long will it take 1 man?

9. If 3 cows eat a ton of hay in 31 days, how long will it last 1 cow? *W.* If 12 cows eat it in 125 days, how long will it last 1 cow?

10. If 3 horses eat 20 bushels of oats in 20 days, how long will it take 1 horse to eat the same quantity? *W.* If 11 horses eat 70 bushels in 37 days, how long will the same quantity last 1 horse?

11. If 30 horses eat 4 bushels of oats in 8 days, how many horses will eat as many in one day? *W.* If 54 horses eat 37 bushels in 137 days, how many horses will eat as much in 1 day?

12. What will 3 sheep cost at 11 dollars each? *W.* At $13?

13. What will 11 pencils cost at 7 cents apiece? *W.* At 27 cents?

14. At 3 cents apiece, what will 4 nutmegs cost? *W.* 57 nutmegs?

15. At 4 cents apiece, what will 5 apples cost? *W.* 89 apples?

16. What will 5 arithmetics cost at 12 cents apiece? *W.* At 129 cents?

---

REVIEW. — What is a unit? (1.) Define a number. (2.) A concrete number. (3.) A denominate number. (4.) An abstract number. (5.) Of what does arithmetic treat? (6.) How many methods of notation are in common use? (7.) Describe the Roman method. (8.) The Arabic. (9.)

17. What will 7 pens cost at 3 cents apiece? *W.* At 127 cents?

18. At 8 cents apiece, what will 3 oranges cost? *W.* 125 oranges?

19. At 7 cents apiece, what will 6 lemons cost? *W.* 314 lemons?

20. How many are 8 times 7? 6? *W.* 952?

21. *W.* If 99 men eat 40 cheeses in 16 days, how many men will be required to eat the same quantity in 1 day?

22. *W.* If 70 cows can eat 9 tons of hay in 8 days, how many cows will be required to eat the same quantity in 1 day?

### WRITTEN PROBLEMS CONTAINING NOT MORE THAN TWO ELEMENTARY QUESTIONS.

23. A man bought 27 cords of wood at $8 per cord, and gave in exchange produce worth $89: how much remained unpaid?

FORMULA.—1. $8 × 27 = $216.
2. $216 — $89 = $127. Ans.

#### ANALYSIS.

ANAL. STEPS.—1. *Find the cost of the wood.*

Since 1 cord costs $8, 27 cords will cost 27 times $8, which is $216.

2. *Find balance unpaid.*

If the wood costs $216, and the produce was worth $89, the balance unpaid will be the difference of these amounts, which is $127.

24. A man bought 27 sheep at $4 apiece, and gave

---

REVIEW.—What is the difference between the simple and local values of a figure? (10, 11.) Repeat the numeration table from right to left. (12.) From left to right. What is meant by numbers increasing in a tenfold ratio? (13.) Give rule for notation. *(15.) Give rule for numeration.* (16.)

in payment a note for $47, and the remainder in cash: how much cash did he pay?

25. A tailor had a piece of broadcloth containing 42 yards; he cut from it 17 yards: how much was the remainder worth at $4 a yard?

26. A man sold 17 sheep at $5 apiece; he received $37 in cash, and the rest in wood: how much did his wood cost him?

27. A man started to make a journey of 279 miles; he travelled 4 days at 38 miles a day: how far was he from the end of his journey?

28. A farmer had 108 bushels of potatoes; he sold 47 bushels: how much was the remainder worth at 7 shillings a bushel?

29. A tailor has a piece of cloth containing 57 yards; he cuts from it 13 suits containing 4 yards each: how many yards remain?

30. I have 337 yards of calico: how much remains if 37 dresses, each containing 9 yards, are cut off?

31. Charles bought 93 oranges at 4 cents apiece; he paid towards them 178 cents: how much did he still owe?

32. A man earned 537 cents in a week; he paid 39 cents a day for board and washing: how much had he left at the expiration of the 7 days?

33. A man owing $189 gave in part payment 23 sheep at $4 per head: how much did he still owe?

34. I bought 28 sheep at $6 per head, and a span of horses for $297: how much more did the horses cost than the sheep?

35. A man paid $189 toward 147 bushels of wheat at $2 a bushel: how much did he still owe?

7 36. I have 7 bags of grain; 6 of them contain 89 pounds each; they all contain 722 pounds: how many pounds does the seventh contain?

37. I have a cistern containing 189 gallons; the faucet will discharge 2 gallons every minute: if the faucet is left open 83 minutes, how much water will remain in the cistern?

38. At 2 cents a foot, how much will 12 boards cost, each board containing 13 feet?

39. What will 17 dozen of eggs cost at 2 cents apiece?

40. At 3 shillings a yard, how many shillings will 13 pieces of cloth cost, if each piece contains 37 yards?

41. What will 7 dozen arithmetics cost at 85 cents apiece?

42. How many pounds in 8 bags of wheat, each bag containing 2 bushels of 60 pounds each?

43. If I buy 6 barrels of pork, at $16 a barrel, and sell it for $105, how much do I gain?

44. If I buy 12 pounds of sugar at 16 cents per pound, and sell it for 205 cents, how much do I gain?

45. In an orchard are 26 peach-trees, and 6 times as many apple-trees as peach-trees: how many trees in the orchard?

46. In an orchard there are 8 rows of trees, and 125 trees in each row: how many trees in the orchard?

47. If 8 horses eat 12 bushels of oats in 6 days, how many bushels will they eat in 32 days?

---

REVIEW.—Define addition. (17) The sum. (18.) What is the answer in addition called? Illustrate the sign of addition. (19.) What is it called? Illustrate the use of the sign of equality. (20.) The dollar-sign. (21.) What is an axiom? (22.) Define a problem. (23.) What is the difference between a simple and a complex problem. (24, 25.) What is meant by an analytical step? (26.)

## *49.* TO MULTIPLY BY 10, 100, 1,000, &c.

### ILLUSTRATION.

347 × 10 = 3,470.
347 × 100 = 34,700.
347 × 1,000 = 347,000.

It is shown, by the above illustration, that a number can be multiplied by *ten* by annexing *one* cipher, also that, by annexing an additional cipher, it is multiplied by *ten* twice, or 100, &c.: hence,

1. *Annexing one cipher to any number multiplies it by* 10.
2. *Annexing two ciphers to any number multiplies it by* 100.
3. *Annexing three ciphers to any number multiplies it by* 1,000, &c.

## *50.* TABLE UNITED STATES CURRENCY.

$$\left.\begin{array}{l}10 \text{ mills } = 1 \text{ cent,}\\ 10 \text{ cents } = 1 \text{ dime,}\\ 10 \text{ dimes } = 1 \text{ dollar,}\end{array}\right\} \text{ or } \left\{\begin{array}{r}1{,}000 \text{ mills } =\\ 100 \text{ cents } =\\ 10 \text{ dimes } =\end{array}\right\} 1 \text{ dollar.}$$

$1 = 100 cents.
½ = 50 cents.
¼ = 25 cents.
¾ = 75 cents.
½ cent = 5 mills.

(GOLD.)

(BRONZE.)

(SILVER.)

48. How many cents in 8 dollars?

FORMULA. — 100 cents × 8 = 800 cents.

ANALYSIS. — Since $1 equals 100 cents, 8 dollars must equal 8 times 100 cents, or 800 cents.

49. How many dimes in 12 dollars? In $36? ***W.*** In $420? In $35?

50. How many mills in 4 dollars? ***W.*** In 20 dollars? *In 75 dollars?*

51. How many mills in 3 cents? *W.* In 250 cents? In 183 cents?

### MISCELLANEOUS WRITTEN PROBLEMS.

52. What will 2 barrels of flour cost at 8 dollars and 10 cents per barrel?

**PROCESS.**

$$\begin{array}{r} \$8.10 \\ 2 \\ \hline \$16.20 \end{array}$$

**ANALYSIS.**—If 1 barrel of flour costs $8.10, 2 barrels will cost twice $8.10, or $16.20.

53. What will 3 pairs of gloves cost at 87 cents a pair? What will 389 pairs cost?

54. Give the cost of each of the following items: 3 pairs of silk hose at $1.20 each, 5 pairs of boots at $3.50 each, 4 pairs of shoes at $1.50 each, 4 pairs of cotton gloves at $.38 each, 7 pairs kid gloves at $1.00 each.

55. Give the cost of the following: 4 First Readers at 30 cents, 5 Second Readers at 50 cents, 3 Third Readers at 70 cents, 9 Fourth Readers at 80 cents, and 8 Fifth Readers at $1.00.

56. Give the cost of the following: 5 pounds of sugar at 11 cents per lb., 3 pounds of tea at 90 cents per lb., 4 pounds of coffee at 22 cents per lb., 8 pounds of raisins at 31 cents per lb., 4 pounds of rice at 16 cents per lb.

57. Bought 3 yards of calico at 16 cents a yard; I gave the clerk a half-dollar: how much change should I receive?

---

**REVIEW.**—Give the order of solving problems. (27.) Add a column of 2's to 50. (29.) Add a column of 3's in the same way. Define subtraction. (31.) Give an example, and name and define *the subtrahend* (32); the minuend (33); the remainder *(34).*

58 Bought a book for $1.30, a slate for 16 cents gave the shopkeeper a 2-dollar bill: how much change should I receive? Gave him a 5-dollar bill? A 10-dollar bill?

59. A little girl went shopping with 1 dollar; she bought 2 spools of thread at 10 cents a spool, 3 spools of silk at 15 cents each: how much did the thread and silk cost? How much money had she left?

60. His father gave Charles $1.50; he spent 41 cents for a knife, 21 cents for a kite, and 11 cents for a top: how much money had he left?

61. What must I give for 3 books at $1.25 each, and 5 slates at 27 cents each?

62. Thomas attended public school 76 days, at a tuition of 3 cents per day: how much was his bill? How much at 2½ cents per day?

**FORMULA.**—2½ cents = $.025. $.025 × 76 = $1.900.

**ANALYSIS.**—1. 2½ cents = 25 mills.

2. Since the tuition for 1 day costs 25 mills, the tuition for 76 days will cost 76 times as much, or 1,900 mills, equal to $1.90.

### ORAL AND WRITTEN PROBLEMS.

63. At 5 cents each, how much should a newsboy receive for 6 papers? *W.* For 876 papers?

64. If a man spend $3½ per day, to how much will his expenses amount in 3 days? *W.* In 1 month, or 30 days? In 1 year, or 365 days?

65. At $6¼ each, how much will 2 army-coats cost? *W.* 5 army-coats cost? 476 army-coats?

---

**REVIEW.**—Illustrate the use of the sign of subtraction. (35.) Repeat the axioms used in subtraction. (36.) Give the analysis of a problem in subtraction. (37.) Illustrate the use of the parenthesis. (39.)

66. *W.* How many pounds of cotton in 2 bales, if 1 bale weighs 478 pounds? In 5 bales? In 384 bales?

67. How much is 1 bale worth at 10 cents per pound? *W.* How much are 5 bales worth? 384 bales?

68. How many gallons are 8 times 3 gallons? 5 gallons? *W.* 47 gallons? 3,947 gallons?

69. How many pounds are 16 times 1 pound? 2 pounds? *W.* 46 pounds? 3,478 pounds?

70. At 3 cents each, what will 2 oranges cost? 5 oranges? *W.* 147 oranges? 1,973 oranges?

71. If 1 bag of apples is worth $2¼, what are 2 bags worth? *W.* 5 bags? 167 bags?

72. If 1 quart of peanuts cost 9 cents, what will 8 quarts, or 1 peck cost? *W.* 16 quarts, or 2 pecks? 56 quarts, or 7 pecks?

73. If 1 peck of apples is worth 60 cents, what are 3 pecks worth? 4 pecks, or 1 bushel? *W.* 8 pecks, or 2 bushels?

74. *W.* A man bought 6 pounds of veal at 11 cents per pound, and 7 pounds of beef at 16 cents per pound: how much did both cost? He gave the shopkeeper a 10-dollar bill: how much change should he receive?

75. At 6 cents per quart, what do 3 quarts of milk cost? 4 quarts, or 1 gallon? *W.* 5 gallons? 16 gallons?

76. *W.* If I have a 10-dollar bill, and pay out 37 cents, how much shall I have left? If I afterwards receive $1.87, how much shall I have? If I then loan Mr. Smith $3 and 4 cents, how much will remain?

77. *W.* 4 dollars and 20 cents, *plus* $1.50, *minus* 10

---

**REVIEW.**—Define multiplication. (40.) Give an example, point out, name, and define the multiplicand. (41.) Multiplier. (42.) Product. (43.) Factors. (44.) Give sign of multiplication. (45.) Illustrate its use.

cents, *plus* $4.13, *minus* $3½, = how much? $100 *plus* 100 cents, *minus* 100 mills, = how much?

78. *W.* 100 cents *plus* 100 dollars *minus* $3¾ = how much?

79. *W.* $300 + 20 cents, – $3.00, + $2.20, – 4½ dollars = ?

80. *W.* $4.20 × 3, + $1.20, – $8.00, + 40 cents = ?

81. *W.* $5 × 20, + 200 cents, – $87½, + 87½ cts. = ?

82. *W.* 40 dollars + 40 cents, + 30 dollars, – 128 cents = ?

83. *W.* $4.17 × 8, + $9¼, – 87½ cents, + $87½ = ?

84. *W.* $0.17 × 9, – 27 cents, + $27, – 27 mills = ?

85. *W.* 300 dollars – 300 cents, + 300 mills = ?

86. *W.* 92 cents × 8, – $6.924, + 17 mills = ?

87. *W.* $100 × 3, – 5½ cents, + 3 dollars = ?

88. *W.* $217 – 1½ cents, × 3, + 7½ cents = ?

## ANALYSIS OF SECTION IV.

### Multiplication.

1. ILLUSTRATION.
2. DEFINITIONS.
3. SIGN.
4. AXIOM.
5. PRINCIPLES.
6. ABSTRACT PROBLEMS.
    - a. *Process.*
    - b. *Analysis.*
    - c. *Proof.*
    - d. *Rule.*
7. CONCRETE PROBLEMS.
    - a. *Formula.*
    - b. *Analytical Steps.*
    - c. *Analysis.*

---

**REVIEW** – Name the axiom used in multiplication. (46.) Repeat the principles, and illustrate each. (47.) Give the analysis of multiplication. (48.) Illustrate the multiplication by 10, 100, 1,000, &c. (49.) Repeat the table of United States currency. (50.)

## SECTION V.

### DIVISION.

ARTHUR and his two sisters are dividing some fruit equally among themselves. They have six apples: into how many equal parts must they separate them?

How many will each have?

They also have 9 pears: into how many equal parts must they be separated?

How many will each have?

Arthur has just received a number of peaches. He commenced dividing them by giving each of his two sisters 4 peaches, and took 4 himself; he then had one peach left. How many times did he take away 4 peaches from the number?

How many were left?

How many peaches did he receive in all?

This process of finding how many times one number is contained in another, or of separating a number of things into equal parts, is called *division*. The number to be divided is called the *dividend*. The number of equal parts into which it is to be divided is called the *divisor*. The result of the division, or the number of things in each of the equal parts, is called the *quotient*. The number of things left after the equal parts are taken out is called the *remainder*.

**51. *Division*** is the process of finding how many times one number is contained in another, or of separating a number into equal parts.

**52. *The Dividend*** is the number divided.

**53. *The Divisor*** is the number by which the dividend is divided.

**54. *The Quotient*** is the result obtained by dividing.

**55. *The Remainder*** is the part of the dividend left undivided.

**56. *The Sign of Division*** is a short horizontal line between two dots; thus, ÷. It shows that the number before it is to be divided by the number after it; thus, $20 \div 4$ means that 20 is to be divided by 4.

**57. *The Fractional Sign of Division*** consists in writing the dividend above a line, and the divisor below it; thus, $\frac{20}{4}$ denotes that 20 is to be divided by 4.

**58.** In ***Short Division,*** the quotient only is written. The intermediate steps are omitted.

**59.** In ***Long Division,*** each step of the process is written.

**60.** The axiom used in division is,—

*If a number be both multiplied and divided by the same number, its value will not be changed.*

---

REVIEW.—What is a unit? (1.) A number? (2.) A concrete number? (3.) A denominate number? (4.) An abstract number?

**61.** PRINCIPLES,—

1. *The dividend may be either abstract or concrete.*

2. *When the divisor is abstract, the quotient is like the dividend.*

3. *When the divisor is concrete, the quotient is abstract.*

### 62. ANALYSIS OF SHORT DIVISION.

1. PROBLEM.—Divide 70,301 by 4.

PROCESS.

Divisor. 4)70,301 Dividend.
17,575¼ Quotient.

*Proof.*
17,575¼ Quotient.
4 Divisor.
70,301 Dividend.

2. ANALYSIS.*—(1.) For convenience write the divisor at the left of the dividend, and commence by dividing the left-hand digit.

(2.) *One-fourth* of 7 tens of thousands is 1 ten of thousands with a remainder of 3 tens of thousands, equal to 30 thousand. Write the 1 in the place of tens of thousands, and add the 30 thousands to the thousands.

(3.) *One-fourth* of 30 thousands is 7 thousands with a remainder of 2 thousands, which is equal to 20 hundreds. Write the 7 in the place of thousands, and add the 20 hundreds to the hundreds: 20 hundreds and 3 hundreds are 23 hundreds.

(4.) *One-fourth* of 23 hundreds is 5 hundreds with a remainder of 3 hundreds, equal to 30 tens. Write the 5 in hundreds' place, and add the 30 tens to the tens.

(5.) *One-fourth* of 30 tens is 7 tens with a remainder of 2 tens, equal to 20 units. Write the 7 in the place of tens, and add the 20 units to the units: 20 units and 1 unit are 21 units.

(6.) *One-fourth* of 21 units is 5 units with a remainder of 1 unit,† which, divided by 4, gives one-fourth of a unit, written ¼. Write the 5 in the place of units, and annex the ¼ of a unit.

3. CONCLUSION.—Therefore *one-fourth* of 70,301 is 17,575¼.

---

* In this analysis, we prefer the *fractional* form, as it is simpler, and more readily understood, although liable to some objection. The teacher will find another form in the analysis of long division (64), which he can use here, if he prefers.

† It will be seen by this that a remainder in division can be changed to a fractional quotient by writing the divisor under it, in the form of a fractional sign of division.

**4. Rule for Short Division.** — I. *Write the divisor at the left of the dividend.*

II. *Beginning at the left, divide each figure of the dividend by the divisor, and set each quotient figure under its dividend.*

III. *If there is a remainder after any division, prefix it to the next figure of the dividend, and divide as before.*

IV. *If any partial dividend is less than the divisor, write a cipher for the quotient, prefix the partial dividend to the next figure of the dividend, and divide as before.*

V. *If there is a remainder after dividing the last figure of the dividend, it can be changed to a fractional quotient by writing the divisor under it, and annexing the result to the quotient.*

**5. Proof.** — *Division may be proved by multiplying the quotient by the divisor, and adding the remainder, if there is any, to the product. The result will be equal to the dividend, if the work is correct.*

### DRILL CARD EXERCISES.

| Table I., *Thousands.* | Table II., *Billions.* |
|---|---|
| Exercise I. — 2 Divisor. | Exercise II. — 2 Divisor. |
| III. — 3 " | IV. — 3 " |
| V. — 4 " | VI. — 4 " |
| VII. — 5 " | VIII. — 5 " |
| IX. — 6 " | X. — 6 " |
| XI. — 7 " | XII. — 7 " |
| XIII. — 8 " | XIV. — 8 " |
| XV. — 9 " | XVI. — 9 " |

---

Review. — What is notation? (7.) What methods are most used? Which nploys letters? Which figures? How many, and what, are the letters used? (8.) hen are numbers written by the use of letters? Give the value of each letter. hat is the effect of repeating a letter? What effect has a letter of less value laced *before* one of greater value? When placed after? How is the value fected when a dash is placed over a letter, or a combination of letters?

## FELTER'S

# ARITHMETICAL DRILL CARDS.

| TABLE I. | | TABLE II. | | TABLE III. |
|---|---|---|---|---|
| Thou. Units.<br>579,438 | | B. Mill. Thou. Units.<br>9,381,908,191 | | Dollars. Cts. m.<br>5,319.43 4 |
| 242,022 | A | 8,448,646,446 | A | 3,402 cts. |
| 442,202 | B | 6,688,286,466 | B | 1,811 m. |
| 442,200 | C | 868,424,246 | C | 60 dol. |
| 240,220 | D | 606,486,644 | D | 403 cts. |
| 402,222 | E | 428,664,246 | E | 10 dol. |
| 331,110 | F | 75,351,535 | F | 121 m. |
| 303,211 | G | 73,735,353 | G | 8 cts. |
| 311,121 | H | 53,533,353 | H | 195 dol. |
| 342,221 | I | 53,131,533 | I | 2,000 m. |
| 221,121 | K | 33,713,555 | K | 715 dol. |
| 550,112 | L | 29,036,632 | L | 86 cts. |
| 404,201 | M | 19,241,747 | M | 7 dol. |
| 040,121 | N | 9,866,474 | N | 9 m. |
| 443,121 | O | 9,673,342 | O | 8,189 cts. |
| 552,232 | P | 9,477,773 | P | 908 m. |
| 243,110 | Q | 8,347,544 | Q | 4,003 cts. |
| 152,231 | R | 887,744 | R | 500 dol. |
| 435,112 | S | 347,654 | S | 6,001 cts. |
| 543,121 | T | 281,263 | T | 2,100 m. |
| 402,301 | U | 187,602 | U | 5,000 cts. |

**N.B.**—**A Key** *containing* the answers to the problems on the *above card may be obtained* by *mail*, by addressing the publishers.

## MENTAL DIVISION.

### ILLUSTRATION.

One.

2 halves.

3 thirds.

4 fourths.

5 fifths.

How many *halves* in one apple? How many *thirds* in one pear? How many *fourths* in one orange? How many *fifths* in one melon? How many *sixths* in one dollar? How many *sevenths* in one dime? How many *eighths* in one pound?

How many halves make one apple? How many thirds make one pear? How many fourths make one orange? How many fifths make one melon? How many tenths make one mile? How many twenty-fifths make one peach?

1 half.

1 third.

2 fourths.

3 fifths.

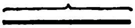
What is meant by one *half?* Ans. — *One of the two equal parts into which a unit, or number of units, is divided.*

---

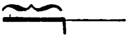
Review. — What characters are used in the Arabic notation? (9.) How is 10 written? Why? How is 12 written? Why? What is the simple value of a figure? (10.) The local value? (11.) What is numeration? (12.) Repeat the table by periods from right to left. From left to right.

What is meant by one *third?* What by 2 fourths? What by 3 fifths? What by 4 sevenths? What by 7 twelfths? What by 4 fifths?

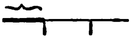
One.



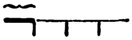
1 half.



1 third.

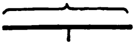


1 fourth.

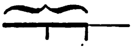


Which is the greater, — one, or one *half?* One *half*, or one *third?* One *third*, or one *fourth?* One *half*, or one *fourth?* *One*, or one *third?* *One*, or one *fourth?* One *fourth*, or one *fifth?* One *seventh*, or one *tenth?* One *sixteenth*, or one *seventeenth?*

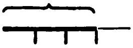
2 halves.



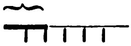
2 thirds.



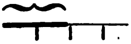
3 fourths.



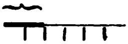
2 sixths.



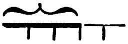
2 fourths.



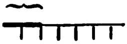
2 sixths.



3 fifths.

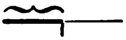


2 sevenths.

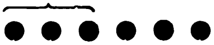


Which is the greater, — 2 halves, or 2 fourths? 2 thirds, or 2 sixths? 3 fourths, or 3 fifths? 2 sixths, or 2 sevenths? 6 sevenths, or 6 eighths? 5 sevenths, or 5 elevenths? 5 ninths, or 5 thirteenths?

1 half of one.



1 half of 6.



What do we mean by one-half of 1? Ans. — *One of the two equal parts into which one is divided.*

What do we mean by one-half of 2? Ans. — *One of the two equal parts into which two is divided.*

What do we mean by one-half of 6? What by one-half of 8? What by one-half of 12? What by one-half of 16? Of 18? Of 20? Of 10?

How many units in one-half of 1? Ans. — *One-half* of one unit.

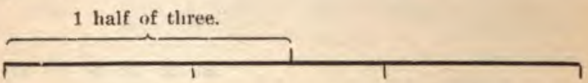
How many units in one-half of 2? Ans. — *One* unit.

How many units in one-half of 6? Ans. — *Three.*

In one-half of 8? In one-half of 12? In one-half of 16? Of 18? Of 20? Of 10?

I wish to divide 3 apples equally between 2 girls: how many whole apples will each have? What must be done with the odd apple? What part of the odd apple will each have? How many apples will each have?

I wish to divide 5 apples equally between 2 boys: how many whole apples will each have? What part of the apples will each have? How many apples will each have?


1 half of three.

How many units in one-half of 3? Ans. — *One unit and a half.*

How many units in one-half of 5? Ans. — *Two units and a half.*

How many units in one-half of 7? In one-half of 9? In one-half of 13? Of 15? Of 17? Of 19?

---

Review. — What is a rule? (14.) Recite the rule for notation. (15.) For *numeration.* (16.) What is addition? (17.) What is meant by the sum of two or *more numbers?* (18.) Describe the sign of addition. (19.) Show its use.

### **63.** TO DIVIDE by 10, 100, 1,000, &c.

ILLUSTRATION.

4,000 ÷ 10 = 400
4,000 ÷ 100 = 40
4,000 ÷ 1,000 = 4

4,346 ÷ 10 = 434 and 6 Rem.
4,346 ÷ 100 = 43 and 46 Rem.
4,346 ÷ 1,000 = 4 and 346 Rem.

It is shown by the above illustrations that a number can be divided by 10 by cutting off one figure from the right, also, that, by cutting off two figures from the right of a number, it is divided by 10 twice, or 100, &c.; and the figures cut off are a remainder.

Hence,

1. *Removing a figure from the right of a number divides it by* 10.
2. *Removing two figures from the right of a number divides it by* 100.
3. *Removing three figures from the right of a number divides it by* 1,000; *&c.*

### UNITED STATES CURRENCY.

*To change cents and mills to dollars.*

ILLUSTRATION.

3,454 mills ÷ 1,000 = $3.454.
3,454 cents ÷ 100 = $34.54.
424 mills ÷ 10 = $ .424.

From the above it is seen that, —

1. *Mills are changed to dollars by pointing off three figures from the right, or dividing by* 1,000.
2. *Cents are changed to dollars by pointing off two figures from the right, or dividing by* 100.
3. *Mills are changed to cents by pointing off* one figure *from the right, or by dividing by* 10.

How many dollars in 4,235 mills?

**FORMULA.**—4,235 mills ÷ 1,000 mills = $4.235.

**ANALYSIS.**—Since in 1,000 mills there is 1 dollar, in 4,235 mills there are as many dollars as 1,000 mills are contained times in 4,235 mills, or 4 dollars with a remainder of 23 cents and 5 mills.

The teacher should dictate problems until the pupils can readily divide or multiply by 10, 100, 1,000, &c.

## EXERCISES.

Change each of the following to dollars, cents, and mills, by dividing by 10, 100, or 1,000, as the case may be.

| PROCESS. | (1.) | (2.) |
|---|---|---|
| 434 cents = $4.34 | 58 mills. | 1,304 mills. |
| 434 mills = .434 | 20 cents. | 2,050 mills. |
| 404 mills = .404 | 938 cents. | 250 cents. |
| 20 cents = .20 | 715 cents. | 452 cents. |

N.B.—Additional problems should be dictated by the teacher.

## ORAL PROBLEMS.

**PROBLEM.**—If 15 apples are divided equally among 3 boys, how many apples does each boy receive? If 3,474 apples are so divided?

PROCESS.

3)3,474 apples.

1,158 apples. Ans.

**ANALYSIS.**—If 3,474 apples are divided equally among 3 boys, 1 boy will have 1 third of 3,474 apples, or 1,158 apples.*

1. If 9 peaches cost 27 cents, what is the cost of 1 peach? If 7 cost 35 cents? If 12 cost 144 cents?

---

* Let the pupil be taught that one-*third* of a thing, or number, is the same as *the quotient of* that thing, or number, divided by 3.

2. If 8 lead-pencils cost 24 cents, how much is that apiece? If 8 cost 88 cents? 72 cents?

3. If I pay $32 for 8 cords of wood, how much is that a cord? If I pay $64? $96?

4. 12 men can perform a certain piece of work in 48 days: what part of it can 1 man perform? 3 men?

5. If 7 men can do a piece of work in 7 days, what part of it can 1 man do in the same time? 5 men?

6. One man can perform a piece of work in 48 days: how long will it take 12 men to do it? 6 men? 3 men?

7. One man can do a piece of work in 56 days: how long will it take 8 men? 7 men? 2 men?

8. One man can build a barn in 63 days: how long will it take 9 men to build it? 3 men? 7 men?

### ORAL AND WRITTEN PROBLEMS CONTAINING NOT MORE THAN TWO ELEMENTARY QUESTIONS.

9. A man earned $36 in 9 days: how much is that per day? For 3 days? *W.* For 37 days?

10. 8 gold chains cost $64: what is the cost of each? What will 3 cost? *W.* 472 cost?

11. 5 barrels of apples are worth $35: what is the value of 1 barrel? Of 7 barrels? *W.* Of 343 barrels?

12. In 7 hours a man rode 84 miles: how far was that an hour? How far for 3 hours? *W.* For 327 hours?

---

**REVIEW.**—Describe the sign of equality. (20.) Illustrate its use. Show the use of the dollar-sign. (21.) What is an axiom? (22.) What is a problem? (23.) What is a simple problem? (24.) A complex problem? (25.) What is an analytical step? (26.) Give the order *of solving* problems. (27.) Give the analysis of an example in *addition. (28.)*

13. 9 pounds of sugar cost 81 cents: how much is that a pound? How much for 2 pounds? *W.* For 380 pounds?

14. In 9 rows of trees there are 72 trees: how many trees in each row? In 4 rows? *W.* In 85 rows?

15. A man travelled 72 miles in 6 hours: how far did he travel in 1 hour? In 7 hours? *W.* In 24 hours?

16. 8 bins of equal size contain 72 bushels of wheat: how many bushels does each contain? How many would 3 contain? *W.* Would 50 contain?

17. If 81 cents be paid for 9 dozen eggs, what will be the cost of 1 dozen? Of 5 dozen? *W.* Of 280 dozen?

18. 11 tons of hay are worth $77: what is the value of 1 ton? Of 9 tons? *W.* Of 386 tons?

PROBLEM. — At 5 cents each, how many lemons can I buy for 20 cents? For $38.75?

PROCESS. 5)3,875

775 lemons.

ANALYSIS. — 1. $38.75 = 3,875 cents.
2. At 5 cents each, as many lemons can be bought as 5 cents are contained times in 3,875 cents, or 775 lemons.

19. How many boxes of cheese at $5 a box can be bought for $30? *W.* For $6,755?

20. If I receive $12 interest in 1 year, in how many years shall I receive $132 interest? *W.* $864 interest?

21. How many pencils can I buy for 360 cents, if 1 pencil costs 3 cents? *W.* How many for 1,041 cents?

---

REVIEW. — What is subtraction? (31.) Define minuend. (33.) Subtrahend (32.) Remainder. (34.) Minus means what? (35.) Recite the axioms of subtraction. (36.)

22. At 7 cents per pound. how many pounds of lead may be bought for $8.40? *W.* For $6.23?

23. How many barrels of apples at $3 a barrel can be bought for $183? *W.* For 2,922 dollars?

24. If 300 men accomplish a certain piece of work in 1 day, how many men will be required to do it in 4 days? In 5 days? *W.* In 10 days?

25. If 168 men use 2 barrels of flour in 1 week, how many men will use the same in 6 weeks? *W.* In 12 weeks?

26. If 750 persons eat 80 loaves of bread in 1 week, how many persons will it require to use the same in 5 weeks? *W.* In 15 weeks?

27. If 3 yards of cloth are enough for a coat, how many coats can be made from 33 yards? *W.* From 384 yards?

28. A man travelled 7 miles in 1 hour: at that rate, how long would it take him to travel 63 miles? *W.* 861 miles?

29. For 49 cents, how many pounds of sugar can be bought at 7 cents a pound? *W.* For 343 cents?

30. For 240 cents, how many oranges can be bought at 3 cents apiece? How many for 9,248 cents?

31. For 360 cents, how many pounds of sugar can be bought at 9 cents a pound? *W.* At 12 cents?

32. If 12 men can build 240 rods of wall in 20 days, how many rods can 1 man build in the same time? *W.* 7 men?

---

**Review.** — Give the first method of analysis of subtraction. (37.) The second. (38.) What is the use of the parenthesis? (39.) Define multiplication. (40.) Multiplicand. (41.) Multiplier. (42.) Product. (43.) Factors. (44.) Describe the sign of multiplication. (45.) Show its use. Give the axiom. (46.) Give the principles. (47.)

## WRITTEN EXAMPLES.

33. If 12 men can build 240 rods of wall in 20 days, how many rods can they build in 1 day?

34. If 9 men can do a piece of work in 3 days, how long will it take 1 man to do it? How long will it take 3 men?

35. Two men own equal shares in a farm worth $3,417: what is the value of each man's share?

**FORMULA.**—$3,417 ÷ 2 = $1,708½.

**PROCESS.**
2)$3,417
1,708½

**ANALYSIS.**—If 2 equal shares are worth $3,417, 1 of the shares is worth one-half as much, or $1,708½.

36. In 3 miles there are 190,080 inches: how many inches in 1 mile?

37. In 4 hogsheads of wine there are 8,064 gills: how many gills in 1 hogshead?

38. In 8 pounds there are 7,680 farthings: how many farthings in 1 pound?

39. How many families are there in a city of 18,000 inhabitants, if there are an average of 6 persons in each family?

40. Sound moves 8,960 feet in 8 seconds: how many feet does it move in 1 second?

41. In 7 equal regiments there are 8,050 men: what number in each?

42. If 1 man can perform a piece of work in 1,728 hours, how long will it take 9 men to do the same?

43. A man dying left an estate valued at $3,874, to be equally divided among his wife and four children: *how much did* each receive?

44. Divide the number 1,347 into 3 equal parts: how many units in each part?

45. The sum of the 4 equal parts of a number is 147,342: what is one of the parts?

46. 5 times a certain number is 41,324: what is the number?

47. How many oranges can be purchased for 3,416 cents at 8 cents each?

**FORMULA.**—3,416 cents ÷ 8 cents = 427, number of oranges.

**ANALYSIS.**—At 8 cents each, as many oranges can be bought as 8 cents are contained times in 3,416 cents, or 427 oranges.

48. How many pounds of beef can be purchased for 4,768 cents at 10 cents a pound?

## *64.* ANALYSIS OF LONG DIVISION.*

1. PROBLEM.—Divide 26,306 by 45.

**STEPS.**—1. Find number of times.
2. Multiply.
3. Subtract.
4. Bring down.

**DIVISION.**

$$\begin{array}{lrrl}
 & \text{Divisor,} & \text{Dividend,} & \text{Quotient.} \\
 & 45\ ) & 26{,}306\ ( & 584\frac{26}{45}\text{, or} \\
 & & 225\ \ ( & 584.57+ \\ \cline{3-3}
 & & 380 & \\
 & & 360 & \\ \cline{3-3}
 & & 206 & \\
 & & 180 & \\ \cline{3-3}
\text{Remainder, units.} & & 26.0 & \\
 & & 22.5 & \\ \cline{3-3}
 & & 3.50 & \\
 & & 3.15 & \\ \cline{3-3}
\text{Remainder, hundredths.} & & .35 &
\end{array}$$

**PROOF.**

$$\begin{array}{rl}
\text{1st Quotient.} & \\
584\frac{26}{45} & \\
45 & \text{Divisor.} \\ \hline
2920 & \\
2336\phantom{0} & \\
26 & \text{Remainder.} \\ \hline
26{,}306 &
\end{array}$$

**PROOF.**

$$\begin{array}{rl}
\text{2d Quotient.} & \\
584.57+ & \\
45 & \text{Divisor.} \\ \hline
2922.85 & \\
23382.8\phantom{0} & \\
.35 & \text{Remainder.} \\ \hline
26{,}306.00 &
\end{array}$$

---

* This analysis may be omitted until a final review of the work.

**2. ANALYSIS.** — (1.) For convenience,* place the divisor at the left of the dividend, and the quotient, when found, at the right.

(2.) 45 is contained in 263 5 times: as the 263 is hundreds, the 5 is also hundreds, and must be written in the quotient in hundreds' place. 5 times 45 are 225, really 225 hundreds, and written to indicate hundreds under the dividend. 225 hundreds from 263 hundreds leave 38 hundreds, equal to 380 tens.

(3.) 45 is contained in 380 tens 8 tens times with a remainder of 20 tens, equal to 200 units. Write the 8 in the tens' place of the quotient, and to the 200 units add the 6 units of the dividend, making 206 units.

(4.) 45 is contained in 206 units 4 times with a remainder of 26 units. Write the 4 in the units' place of the quotient, and indicate the division of 26 by 45 (**57**) as $\frac{26}{45}$ of a unit, to be written at the right of the quotient.

(5.) If desired, the division may be carried still further: thus, the remainder, 26 units, equals 260 tenths: 45 is contained in 260 tenths 5 tenths times with a remainder of 35 tenths, equal to 350 hundredths. Write the 5 tenths in the tenths' place, &c.

**3. RULE FOR LONG DIVISION.** — I. *Write the divisor at the left of the dividend, as in short division.*

II. *Find how many times the divisor is contained in the fewest left-hand figures of the dividend that will contain it, and place the quotient at the right of the dividend.*

III. *Multiply the divisor by this quotient figure; subtract the product from the partial dividend; and to the remainder annex the next figure of the dividend.*

IV. *Divide as before, until all the figures of the dividend have been annexed and divided.*

V. *If any partial dividend does not contain the divisor, place a cipher in the quotient, annex the next figure of the dividend, and divide as before.*

---

REVIEW. — Give analysis of multiplication. (48.) Illustrate the method of multiplying by 10, 100, &c. (49.) Recite the table of United States currency. (50.) How many cents make a dollar? How many mills make a dollar?

* Those who prefer may select any other place, not in the way of subsequent work, for either divisor or quotient. The class should have a method, and adhere to it.

VI. *If there is a remainder after dividing all the figures of the dividend, it may be changed to a fractional quotient by writing the divisor underneath it.*

4. PROOF. — *The proof of long division is the same as that of short division.*

NOTES. — 1. The first step in long division is to find the quotient figure of the highest denomination.

2. Multiply the divisor by that figure, and write the product under the first partial dividend; then observe, that, if the product is greater than the partial dividend, the quotient figure is too large.

3. Subtract that product from the first partial dividend, and observe, that, if the remainder is equal to or greater than the divisor the quotient figure is too small.

4. Change the remainder to the next lower denomination, and add to it the corresponding denomination in the dividend.

### WRITTEN EXAMPLES.

49. When 16 acres of land cost $2,304, what is the cost per acre?

50. If 21 bushels of wheat weigh 1,260 pounds, what is the weight of 1 bushel? Of 5 bushels?

51. How many bushels in 5,040 pounds of wheat, if 1 bushel weighs 60 pounds?

52. There are 25 barrels of cider, each barrel contains 40 gallons: how many gallons in all?

53. I have 1,000 gallons of wine: how many barrels must I have to contain it, if each barrel contains 40 gallons?

54. I have 1,000 gallons of vinegar which I wish to put into barrels, each holding 25 gallons: how many barrels shall I require?

---

REVIEW. — Define division. (51.) Dividend. (52.) Divisor. (53.) Quotient. (54.) Remainder. (55.) Describe the sign. (56.) Show its use. Describe the fractional sign. (57.) What is meant by short division? (58.) By long division. (59.) What is the axiom of division? (60.) Give the principles. (61.)

55. In every mile there are 320 rods: how many rods in 2 miles? In 42 miles?

56. Since every 320 rods make a mile, how many miles in 640 rods? In 7,234 rods?

57. $4 \div 2 =$ what? $14 \div 7 =$ what? $473 \div 9 =$ what? $9{,}134 \div 46 =$ what?

58. $55 \times 2 \times 4 - 26 + 47 \div 46 =$ what?

59. The dividend is 25, the divisor is 5: what is the quotient?

Point out and name the terms, and give the answers, in the following:—

60. $4 + 3 + 4 + 25 =$ what?
61. $115 - 63 =$ what?
62. $137 \times 14 =$ what?
63. $571 \div 92 =$ what?
64. $15 \times 13 =$ what?
65. $374 + 4 =$ what?
66. $913 \div 21 =$ what?
67. $517 - 118 =$ what?

What is the answer to each of the following?

68. $\overline{29 + 7} + \overline{56 - 23}$?
69. $\overline{29 + 7} - \overline{56 - 23}$?
70. $27 + \overline{84 \div 4} - \overline{0 + 8}$?
71. $27 + \overline{84 \times 4} - \overline{0 \times 8?}$
72. $6 + 12 - 11 \times \overline{15 \div 3}$?
73. $12 \times 3, + 4, - 6, \times 1, \div 4$?
74. $\overline{12 \times 3} + 4, - 6, \times 1, \div 4$?
75. $12 \times \overline{3 + 4} - 6, \times 1, \div 4$?

---

**REVIEW.**—Give the analysis of an example in short division. (62.) Illustrate the method of dividing by 10, 100, &c. (63). Give the analysis of an example in long division. (64.)

76. $12 \times 3, + (14 - 6) \times 1, \div 4$?
77. $12 \times 3, + 14 - (6 \times 1) \div 4$?
78. $12 \times 3, + 14, - 6 \times \overline{12 \div 4}$?
79. $25 \times 0, + (86 \div 2) - \overline{7 - 2}$?

Indicate in the proper manner, by signs, the following, and give the results: —

80. Multiply the sum of 10 and 8 by 3, and divide the product by 9.

81. From the product of 8 and 7 subtract 25, and divide the remainder by 11.

82. Multiply by 35 the quotient of 60 less 8 divided by 4.

83. How much is 3 less than the quotient of 408 divided by 12?

Write the formula for each of the following problems: —

84. A boy spent 20 cents on Monday, $1.30 on Tuesday, 87½ cents on Wednesday, and $2.30 on Thursday: how much did he spend on the three days?

85. A boy earned 80 cents one time, and 40 cents at another; he spent a dollar: how much had he left?

86. What will 8 pounds of rice cost at 22 cents per pound?

87. What is the value of 5 tons of hay, at the rate of $72 for 12 tons?

88. A man bought 18 yards of cloth for $54, and 12 yards of cheaper cloth for $25: what was the difference of the price per yard?

## DRILL CARD EXERCISES.

### LONG DIVISION.

| TABLE I., *Thousands.* | | TABLE II., *Thousands.* |
|---|---|---|
| Exer. I. — | Divide A to K inclusive, by 10. | Exer. II. |
| | " L to U " " 11. | |
| III. — | Divide by 11, 21, 31, 41, &c., to 101 inclusive. | IV. |
| | " " 12, 22, 32, 42, &c., to 102 " | |
| V. — | Divide by 13, 23, 33, 43, &c., to 103 " | VI. |
| | " " 14, 24, 34, 44, &c., to 104 " | |
| VII. — | Divide by 20, 30, 40, 50, &c., to 110 " | VIII. |
| | " " 19, 20, 39, 49, &c., to 109 " | |
| IX. — | Divide by 18, 28, 38, 48, &c., to 108 " | X. |
| | " " 17, 27, 37, 47, &c., to 107 " | |
| XI. — | Divide by 16, 26, 36, 46, &c., to 106 " | XII. |
| | " " 15, 25, 35, 45, &c., to 105 " | |
| XIII. — | Divide by 100, 101, 102, &c., to 109 " | XIV. |
| | " " 210, 211, 212, &c., to 219 " | |
| XV. — | Divide by 760, 761, 762, &c., to 769 " | XVI. |
| | " " 870, 871, 872, &c., to 879 " | |
| XVII. — | Divide by 980, 981, 982, &c., to 989 " | XVIII. |
| | " " 990, 991, 992, &c., to 999 " | |

TABLE II., *Billions.*

| | |
|---|---|
| Exercise XIX. — | Divide by 5009, 5109, 5209, &c., to 5909 inclusive. |
| | " " 6090, 6190, 6290, &c., to 6990 " |

XX. — Divide Table II. by the corresponding numbers of Table I., thus: — A (Table II.) ÷ A (Table I.), B ÷ B, C ÷ C, &c., to R.

### UNITED STATES CURRENCY. — TABLE III.

| | | |
|---|---|---|
| Exer. I. — 2 divisor. | Exer. V. — 6 divisor. | Exer. IX. — 10 divisor. |
| II. — 3 " | VI. — 7 " | X. — 25 " |
| III. — 4 " | VII. — 8 " | XI. — 84 " |
| IV. — 5 " | VIII. — 9 " | XII. — 59 " |

DIRECTIONS. — EXER. III. Divide A (Table I.) by 11, B by 21, C by 31, &c., and K by 101. Then divide L by 12, M by 22, N by 32, &c., and U by 102.

EXER. IV. Divide A (Table II.) by 11, B by 21, C by 31, D by 41, E by 51, &c., as in Exercise III. (See Key.)

## FELTER'S

# ARITHMETICAL DRILL CARDS.

| TABLE I. | | TABLE II. | | TABLE III. |
|---|---|---|---|---|
| Thou. Units. | | B. Mill. Thou. Units. | | Dollars. Cts. m. |
| 579,438 | | 9,381,908,191 | | 5,319.43 4 |
| 242,022 | A | 8,448,646,446 | A | 3,402 cts. |
| 442,202 | B | 6,688,286,466 | B | 1,811 m. |
| 442,200 | C | 868,424,246 | C | 60 dol. |
| 240,220 | D | 606,486,644 | D | 403 cts. |
| 402,222 | E | 428,664,246 | E | 10 dol. |
| 331,110 | F | 75,351,535 | F | 121 m. |
| 303,211 | G | 73,735,353 | G | 8 cts. |
| 311,121 | H | 53,533,353 | H | 195 dol. |
| 342,221 | I | 53,131,533 | I | 2,000 m. |
| 221,121 | K | 33,713,555 | K | 715 dol. |
| 550,112 | L | 29,036,632 | L | 86 cts. |
| 404,201 | M | 19,241,747 | M | 7 dol. |
| 040,121 | N | 9,866,474 | N | 9 m. |
| 443,121 | O | 9,673,342 | O | 8,189 cts. |
| 552,232 | P | 9,477,773 | P | 908 m. |
| 243,110 | Q | 8,347,544 | Q | 4,003 cts. |
| 152,231 | R | 887,744 | R | 500 dol. |
| 435,112 | S | 347,654 | S | 6,001 cts. |
| 543,121 | T | 281,263 | T | 2,100 m. |
| 402,301 | U | 187,602 | U | 5,000 cts. |

N.B. — A **Key** containing the answers to the problems on the above card may be obtained by *mail*, by addressing the publishers.

## MISCELLANEOUS WRITTEN PROBLEMS WHICH CONTAIN NOT MORE THAN THREE ELEMENTARY QUESTIONS.

89. A farmer bought a horse for $65, a saddle for 16 dollars, and a bridle for 8 dollars: how much did he pay for all?

90. A lady paid $200 for a piano, $37 for a sofa, and $20 for an easy-chair: how much did she pay for all? How much had she left of a five-hundred-dollar bill?

91. Julia bought 2 pieces of tape for 20 cents each, 2 papers of pins for 26 cents, and a pair of scissors for 50 cents: how much did she pay for all? How much change should she receive for a ten-dollar bill?

92. A farmer has 115 sheep in one lot, 75 in another, and 40 in his barnyard: how many sheep has he? How many had he after selling 37 sheep?

93. James has 39 nuts in one pocket, and 14 less in another: how many has he in both?

ANAL. STEPS.—1. *Find how many nuts in the second pocket.*

If James has 14 nuts less in the second pocket than in the first, he must have in the second pocket the difference of these numbers, or 25 nuts.

2. *Find how many nuts in both.*

If James has 30 nuts in one pocket, and 25 nuts in another, he must have in both the sum of these numbers, or 64 nuts.

94. A man bought 40 sheep of three farmers. He bought 9 of the first, and 13 of the second: how many did he buy of the third? How many more must he buy to have 250 sheep?

95. A grocer had a tub of butter containing 48 pounds. He sold 13 pounds to Mr. Jacob Wilson, and

15 pounds to John Huntsman: how many pounds were left?

96. Mr. Clark paid $25.50 for a cow, and for a horse $56 more than he paid for the cow: how much did he pay for both?

97. What is the difference between the cost of 6 pieces of tape at four cents apiece, and of 4 yards of ribbon at 6 cents a yard?

ANAL. STEPS.—1. *Find cost of tape.*
2. *Find cost of ribbon.*
3. *Find difference of cost.*

98. How many cents must a boy have to buy 18 apples at 3 cents apiece, and 14 cakes at 2 cents apiece? How much change should he receive for a 5-dollar bill?

99. I bought 6 barrels of pork at 11 dollars a barrel, and sold it for $89.47: how much did I gain?

100. What is the cost of 13 sticks of candy at 2 cents apiece, and 12 pounds of raisins at 15 cents a pound? I paid for these from a $5 bill, and spent the remainder for rice at 12 cents a pound: how many pounds did I buy?

101. If 3 dresses of 16 yards each are cut from 86 yards of calico, how many yards will be left?

102. A man started from home, and travelled 5 miles an hour for 13 hours, and then set out to return, at the rate of 6 miles per hour: how far was he from home, after having travelled 9 hours?

103. Henry and John started together, and travelled in the same direction. Henry walked 3 miles an hour, and John 4 miles an hour: how far apart were they at the end of 9 hours? At the end of 48 hours?

104. Robert and Joseph started together from home, and travelled in the same direction. Robert travelled 2 miles an hour, and Joseph 4 miles an hour: how far were they apart at the end of 36 hours? At the end of 5 days, travelling 8 hours each day?

105. The next week, Robert and Joseph started together from home, and travelled in opposite directions. Robert travelled 2 miles an hour, and Joseph 3 miles an hour: how far were they apart at the end of 19 hours? At the end of 49 hours?

106. If 3 yards of cloth cost $27, what will 967 yards cost?

ANAL. STEPS. — 1. *Find the cost of 1 yard.*
2. *Find the cost of 967 yards.*

**If 3 yards cost $27, 1 yard will cost 1 third of $27, which is $9, and 967 yards will cost 967 times $9, or $8,703.**

107. If 7 oranges cost 63 cents, how much will 127 oranges cost? 149 oranges?

108. If 9 yards of tape cost 27 cents, how much will 120 yards cost? 346 yards?

109. If 7 sheep cost $56, how much will 321 sheep cost? 7,487 sheep?

110. Susan can solve 80 problems in 8 days: how many can she solve in 76 days? In 200 days, or a school-year?

111. I pay $84 for 12 sheep: how much must I pay for 196 sheep? For 9,327 sheep?

112. A man rode 75 miles in 25 hours: how far, at that rate, can he ride in 324 hours? In 93 days of 24 hours each?

113. A man travels 63 miles in 7 hours: how far, at that rate, can he ride in 72 hours? In 297 days of 24 hours each?

114. If 84 dollars are paid for 21 barrels of flour, how much must be paid for 9,386 barrels?

115. How much will 8,437 hats cost, if 7 cost $21?

116. How much can a person earn in 114 days, if he can earn $48 in 24 days? How much in 30 days?

117. What must I pay for 2,746 apples, if 5 cost me 25 cents? If 9 cost 18 cents?

118. A farmer sold 114 calves at the rate of 2 calves for $24: what did he receive for them?

119. If 36 men can dig a cellar in 18 days, how many men will be required to dig it in 3 days?

ANALYSIS. — 18 days are 6 times 3 days: therefore to do the work in 3 days will take 6 times 36 men, or 216 men.

120. If 4 men can dig a ditch in 40 days, how many men will be required to dig it in 5 days?

121. How long will it take 7 men to reap a field that 28 men can reap in 2 days? That 42 men can reap in 5 days?

122. Two ships 168 miles apart sail directly toward each other, — one at the rate of 8 miles an hour, and the other at the rate of 4 miles an hour: how soon will they meet? If they are 8,904 miles apart?

123. A man bought a number of lemons at the rate of 11 for 25 cents: how many could he buy for 250 cents?

ANAL. STEPS. — 1. *Find how many times 11 lemons can be bought for 250 cents.*
2. *Find the number.*

124. When potatoes are sold at the rate of 4 barrels for $8, how many barrels must be given for 4 cwt. of sugar at $14 a hundred?

743365

ANAL. STEPS. — 1. *Find value of 1 barrel of potatoes.*
2. *Find entire value of the sugar.*
3. *Find number of barrels required.*

125. How many dozen of eggs at 18 cents a dozen will pay for 9 gallons of molasses at 36 cents a gallon? How many will pay for 36 gallons?

126. When eggs are sold at 24 cents for 8, how many will it take to pay for 13 pounds of sugar at 6 cents a pound? For 926 pounds at 12 cents?

127. How many gallons of molasses at 35 cents a gallon will pay for 70 pounds of butter at 30 cents a pound? 67,280 pounds of butter?

128. A man received 32 pounds of sugar in exchange for 920 pounds of butter at 16 cents a pound: what did the sugar cost him per pound?

129. A man bought 80 turkeys at the rate of 4 for 3 dollars; he sold them for $80.30: did he make, or lose? and how much?

ANAL. STEPS. — 1. *Find cost 1 turkey.*
2. *Find cost of 80 turkeys.*
3. *Find gain or loss.*

130. A farmer bought 24 sheep for 60 dollars. At how much per head must he sell them in order to gain 12 dollars? To gain $36?

131. 5 men bought a threshing-machine for $150. They rented it 4 weeks at $10 a week, and then sold it for $100: did they make, or lose? and what was each man's share?

132. 5 barrels of flour are worth $60: how many hundred weight of pork at $18 a hundred will 3 barrels buy? How many will 6 barrels buy?

133. 4 men agree to build a barn for $160; but, when

it is half built, 2 of the men quit work, and the remainder complete the job: how much should each man receive?

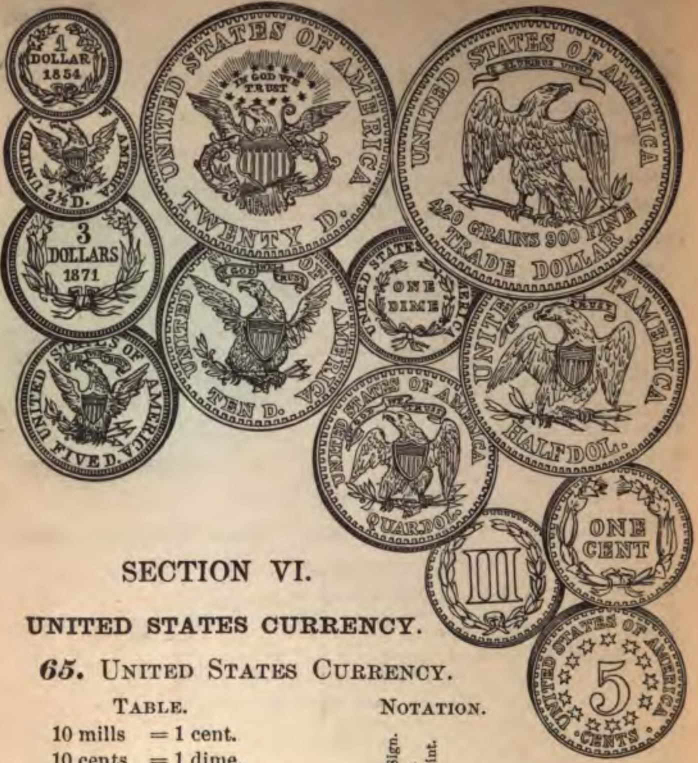
**ANAL. STEPS.** — 1. *Find what each man should receive for building the first half.*
2. *Find what each man should receive for building the second half.*
3. *Find the entire amount each man should receive.*

134. 4 men agree to chop 25 cords of wood for $48; but, when it is half done, 3 of the men quit, leaving the remaining man to finish the work. How shall the money be divided?

## ANALYSIS SECTION V.

### Division.

1. ILLUSTRATION.
2. DEFINITIONS
3. SIGNS.
4. KINDS.
5. AXIOMS.
6. PRINCIPLES.
7. ABSTRACT PROBLEMS.
    - a. *Process.*
    - b. *Analysis.*
    - c. *Rule.*
    - d. *Proof.*
8. CONCRETE PROBLEMS.
    - a. *Formula.*
    - b. *Analytical Steps.*
    - c. *Analysis.*


1 DOLLAR 1854
UNITED STATES OF AMERICA 2½ D.
3 DOLLARS 1871
UNITED STATES OF AMERICA FIVE D.
UNITED STATES OF AMERICA IN GOD WE TRUST TWENTY D.
UNITED STATES OF AMERICA TEN D.
UNITED STATES OF AMERICA ONE DIME
UNITED STATES OF AMERICA 420 GRAINS 900 FINE TRADE DOLLAR
UNITED STATES OF AMERICA HALF DOL.
UNITED STATES OF AMERICA QUAR. DOL.
III
ONE CENT
UNITED STATES OF AMERICA 5 CENTS

## SECTION VI.

## UNITED STATES CURRENCY.

### *65.* UNITED STATES CURRENCY.

TABLE.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes = 1 dollar.
10 dollars = 1 eagle.

NOTATION.

Dollar Sign. Dollars. Dec. Point. Cents. Mills.
$11 . 111

### *66.* TABLE OF ALIQUOT PARTS.

$\frac{1}{2}$ dollar = 50 cents.
$\frac{1}{4}$ dollar = 25 cents.
$\frac{1}{5}$ dollar = 20 cents.
$\frac{1}{8}$ dollar = $12\frac{1}{2}$ cents.
$\frac{1}{10}$ dollar = 10 cents.

$\frac{3}{4}$ dollar = 75 cents.
$\frac{1}{2}$ cent = 5 mills.
$\frac{1}{4}$ cent = $2\frac{1}{2}$ mills.
$\frac{1}{3}$ dollar = 33 cts. 3 mills.

**67.** The ***coins**** of the United States are made of gold, silver, nickel, and bronze (or copper), as follows: —

---

* The silver five and three cent pieces, the bronze two cent, and the nicke one cent, are no longer coined.

***Gold,*** the double-eagle, eagle, half-eagle, quarter-eagle, three-dollar and one-dollar pieces.

***Silver,*** the dollar, half-dollar, quarter-dollar and dime.

***Nickel,*** the five-cent and three-cent pieces.

***Bronze,*** one cent.

***68. Paper money*** consists of notes issued by the United States Treasury and banks as substitutes for coin.

### WRITTEN PROBLEMS. — REVIEW OF UNITED STATES CURRENCY.

PROBLEM. — (*a.*) How many times are 12½ cents contained in $4?
(*b.*) In $3¾?

| (*a.*) PROCESS. | (*b.*) PROCESS. |
|---|---|
| $0.125)$4,000(32 Ans. | $0.125)$3,750(30 Ans. |
| 375 | 375 |
| 250 | 0 |
| 250 | |

ANALYSIS. — (*a.*) 12½ cents = 125 mills. $4 = 4,000 mills. 125 mills are contained in 4,000 mills 32 times.

(*b.*) 12½ cents = 125 mills. $3¾ = 3,750 mills. 125 mills are contained in 3,750 mills 30 times: hence the following

RULE. — *Change the divisor and dividend to the lowest denomination found in either, and divide as in simple division.*

1. How many times are 3 cents contained in $3? In $3.33?

2. How many times are 5½ cents contained in $5½? In $203½?

3. How many times are 20 cents contained in $20? In $2? In $5.20?

4. How many times are $1.12½ contained in $10.12½? In $12.37½?

5. $4.40 are how many times 4 cents? 40 cents? 5½ cents?

6. $6¾ are how many times 25 cents? 5 cents? 9 cents?

### BUSINESS PROBLEMS.

7. A boy bought a book for $1, a slate for 50 cents, and a sponge for 12½ cents: how much did he give for all?

8. A lady bought some calico for 37½ cents, and some lace for $1¾: what did she give for both?

9. A boy had a three-dollar bill; he paid out $1.37½: how much had he left?

NOTE FOR THE TEACHER.—The analyses to be used in the solution of these problems are the same as those used in simple numbers.

10. Lucy bought a book for 30 cents, an inkstand for 62½ cents, and some paper for 12½ cents: how much did she spend?

11. John had $11.37½; he gave 2½ dollars to William: how much had he left?

12. A boy earned $5¾ in a week; he gave his father $4½: how much had he left?

13. A boy found 30 cents; he then earned 2½ dollars: how much had he in all?

14. What will 120 apples come to at 2 cents apiece? At 3½ cents? At 2½ cents?

---

REVIEW.—Recite the table of United States currency. (65.) Give the table of aliquot parts. (66.) Name the gold coins of the United States. (67.) The silver coins. The nickel. The bronze.

15. If the interest on 1 dollar for one year is 6 cents, what is the interest on $347? On $276?

16. If 1 eighth of a house is worth $371.26, what is 8 eighths, or the whole house, worth?

17. A man gave $387.27½ for 1 eighteenth of a ship: what is 15 eighteenths of the ship worth?

18. 1 sixth of a church cost $1397.87½: what did 5 sixths of the church cost? 6 sixths?

19. 1 ninth of a factory is worth $876.43: what are 5 ninths worth? 9 ninths, or the whole?

20. Seven boys had $12½: how much is that apiece?

**ANALYSIS.**—$12½ are equal to 12,500 mills. 1 boy must have had 1 seventh of 12,500 mills, which is 1,785+ mills, or $1.785.

21. If 9 boys earned $4¼, how much did 1 boy earn? If they earned 5¾? $10.87½?

22. If I give $1.37½ for a book, how much will 2 copies cost? 27 copies? 1,002 copies?

23. If 1 pound of beef cost 16½ cents, how much will 3 pounds cost? 307 pounds? 4,003 pounds?

24. 7 pounds of sugar cost $1.17: how much is that per pound? If it cost $1.20? If it cost $1¼?

25. 8 pounds of cheese cost ¾ of a dollar: how much was that a pound? If it cost $1¼?

26. I have a dollar-bill: how many oranges can I buy with it at 5 cents apiece? At 2½ cents?

27. How many pounds of beef can be bought for $91.62½ at 12½ cents per pound?

28. How many oranges can be bought for $14.80 at 5 cents apiece? At 2½ cents?

29. How many pounds of rice can be bought for $4.32 at 4 cents per pound? At 8 cents per pound?

30. At 50 cents a yard, how many yards of cloth can be bought for $12? For $816? For $9,218½?

31. How much will 75 baskets of peaches cost at $1.62½ per basket? At $1.87½? At 90½ cents?

32. What will 117 pounds of beef cost at 16½ cents per pound? At 12½ cents? At 16 cents?

33. What must I give for 1 yard of lace, if 7 yards cost $7.50? $13¾? $9.87½?

34. What must I give for 93 bushels of potatoes, if 13 bushels cost $8.97? $11.882? $13?

35. What will 17 pounds of sugar cost, if 190 pounds cost $38? $36.10? $28½? $33.44?

36. What must I pay per gallon for vinegar, if 63 gallons cost $50? $35?

37. A boy had $4.00; he gave away 12½ cents: how much had he left?

38. If the interest on 1 dollar for a year is 5½ cents, how much will it be on $20? On $75? On $375?

39. If the interest on 1 dollar for a year is 7½ cents, how much will it be for 3 years? For 30 years?

40. If the interest on 1 dollar for a year is 7½ cents, how much will it be for $3? For $2,003?

41. What cost 15 pounds of tea at 37 cents per pound, 19 pounds of coffee at 13½ cents per pound, 46 gallons of molasses at $0.37½ per gallon, and 37 gallons of vinegar at 11 cents per gallon?

42. What cost 37 chests of tea at $37.46 each; 37 bags of coffee, 97 pounds each, at $0.12½ per pound; 49 casks of rice at $42 each; and 60 barrels of oil, 38 gallons each, at $1.08 per gallon?

43. What cost 49,748 feet of boards at $0.03 per foot, *18,764 shingles* at ½ cent each, 987,864 square feet of

boards at 3 cents per foot, and 49,187 lath at 3½ cents each?

44. What cost 46 bushels of turnips at $0.36 per bushel, 97 bushels of carrots at $0.25 per bushel, 47 bushels of beets at 63 cents per bushel, and 138 bushels of potatoes at $0.87½ per bushel?

## BILLS.

**69.** ***A Bill or Invoice of Goods*** is a written statement of the goods purchased, given by the seller to the buyer.

***The Debtor*** is the party who owes.

***The Creditor*** is the party who is owed.

***An Account*** is a bill which contains both debit and credit items.

**The teacher should dictate simple bills like the following, requiring, in the solution, the business form to be strictly observed.**

**NEW YORK, July 1, 1875.**

**J. W. SCHERMERHORN & CO.,**

**TO SCRIBNER, ARMSTRONG, & CO., DR.**

| | | | | | |
|---|---|---|---|---|---|
| **1875.** | | | | | |
| **April 10.** | **To 25 Sheldon's Ob. Less. at $1.75** | **$43** | **75** | | |
| **" 16.** | **" 200 First Lessons at 25c** | **50** | | | |
| **May 10.** | **" 500 Primary Arith. at 30c** | **150** | | | |
| | | **243** | **75** | | |
| | **Less 20 per cent** | **48** | **75** | **195** | |
| | **CR.** | | | | |
| **May 25.** | **By School apparatus** | **137** | | | |
| | **Less 10 per cent** | **13** | **70** | **123** | **30** |
| | **Balance due S. A. & Co.** | | | **$71** | **70** |

***Received Payment,***

**SCRIBNER, ARMSTRONG, & CO.,**

**per VALENTINE.***

---

*** When a clerk or agent receipts a bill, he writes the signature of the firm, and attaches *his own as factor or agent.***

45.

TOPEKA, KAN., Sept. 3, 1875.

Mr. T. H. CHURCH,

Bo't of L. T. GAGE,

36 pounds of sugar, at $0.09½ per pound,
3 hhd. molasses, 63 gallons each, at $0.27 per gal.,
5 casks of rice, 285 pounds each, at $0.05 per pound,
2 chests of tea, 86 pounds each, at $0.96 per pound,
4 barrels of oil, 31 gallons each, at $0.87½ per gal.

$

46.

MANCHESTER, N.H., June 17, 1874

Mr. Z. F. RILEY,

Bo't of WILLIAM PRATT,

| | | | | | |
|---|---|---|---|---|---|
| 69 | pieces | of carpeting, 47 | yards each, | at $1.13 | per yd. |
| 87 | " | of Brussels do. 35 | " | at $1.87½ | " |
| 147 | " | of sheeting, 47 | " | at $0.05½ | " |
| 378 | " | of calico, 39 | " | at $0.08 | " |

$

47.

LEAVENWORTH, April 27, 1875

Mr. THOS. SCHOLES,

To T. W. SCUDDER, DR.

| | | |
|---|---|---|
| 1875. | | |
| Jan. 17. | To 72 tons coal at $8.62½ per ton. | |
| April 8. | " 15 cords oak wood at $7.25 per cord. | |
| " 25. | " 18 cords pine wood at $6.33⅓ per cord. | |
| | CR. | |
| Feb. 17. | By merchandise, as by his bill | $550.50 |
| " 24. | " cash, | 200.00 |

What is the balance due T. W. Scudder?

48.

SOUTHAMPTON, MASS., March 3, 1875.

HENRY ROSE & Co.,

Bo't of MOSES ATWOOD & Co.

86 shovels, at $0.50.
96 spades, at $0.86.
18 ploughs, at $11.00.
23 handsaws, at $3.50.
14 hammers, at $0.62.

$

Received payment,

MOSES ATWOOD & CO.

49.

NEWARK, N.J , Sept. 29, 1875.

Mr. J. F. HUNTSMAN,

Bo't of A. WARD & SON,

37 chests of green tea, at $23.75.
42 chests of black do., at $17.50.
43 casks of wine, at $99.00.
12 crates of Liverpool ware, at $75.00.
19 barrels Genesee flour, at $7.00.
23 bushels rye, at $1.52.

$

50.

BUTTERNUTS, N.Y., May 3, 1875.

Mr. LYMAN A. BURR,

Bo't of HENRY FISK,

69 chests of tea, at $55.65.
126 bags of coffee, 100 pounds each, at $0.12½.
167 boxes of raisins, at $2.75.
800 bags of almonds, at $18.50.
9,004 barrels of shad, at $7.50.

$

Received payment,

HENRY FISK.

## ANALYSIS, SECTION VI.

### United States Currency.

1. ILLUSTRATION.
2. TABLE.
3. COINS.
4. PAPER CURRENCY.
5. INVOICE, OR BILL.
6. FUNDAMENTAL OPERATIONS.
   a. *Process.*
   b. *Analysis.*
7. APPLICATIONS.
   a. *Formula.*
   b. *Analytical Steps.*
   c. *Analysis.*

## TEST QUESTIONS FOR EXAMINATION.

TO THE TEACHER. — It is very important that each pupil should be subjected to a searching examination before leaving the Fundamental Rules and United States currency. A standard of about *eighty-five* per cent should be required to entitle him to proceed. The examination should test the pupils upon the following points: —

1. *Rapidity and accuracy in abstract operations.*
2. *Familiarity with the terms and definitions*, and knowledge of their *meaning.*

3. *Ability to analyze problems of not more than three elementary questions.*

The pupil must not be expected to understand the principles of notation, or to explain multiplication and division. The task is too severe. Children easily commit to memory, but do not readily understand principles. Better leave these explanations until the mind is more matured.

**40.** What is multiplication? Multiplication is a short method of what? Can all problems in addition be done by multiplication? Can all problems in multiplication be done by addition? What is the difference between a problem that can be done only by addition, and one that can be more easily done by multiplication? ANS.—*In one, the numbers to be added are equal: in the other, they are unequal.* What is the answer called in addition? What is it called in multiplication?

**41.** What is the multiplicand? Has the multiplicand any corresponding number in addition? Add 184 to itself. What is the 184 called in addition? Can the problem be done by multiplication? What is the 184 called in multiplication?

**42.** What is the multiplier? What does the multiplier show? What do we call the number that shows how many times the 184 is taken?

**43.** What is the product? To what in addition does the product correspond? [The teacher should show at the blackboard the real difference between addition and multiplication.]

**44.** Which are the factors? What is the meaning of "factor"? How many terms in multiplication? Which two do we multiply together to make the third? Which two terms must be given in order to find the third? If the multiplicand and multiplier are given, how can the product be found? If the product and multiplier are given, how can the multiplicand be found? If the product and multiplicand are given, how can the multiplier be found? Illustrate each.

**45.** What is the sign of multiplication? How is it used? Should the multiplier be placed before, or after, the sign? Where should the product be placed? What sign should be placed between the factors and the product?

**46.** What is the axiom of multiplication? Illustrate it. What is the axiom of addition? Can unlike things be added together? Can things be subtracted from unlike things? Give an illustration.

**47.** Give an example in which the multiplicand is abstract. In which it is concrete. What is always abstract? What is like the multiplicand?

**48.** How should the numbers be written in multiplication? Must the multiplier be written under the multiplicand? Is the less number always the multiplier? Does it make any difference in the result, which number is used as the multiplier? Where should the products of the figures be placed? Why? What is a rule? What is the rule for multiplication? How may you prove an answer in multiplication?

**49.** How may numbers be multiplied by 10? by 100? by 1,000, &c.?

**50.** Give the table of United States currency. Show how dollars can be changed to mills; to cents.

**51.** What is division? Is there any other way of finding how many times one number contains another? ANS.—*By subtraction.* Illustrate.

---

*REVIEW.—What can* be said of paper money? (68) What is an invoice, or bill? *(69.) What is meant* by debtor? By creditor? An account?

**52.** What is the dividend?

**53.** What is the divisor?

**54.** What is the quotient? To what term in division does the *minuend* correspond? To what does the *subtrahend* correspond? How many times is 6 contained in 18? How many times can 6 be subtracted from 18? Has the quotient 3 any corresponding number in subtraction? In subtraction, how many times is the subtrahend taken from the minuend? In division, how many times is the divisor contained in the dividend? To what does the remainder in subtraction correspond in division?

**55.** What is the remainder in division? What is the remainder in subtraction?

**56.** What is the sign of division? Illustrate the use of the sign of division. Is the sign placed before, or after, the divisor? What sign is placed between the divisor and quotient, when it is to be shown that 18 divided by 6 is 3?

**57.** In what does the fractional sign of division consist? How are the numbers written? Which is the dividend? Which is the divisor? Illustrate the use of both signs, and show their difference.

**58 & 59.** What is the difference between short division and long division?

**60.** Give and illustrate the axiom of division.

**61.** Give an example in which the dividend is abstract; one in which it is concrete When the divisor is abstract, what is the quotient like? When the divisor is concrete, what is the quotient?

**62.** Give the analysis of a problem in short division. Repeat the rule. State the method of proof. What is the remainder in division? How can a remainder be changed to a fractional quotient? What is the answer called in division? What is to be done with the fractional quotient?

**63.** How may a number be divided by 10, 100, 1,000, &c. By this means change mills to dollars, cents to dollars, mills to cents.

**64.** What is the method of long division? Repeat the rule for long division. What is the first step in long division? The second? The third? The fourth? When the next figure is brought down to the remainder, and it will not then contain the divisor, what is to be done? If the product of the divisor and the quotient figure is more than the partial dividend, what does it show? If any remainder equals or exceeds the divisor, what does it show? What is the opposite of division? How can division be proved? How can multiplication be proved? How does multiplying the divisor affect the quotient? How does multiplying the dividend affect the quotient? How does dividing the divisor affect the quotient?

**65.** What is the currency of the United States? When was it adopted? ANS.—1786. What are the denominations? In business, how are eagles read? How are dimes read? Which denomination of United States money is the unit? How many cents make a dollar? How many mills make a cent? A dime? A dollar?

**66.** How many cents in a half-dollar? In a quarter-dollar? In three quarter-dollars? How many cents in 1 eighth of a dollar? In 1 tenth? In 1 fifth? In 2 fifths? A mill is what part of a cent? A cent is what part of a dollar? Into how many parts is a cent divided? What are the parts called?

**67.** What are coins? Of what are coins made? What is the place called where they are made? What metals are called precious? Which precious metal is worth the most? What are the gold coins of the United States? The silver coins? The nickel? The bronze?

**68.** What is meant by paper money?

**69.** What is a bill or invoice of goods? By whom are bills given? What is their use? What is a debtor? What is a creditor? When is a bill called an account? Write a bill on the blackboard. Write an account. What is it to receipt a bill? Is the bill receipted by the debtor, or by the creditor? How is a bill receipted by a clerk or agent?

### GENERAL QUESTIONS IN UNITED STATES CURRENCY.

How are dollars reduced to cents? Which of the fundamental rules is used to reduce dollars to cents? Cents to mills? Mills to cents? Mills to dollars? Cents to dollars? Cents to mills? Give the formula for reducing $2 to cents. 500 cents to dollars. 4 cents to mills. 80 mills to cents. 5,000 mills to dollars. Recite the rule for reducing dollars to cents. Dollars to mills. Mills to cents. Mills to dollars. Cents to dollars. Cents to mills.

How are numbers in Federal money added? Can dollars be added to cents? Cents to mills? Mills to dollars? How are the numbers written? Why? What is the answer called? How may the answer be proved? How many places do mills take? Cents? Where is the decimal point placed in the answer? What sign is placed before dollars?

What is subtraction of United States money? How are the numbers written? Which is the subtrahend? Which the minuend? Which the remainder? Can cents be taken from dollars? Mills from cents. Cents from cents? Show at the blackboard how 1,000 mills can be taken from a thousand cents. A thousand mills from a thousand dollars. A thousand cents from a thousand dollars. How may subtraction be proved? What is the opposite of subtraction?

What is multiplication? What do we call the number to be multiplied? Show at the blackboard how 1 dollar 1 cent and 1 mill can be multiplied by 100? Which is the multiplicand? Which the multiplier? Which the product? How should the product be written? The cost of 2 apples is how many times as much as the cost of 1 apple? 20 cows are how many times as many as 1 cow? The cost of 20 cows is how many times as much as the cost of 1 cow? How may multiplication be proved?

What is division? What do we call the number to be divided? What do we call the number by which we divide? What is the answer called? When the dividend is divided into equal parts, of what denomination is the quotient? Is it like, or unlike, the dividend? When the dividend is cents, what is the quotient? When the dividend is mills what is the quotient? When we want to find how many times one sum of money is contained in another, what must first be done? ANS. — *If they are unlike, reduce them to the same denomination.* If the divisor is mills, and the dividend is cents, what must be done? What if the divisor is dollars and cents, and the dividend is dollars? If the divisor is cents and mills, and the dividend is dollars and cents? If the divisor is mills, and the dividend is dollars? If the dividend is cents, and the divisor is dollars? How many times are 6 cents contained in 6 dollars? 20 mills in 20 dollars? 5 dimes in 5 eagles? 1 mill in 1 eagle? 1 dime in 1 dollar? Divide 20 dollars into 5 equal parts? Into 4 equal parts? Into 3 equal parts? ANS. — $6.666+. Why reduce the dollars to mills before dividing? ANS. — *That the remainder may be mills, and therefore of small value.* What does the sign *plus* after the answer show? What is 1 *third* of 30 cents? 1 *third* of 10 mills? What is it customary in business to do with the remainder of mills? ANS. — *When it equals or exceeds* 5, *add* ONE *to the mills; if less, reject it as of no value.* In the final answer, what is done with the mills? ANS. — *When the mills equal or exceed* 5, *add* ONE to the cents; if less, reject them.

# SECTION VII.

## THE GREATEST COMMON DIVISOR.

**Note to the Teacher.** — There are many who prefer to teach compound numbers, and their application to the transactions of the shop, store, and market, *before* common and decimal fractions. This book is so arranged, that no difficulty will be experienced by the teacher or pupil in omitting the latter subjects (Sections VII., VIII., IX.), and in taking them up after denominate numbers.

The practical value of the greatest common divisor and the least common multiple is very slight indeed, as they are seldom used, except in school-books. Cancellation is of more utility; but in business transactions its importance has been overrated.

When 2 and 3 are multiplied together, the product, 6, is said to be a composite number; and the numbers multiplied together are said to be factors of 6. A number that cannot be so produced by the multiplication of factors is said to be a *prime number*, and can have no *divisor;* as, 2, 3, 7, 11, &c. Since 2 and 3, the factors of 6, are prime numbers, they are said to be the *prime factors* of 6.

***70. A Composite Number*** is one produced by the multiplication of two or more factors.*

***71. A Prime Number*** is one that cannot be produced by the multiplication of factors. Numbers are said to be prime to each other when they have no common factor or divisor.

***72. A Factor, or Divisor,*** of a number is one of the numbers which multiplied together will produce it. When the factor is a prime number, it is said to be a ***prime factor.***

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* The words *factor* and *divisor* refer to factors and divisors that are integral only.

## *73.* TO RESOLVE A NUMBER INTO ITS PRIME FACTORS.

PROBLEM. — What are the prime factors of 60?

PROCESS.

$$\begin{array}{r|l} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

$60 = 2 \times 2 \times 3 \times 5$

ANALYSIS. — (1.) By trial, 60 is found to be the product of the prime factor 2, and the composite factor 30.

(2.) 30 is the product of the prime factor 2, and the composite factor 15.

(3.) 15 is the product of the prime factors 3 and 5.

CONCLUSION. — Therefore by trial the prime factors of 60 are found to be $2 \times 2 \times 3 \times 5$.

RULE. — *Divide by the least prime factor, and each quotient by its least prime factor, until a quotient is found which is a prime factor. The several divisors and the last quotient are the factors required.*

What are the prime factors of the following numbers?

1. 24.
2. 36.
3. 48.
4. 144.
5. 120.
6. 136.
7. 145.
8. 225.
9. 796.
10. 576.
11. 256.
12. 344.
13. 864.
14. 945.
15. 320.
16. 500.
17. 965.
18. 7007.
19. 8148.
20. 719.
21. 913.
22. 24024.
23. 645.
24. 840.

The numbers 8 and 12 can both be divided by 2 and 4: hence 2 and 4 are said to be *common factors*, or *common divisors*, of 8 and 12. Since 4 is the greatest number that will divide both 8 and 12, 4 is said to be the *greatest common divisor*, or factor, of 8 and 12: hence,

**74.** ***The Greatest Common Divisor,*** or factor, of two or more numbers is the greatest number that will divide each of them without a remainder.

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*REVIEW.* — Define a unit. (1.) A number. (2.) A concrete number. (3.) A *denominate number*. (4.) An abstract number. (5.)

### TO FIND THE GREATEST COMMON DIVISOR OF TWO OR MORE NUMBERS.

**PROBLEM.**—Find the greatest common divisor of 6, 12, and 18.

PROCESS.

$6 = 3 \times 2$

$12 = 3 \times 2 \times 2$

$18 = 3 \times 2 \times 3$

**ANALYSIS.**—By inspection we find that 3 and 2 are the only common factors, or divisors, of 6, 12, and 18: hence the product of 3 and 2, or 6, is the greatest common divisor of 6, 12, and 18.

**RULE.**—*Resolve the numbers into their prime factors. The product of the factors which are common to all the numbers will be the greatest common divisor.*

Find the greatest common divisors

25. Of 6, 12. Of 8, 12, 16. Of 24, 30, 36.
26. Of 9, 15, 21. Of 16, 20, 28. Of 36, 42, 88.
27. Of 36, 81, 135. Of 39, 65, 104. Of 72, 168.
28. Of 78, 234, 468. Of 16, 60, 72, 96.

When numbers cannot be easily factored, the pupil may use the following

**RULE.**—I. *Divide the greater number by the less: if a remainder appears, divide the preceding divisor by it; and so continue to divide the last divisor by the last remainder, until nothing remains. The last divisor used is the greatest common divisor.*

II. *When there are more than two numbers, find the greatest common divisor of two of them, then of that divisor and one of the other numbers, &c. The last divisor will be the greatest common divisor.*

---

**REVIEW.**—Define notation. (7.) For what is the Roman notation used? (8.) What letters are used? *What is the* value of each? How does repeating a letter *affect its value? How is 9 represented?* How 11?

### EXAMPLES.

29. Find the greatest common divisor of 2219 and 4501.

30. Find the greatest common divisor of 16082 and 25762.

31. Find the greatest common divisor of 1496, 2312, and 10761.

32. Find the greatest common divisor of 3298 and 10761.

## TO FIND THE LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS.

When a number, as 20, can be divided by an integral number, as 5, without a fractional quotient, it is said to be a *multiple* of that number. When a number, as 20, can be so divided by two or more numbers, as by 4 and 5, it is said to be a *common multiple* of those numbers. Since 20 is the least number that can be so divided by 4 and 5, 20 is said to be the *least common multiple* of 4 and 5.

**75. *A Common Multiple*** of two or more numbers is a number which can be divided by each of them without a remainder.

**76. *A Least Common Multiple*** of two or more numbers is the least number which can be divided by each of them without a remainder.

PROBLEM.—What is the least common multiple of 6, 12, and 18?

PROCESS.

$6 = 3 \times 2$

$12 = 3 \times 2 \times 2$

$18 = 3 \times 3 \times 2$

$3 \times 3 \times 2 \times 2 = 36$

ANALYSIS.—Resolve the numbers 6, 12, and 18 into their prime factors. A multiple to be divided by 18 must contain all the factors of 18, or $3 \times 3 \times 2$: to be divided by 12, it must contain also the additional factor 2, not found in 18. All the factors of 6 are found in 12 and 18: hence the factors of the multiple are $3 \times 3 \times 2 \times 2 = 36$. Since 36 is

**the least number divisible by 6, 12, and 18, it is also the least common multiple of these numbers.**

**The following *process* and *rule* have the advantage of brevity.**

PROBLEM. — What is the least common multiple of 24, 16, 15, 20, and 8?

**PROCESS.**

$$\begin{array}{r|lllll} 2 & 24 & 16 & 15 & 20 & \cancel{8}^{*} \\ \hline 2 & 12 & 8 & 15 & 10 & \\ \hline 2 & 6 & 4 & 15 & \cancel{5}^{*} & \\ \hline & \cancel{3}^{*} & 2 & 15 & & \end{array}$$

**$2 \times 2 \times 2 \times 2 \times 15 = 240$ least common multiple.**

RULE. — I. *Write the numbers in a horizontal line, and cancel those which are factors of any of the rest.*

II. *Divide by the smallest number that will divide them without a remainder, and write the quotients and undivided numbers below.*

III. *Cancel and divide as before, until no number will divide two of them without a remainder. The product of the divisors and undivided numbers is the least common multiple.*

Find the least common multiple of

33. Of 10, 6, 18, 15.
34. Of 24, 16, 15, 20.
35. Of 25, 60, 72, 35.
36. Of 63, 12, 84, 72.
37. Of 54, 81, 14, 63.
38. Of 12, 72, 36, 144.
39. Of 1, 2, 3, 4, 5, 6, 7, 8, 9.
40. Of 8, 9, 55, 49.
41. Of 720, 336, 1,736.
42. Of 8, 12, 16, 24, 36, 48, 72, 144.
43. Of 3, 7, 5, 9, 186.
44. Of 5, 3, 7, 9, 91.
45. Of 4, 3, 6, 7, 8, 16, 19.
46. Of 4, 8, 12, 24, 48, 84.

---

* *See process.* Cancel the 8 because it is a factor of 24, also 5 and 3 because *they are factors of 15.*

## CANCELLATION.

**77. *Cancellation.*** When one number is to be divided by another, the process may often be shortened by striking out factors common to both divisor and dividend. This process is called *cancellation.*

PROBLEM. — Divide 28 by 14.

PROCESS.

| Dividend. | Divisor. | Quotient. |
|---|---|---|
| 28 ÷ | 14 = | 2 |

$(\cancel{7}\times\cancel{2}\times 2)\div(\cancel{7}\times\cancel{2})=2$

ANALYSIS. — Since the dividend, 28, is composed of the factors 7 × 2 × 2, and the divisor, 14, is composed of the factors, 7 × 2, if the factors 7 × 2 in the dividend are cancelled, and the equal factors, 7 × 2, in the divisor, are cancelled, the quotient of the remaining factor will still be 2: hence,

**78.** PRINCIPLES. — I. *Cancelling a factor of a number divides the number by that factor.*

II. *Cancelling equal factors in both numbers will not change the quotient.*

PROBLEM. — Divide 11 × 3 × 4 by 11 × 6.

PROCESS.

$$\begin{array}{l} \text{Dividend} \\ \text{Divisor} \end{array} \frac{\cancel{11}\times\cancel{3}\times\overset{2}{\cancel{4}}}{\cancel{11}\times\underset{\cancel{2}}{\cancel{6}}}=2 \text{ Quotient.}$$

RULE. — *Write the dividend over the divisor, and cancel all common factors.*

47. Divide 48 × 3 × 4 by 8 × 7 × 5.
48. Divide 40 × 10 × 3 by 5 × 6 × 2.
49. Divide 25 × 7 × 19 by 5 × 48 × 2.
50. Divide 21 × 21 × 25 by 7 × 5 × 5.
51. Divide 4 × 13 × 8 by 2 × 6.
52. Divide 81 × 8 × 3 by 2 × 27.
*53.* *(6 × 3* × 4 × 9) ÷ (4 × 3 × 8) = what?

54. $(7 \times 4 \times 8 \times 2) \div (3 \times 4 \times 2) =$ what?

55. $144 \div (3 \times 4 \times 2) =$ what?

56. $(5 \times 8 \times 9) \div 120 =$ what?

57. $\frac{6 \times 8 \times 4 \times 9 \times 3}{324} =$ what?

58. A farmer sells 45 bushels of apples worth 40 cents a bushel, and receives in payment 20 pounds of tea. What is the tea worth per pound?

59. How many tons of hay, at $16 per ton, must be given for 12 barrels of flour, at $8 per barrel?

60. A farmer exchanged 9 tubs of butter, each containing 56 pounds, worth 25 cents per pound, for 4 chests of tea containing 42 pounds each. What was the price per pound of the tea?

NOTE.—Additional examples should be dictated by the teacher, or placed on the blackboard to be copied, and solved, at seats.

## ANALYSIS OF SECTION VII.

### I. Composite Numbers.

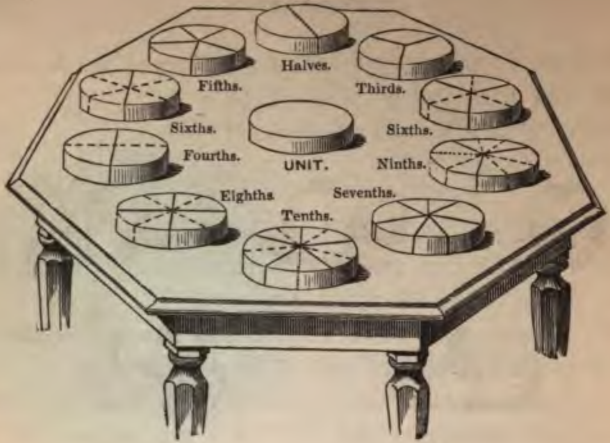
1. ILLUSTRATION.
2. PRIME NUMBERS.
3. PRIME FACTORS.
4. COMPOSITE NUMBERS.
5. GREATEST COMMON DIVISOR.
   (*a.*) *Definition.*
   (*b.*) *Problems.*
   (*c.*) *Analysis.*
   (*d.*) *Rule.*

### II. Least Common Multiple.

1. ILLUSTRATION.
2. DEFINITIONS.
3. PROBLEMS.
   (*a.*) *Analysis.*
   (*b.*) *Rule.*

### III. Cancellation.

1. ILLUSTRATION.
2. PRINCIPLES.
3. PROBLEMS.
   (*a.*) *Analysis.*
   (*b.*) *Rule.*



## SECTION VIII.

### FRACTIONS.

1. WHAT is meant by halves? By fifths? By thirds? By eighths? By ninths? By sevenths?

2. How are halves obtained? How thirds? How fifths? How sevenths?

3. In how many ways are sixths obtained?

ANS. — *By dividing a unit into six equal parts, or dividing halves each into* 3 *equal parts, or thirds each into* 2 *equal parts.*

How can quarters be obtained? eighths? tenths?

4. Point out the unit in the picture, halves, thirds, fifths, eighths, tenths, ninths, fourths, sevenths.

5. Into how many equal parts must a unit be divided to make halves? thirds? fifths? tenths? eighths, &c.?

6. How do halves compare in size with thirds? Halves with fourths? Thirds with sixths? Fifths with tenths? Ninths with sevenths? Fourths with eighths?

7. How is the size of the parts affected by cutting the unit into a larger number of equal parts? By cutting into a less number of equal parts?

One.

2 halves ($\frac{2}{2}$).

3 thirds ($\frac{3}{3}$).

4 fourths ($\frac{4}{4}$).

5 fifths ($\frac{5}{5}$).

8. When an apple is divided into two equal parts, what are the parts called? What is one of the parts called? What is each of the parts called?

9. When a pear is divided into three equal parts, what are the parts called? What is any *one* of the parts called?

10. When an orange is divided into four equal parts, what are the parts called? What is each of the parts called?

11. Three thirds of a pear are equal to how many whole pears? Five fifths of a melon are equal to what?

When any thing, or number, is divided into equal parts, one or more of these equal parts is called a *fraction:* one of these equal parts is sometimes called a *fractional unit.*

That which shows the number of equal parts into which a thing, or number, is divided is called the *denominator*, and shows the *size* of the parts.

1 half ($\frac{1}{2}$)

1 third ($\frac{1}{3}$).

2 fourths ($\frac{2}{4}$).

3 fifths ($\frac{3}{5}$).

One of the equal parts of a thing, or number, is called *one-half*, or *one-third*, or *one-fourth*, &c. Two or more of the equal parts are called *two-thirds, two-fourths, three-fifths*, &c.

That which shows *how many* of the equal parts are expressed is called the *numerator.*

When all the parts represented are *less* than a single thing, or unit, the expression is said to be a *proper fraction ;* as, 1 third, 2 fourths, 3 fifths. When all the parts represented equal, or exceed, a single thing, the expression is said to be an improper fraction; as, 5 halves, 3 thirds. When one or more single things, and one or more parts of a single thing, are used together, the expression is called a *mixed number ;* as, $2\frac{1}{2}$ apples.

**79. *A Fraction*** is one or more equal parts of a unit, as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{8}$.

**80. *The Denominator*** shows into how many equal parts the thing, or number, is divided ; as, in $\frac{3}{4}$, the denominator 4 shows that the unit is divided into four equal parts.

EXERCISE. — Write twenty fractions, point out the denominator of each, and tell its use.

**81. *The Numerator*** shows how many of the equal parts are included in the fraction ; as, in $\frac{3}{5}$, the numerator 3 denotes that three of the five equal parts are expressed by the fraction.

EXERCISE. — Write twenty fractions, point out the numerator of each, and tell its use.

**82. *A Proper Fraction*** is one whose value is less than a unit ; as, $\frac{2}{3}$, $\frac{5}{8}$, $\frac{3}{9}$.

*EXERCISE.* — Write twenty proper fractions, and tell why so called.

**83. *An Improper Fraction*** is one whose value is equal to, or greater than, a unit; as, $\frac{4}{3}$, $\frac{17}{13}$, $\frac{5}{5}$.

EXERCISE. — Write twenty improper fractions, and tell why so called.

**84. *A Mixed Number*** consists of an integer and a fraction; as, $2\frac{1}{2}$, $5\frac{7}{8}$. The sign $+$ is understood between the integer and the fraction.

EXERCISE. — Write twenty mixed numbers, and tell why so called.

**85.** PRINCIPLES. — I. *The smaller the number of equal parts into which the unit is divided, the greater is the size of each part.*

II. *The greater the number of equal parts into which a unit is divided, the smaller is the size of each part.*

Fractions may be written in words; as, four-tenths, five-sevenths; or by figures, thus, $\frac{3}{7}$, $\frac{4}{11}$, $\frac{5}{7}\begin{matrix}\text{Numerator.}\\\text{Denominator.}\end{matrix}$

### ORAL EXERCISES.

1. When an apple is divided into 3 equal parts, what are the parts called? When divided into 4 equal parts? When into 7?

2. When an orange is divided into 8 equal parts, what are the parts called? When divided into 4 equal parts? 11 parts? 9 parts? 6 parts?

3. When a peach is divided into 2 equal parts, what is each one of the parts called? When divided into 5 equal parts? Into 3 equal parts? Into 7 equal parts? Into 11 equal parts?

4. When a melon is divided into 5 equal parts, what is 1 of the parts called? What are 2 parts called? 3 parts? 5 parts?

5. When a pie is cut into 8 equal parts, what are 2 parts called? 6 parts? 3 parts? 1 part? 7 parts? 4 parts?

6. What is meant by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{6}$, &c.?

ANS. — *By one-half is meant* ONE *of the* TWO *equal parts into which a thing, or number, is divided; one-third is* ONE *of the* THREE *equal parts; one-fifth is* ONE *of the* FIVE *equal parts; &c.*

7. What is meant by $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{9}$, &c.?

ANS. — *By two-thirds is meant* TWO *of the* THREE *equal parts into which a unit, or number of units, is divided. The term three-fifths means* THREE *of the* FIVE *equal parts into which a unit, or number of units, is divided, &c.*

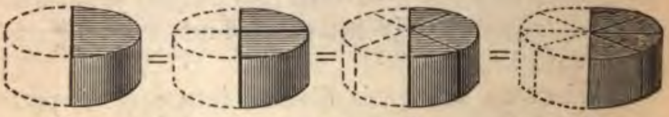
NOTE. — When no definite number of units is expressed, *one unit is always understood.*

8. When a melon is divided into six equal parts, what is one of the parts called?

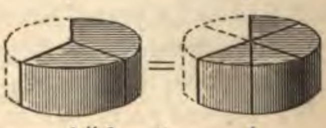
ANS. — One-sixth of a melon.

## REDUCTION OF FRACTIONS TO HIGHER TERMS.

### ILLUSTRATIONS.



$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$



$$\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

From the above illustrations is deduced the following: —

**86.** PRINCIPLE. — *Multiplying both terms of a fraction by the same number does not change its value.*

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

Write a series of ten equivalent fractions from each of the following, to be read as a class-exercise.

PROCESS.

$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}=\frac{7}{14}=\frac{8}{16}=\frac{9}{18}$, &c.
$\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}=\frac{6}{18}=\frac{7}{21}=\frac{8}{24}$, &c.
$\frac{2}{3}=\frac{4}{6}=\frac{6}{9}=\frac{8}{12}=\frac{10}{15}=\frac{12}{18}=\frac{14}{21}=\frac{16}{24}$, &c.
$\frac{3}{5}=\frac{6}{10}=\frac{9}{15}=\frac{12}{20}=\frac{15}{25}=\frac{18}{30}$, &c.

| (1.) | (2.) | (3.) | (4.) | (5.) | (6.) | (7.) | (8.) | (9.) | (10.) | (11.) | (12.) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{2}$ | $\frac{1}{7}$ | $\frac{1}{12}$ | $\frac{2}{11}$ | $\frac{3}{10}$ | $\frac{4}{11}$ | $\frac{5}{11}$ | $\frac{7}{9}$ | $\frac{8}{11}$ | $\frac{1}{13}$ | $\frac{6}{13}$ | $\frac{11}{13}$ |
| $\frac{1}{3}$ | $\frac{1}{8}$ | $\frac{2}{3}$ | $\frac{3}{4}$ | $\frac{3}{11}$ | $\frac{5}{6}$ | $\frac{5}{12}$ | $\frac{7}{10}$ | $\frac{9}{10}$ | $\frac{2}{13}$ | $\frac{7}{13}$ | $\frac{12}{13}$ |
| $\frac{1}{4}$ | $\frac{1}{9}$ | $\frac{2}{5}$ | $\frac{3}{5}$ | $\frac{4}{5}$ | $\frac{5}{7}$ | $\frac{6}{7}$ | $\frac{7}{11}$ | $\frac{9}{11}$ | $\frac{3}{13}$ | $\frac{8}{13}$ | $\frac{1}{14}$ |
| $\frac{1}{5}$ | $\frac{1}{10}$ | $\frac{2}{7}$ | $\frac{3}{7}$ | $\frac{4}{7}$ | $\frac{5}{8}$ | $\frac{6}{11}$ | $\frac{7}{12}$ | $\frac{10}{11}$ | $\frac{4}{13}$ | $\frac{9}{13}$ | $\frac{3}{14}$ |
| $\frac{1}{6}$ | $\frac{1}{11}$ | $\frac{2}{9}$ | $\frac{3}{8}$ | $\frac{4}{9}$ | $\frac{5}{9}$ | $\frac{7}{8}$ | $\frac{8}{9}$ | $\frac{11}{12}$ | $\frac{5}{13}$ | $\frac{10}{13}$ | $\frac{5}{14}$ |

**Problem.**—Change $\frac{5}{6}$ to an equivalent fraction of the denomination of 30ths.

PROCESS.

$\frac{5\times5}{6\times5}=\frac{25}{30}$

**Analysis.**—Since $\frac{1}{6}$ of one unit is equal to $\frac{5}{30}$, $\frac{5}{6}$ of a unit will equal five times as many thirtieths, or $\frac{25}{30}$.

In the same manner, reduce and analyze each of the following:—

1. $\frac{4}{7}$ to 21sts.
2. $\frac{5}{8}$ to 40ths.
3. $\frac{6}{7}$ to 35ths.
4. $\frac{2}{11}$ to 22ds.
5. $\frac{4}{13}$ to 26ths.
6. $\frac{3}{4}$ to 36ths.
7. $\frac{3}{5}$ to 60ths.
8. $\frac{4}{7}$ to 63ds.
9. $\frac{5}{9}$ to 72ds.

---

**Review.**—For what is the Arabic notation used? (9.) How many figures are there? What is the value of each? What is the use of the cipher? Which are called *digits? What is the simple* value of a figure? (10.) What the local value? (11.)

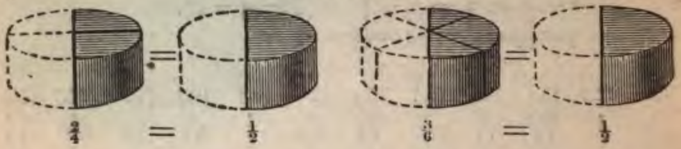
## TO REDUCE A FRACTION TO ITS LOWEST TERMS.

**87.** A *fraction* is at its *lowest terms* when its numerator and denominator are prime to each other.

### ORAL AND WRITTEN EXERCISES.

How many halves in $\frac{2}{4}$? In $\frac{3}{6}$?

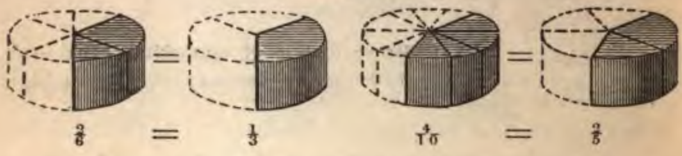
ILLUSTRATION.



1. How many halves in $\frac{3}{6}$? In $\frac{4}{8}$? In $\frac{8}{16}$? In $\frac{25}{50}$? In $\frac{6}{12}$? In $\frac{3}{6}$? In $\frac{12}{24}$? In $\frac{5}{10}$? In $\frac{7}{14}$? *W.* In $\frac{17}{34}$? In $\frac{11}{22}$? In $\frac{71}{142}$? In $\frac{95}{190}$? In $\frac{62}{124}$?

2. Reduce $\frac{2}{6}$ and $\frac{4}{10}$ to their lowest terms.

ILLUSTRATION.



**Analysis.**—Since every 2 *tenths* equals 1 *fifth*, in 4 *tenths* there are as many *fifths* as 2 *tenths* are contained times in 4 *tenths*, or $\frac{2}{5}$.

3. Reduce $\frac{5}{15}$ to its lowest terms. $\frac{3}{9}$. $\frac{6}{18}$. $\frac{4}{12}$. $\frac{7}{21}$. $\frac{11}{33}$. $\frac{14}{21}$. $\frac{16}{32}$. $\frac{4}{12}$. *W.* $\frac{21}{63}$. $\frac{15}{45}$. $\frac{31}{93}$. $\frac{48}{144}$. $\frac{16}{48}$. $\frac{24}{72}$.

---

**Review.**—What is numeration? (12.) Repeat the table from right to left. From left to right by periods. How do numbers increase? (13.) What is meant by tenfold ratio? What is a rule? (14.) Give the rule for notation. (15.) Give the rule for numeration. (16.)

**PROBLEM.** — (*a.*) How many fifths in $\frac{4}{10}$? (*b.*) In $\frac{24}{60}$?

**FORMULA.** — *b.* $\frac{24}{60} \div \frac{12}{12} = \frac{2}{5}$; or,

$12)\frac{24}{60} = \frac{2}{5}$.

(*b.*) **ANALYSIS.** — Since every 12 sixtieths is equal to 1 fifth, in 24 sixtieths, there are as many *fifths* as 12 sixtieths are contained times in 24 sixtieths, or 2 *fifths*, ($\frac{2}{5}$).

From the above is deduced the following: —

*88.* **PRINCIPLE.** — *Dividing both terms of a fraction by the same number does not change its value.*

To reduce a fraction to its lowest terms, use the following: —

**RULE.** — *Divide the numerator and the denominator of the fraction by any number that will divide them both without a remainder. Continue dividing in this way until no number greater than* ONE *will divide them.*

4. How many fourths in $\frac{6}{8}$? In $\frac{4}{16}$? *W.* In $\frac{54}{135}$?

**PROCESS.** — $3)\frac{54}{135}$ $3)\frac{18}{45}$ $3)\frac{6}{15}$ $\frac{2}{5}$.

5. Change $\frac{8}{12}$ to thirds. $\frac{4}{14}$ to sevenths. *W.* $\frac{18}{45}$ to fifths. $\frac{24}{36}$ to thirds.

6. Reduce $\frac{7}{63}$ to ninths. $\frac{4}{10}$ to fifths. *W.* $\frac{13}{91}$ to sevenths. $\frac{24}{84}$ to sevenths.

### ORAL, SLATE, AND BLACKBOARD EXERCISES.*

Reduce each of the following fractions to its lowest terms, to be read as a class-exercise.

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) | (6.) | (7.) |
|---|---|---|---|---|---|---|---|
| $\frac{2}{4} = \frac{1}{2}$ | $\frac{5}{10}$ | $\frac{4}{6}$ | $\frac{14}{77}$ | $\frac{21}{28}$ | $\frac{20}{25}$ | $\frac{10}{12}$ | $\frac{18}{21}$ |
| $\frac{2}{8} = \frac{1}{4}$ | $\frac{8}{16}$ | $\frac{6}{9}$ | $\frac{16}{88}$ | $\frac{21}{49}$ | $\frac{24}{30}$ | $\frac{15}{18}$ | $\frac{28}{32}$ |
| $\frac{4}{6} = \frac{2}{3}$ | $\frac{7}{14}$ | $\frac{8}{12}$ | $\frac{4}{26}$ | $\frac{15}{50}$ | $\frac{36}{45}$ | $\frac{20}{28}$ | $\frac{14}{84}$ |
| $\frac{9}{12} = \frac{3}{4}$ | $\frac{11}{33}$ | $\frac{6}{21}$ | $\frac{6}{8}$ | $\frac{27}{99}$ | $\frac{16}{36}$ | $\frac{15}{27}$ | $\frac{42}{66}$ |
| $\frac{12}{16} = \frac{3}{4}$ | $\frac{14}{42}$ | $\frac{16}{56}$ | $\frac{15}{20}$ | $\frac{24}{104}$ | $\frac{20}{45}$ | $\frac{20}{44}$ | $\frac{56}{96}$ |
| $\frac{10}{15} = \frac{2}{3}$ | $\frac{9}{45}$ | $\frac{18}{81}$ | $\frac{24}{32}$ | $\frac{6}{26}$ | $\frac{24}{42}$ | $\frac{10}{26}$ | $\frac{14}{26}$ |

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* Since the fractions used in business transactions are few, and the terms of most are small, it is of great importance that the pupils should be able to recognize the lowest terms of each at a glance: hence the large number of these exercises.

**Suggestion.** — The greatest common divisor can be readily discovered by finding what part of the numerator is contained in the denominator; as, the whole, a half, a third, a fourth, &c.

The following suggestions are valuable in discovering the divisor of a number: —

1. Every even number is divisible by 2; as, 384, 672.
2. Every number ending with 5 or a zero is divisible by 5; as, 305, 240.
3. If the sum of the digits of any number is divisible by 3, the number itself is divisible by 3; as, 612, 822.

7. What are the lowest terms of $\frac{4}{8}$? Of $\frac{5}{10}$? Of $\frac{6}{14}$? Of $\frac{7}{21}$? Of $\frac{8}{10}$? Of $\frac{9}{12}$? *W.* Of $\frac{64}{72}$? Of $\frac{27}{36}$? Of $\frac{29}{232}$? Of $\frac{31}{279}$?

8. What are the lowest terms of $\frac{9}{12}$? Of $\frac{8}{16}$? Of $\frac{21}{28}$? Of $\frac{16}{48}$? Of $\frac{25}{30}$? Of $\frac{22}{27}$? Of $\frac{88}{99}$? *W.* Of $\frac{63}{105}$? Of $\frac{121}{132}$? Of $\frac{100}{120}$?

9. What are the lowest terms of $\frac{56}{24}$? Of $\frac{81}{27}$? Of $\frac{120}{180}$? Of $\frac{48}{96}$? Of $\frac{36}{84}$? Of $\frac{45}{90}$? *W.* Of $\frac{84}{132}$? Of $\frac{121}{220}$? Of $\frac{750}{890}$?

10. What are the lowest terms of $\frac{7}{14}$? $\frac{25}{100}$? $\frac{20}{100}$? $\frac{40}{100}$? $\frac{45}{100}$? $\frac{75}{100}$? $\frac{15}{100}$? *W.* $\frac{30}{100}$? $\frac{35}{100}$? $\frac{132}{240}$? $\frac{220}{1230}$?

11. What are the lowest terms of $\frac{38}{100}$? $\frac{4}{100}$? $\frac{5}{100}$? $\frac{6}{100}$? $\frac{8}{100}$? $\frac{12}{100}$? $\frac{14}{100}$? $\frac{16}{100}$? *W.* $\frac{18}{100}$? $\frac{88}{100}$? $\frac{55}{200}$? $\frac{128}{448}$? $\frac{81}{108}$? $\frac{678}{321}$?

12. What are the lowest terms of $\frac{46}{100}$? $\frac{54}{100}$? $\frac{64}{100}$? $\frac{22}{100}$? $\frac{24}{100}$? $\frac{26}{100}$? *W.* $\frac{75}{100}$? $\frac{88}{100}$? $\frac{95}{100}$? $\frac{75}{100}$? $\frac{402}{807}$? $\frac{837}{534}$? $\frac{3003}{5005}$?

ALWAYS CHANGE FRACTIONS TO THE LOWEST TERMS PERMITTED BY THE CONDITIONS OF THE WORK.

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**Review.** — What is addition? (17.) What is the sum of two or more numbers? (18.) Describe the sign of addition. (19.) Show its use. Describe the sign of equality. (20.) Show its use. Describe the dollar-sign. (21.)

### *89.* TO CHANGE INTEGRAL OR MIXED NUMBERS TO IMPROPER FRACTIONS.

**PROBLEM.**—How many quarters in $2\frac{3}{4}$ apples? *W.* In $37\frac{1}{4}$ oranges?

**WRITTEN PROCESS.**

$$\begin{array}{r} 37\frac{1}{4} \\ 4 \\ \hline 149 \\ \hline 4 \end{array}$$

**ANALYSIS.**—Since in 1 apple there are 4 quarters, in 2 apples there are 2 times 4 quarters, or 8 quarters, and 8 quarters and 3 quarters are 11 quarters: hence $2\frac{3}{4} = \frac{11}{4}$.

**RULE.**—*Multiply the integral number by the denominator of the fraction, and to the product add the numerator, and write the result over the denominator.*

#### ORAL, SLATE, AND BLACKBOARD EXERCISES.

Write the corresponding improper fraction for each of the following mixed numbers.

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) | (6.) |
|---|---|---|---|---|---|---|
| $3\frac{1}{4} = \frac{13}{4}$ | $4\frac{3}{7}$ | $4\frac{3}{4}$ | $6\frac{3}{4}$ | $14\frac{3}{2}$ | $5\frac{3}{7}$ | $7\frac{2}{3}$ |
| $4\frac{1}{7} = \frac{29}{7}$ | $6\frac{7}{8}$ | $8\frac{6}{7}$ | $4\frac{5}{9}$ | $11\frac{4}{7}$ | $2\frac{5}{4}$ | $8\frac{6}{7}$ |
| $3\frac{1}{8} = \frac{25}{8}$ | $5\frac{5}{7}$ | $5\frac{8}{9}$ | $7\frac{5}{8}$ | $16\frac{3}{7}$ | $6\frac{3}{17}$ | $9\frac{5}{11}$ |
| $5\frac{1}{9} = \frac{46}{9}$ | $3\frac{3}{8}$ | $4\frac{3}{7}$ | $8\frac{6}{7}$ | $14\frac{1}{6}$ | $8\frac{6}{13}$ | $8\frac{17}{14}$ |
| $5\frac{4}{7} = \frac{39}{7}$ | $7\frac{1}{8}$ | $6\frac{6}{7}$ | $13\frac{1}{4}$ | $13\frac{5}{8}$ | $4\frac{7}{11}$ | $6\frac{12}{17}$ |
| $4\frac{5}{8} = \frac{37}{8}$ | $6\frac{8}{9}$ | $5\frac{3}{7}$ | $17\frac{2}{7}$ | $6\frac{7}{8}$ | $8\frac{3}{16}$ | $5\frac{2}{7}$ |

#### WRITTEN EXAMPLES.

Change each of the following numbers to fractions.

| | | | |
|---|---|---|---|
| 13. $42\frac{3}{4}$. | 17. $59\frac{7}{9}$. | 21. $413\frac{1}{2}$. | 25. $917\frac{5}{11}$. |
| 14. $28\frac{5}{7}$. | 18. $46\frac{3}{4}$. | 22. $682\frac{5}{8}$. | 26. $763\frac{9}{17}$. |
| 15. $91\frac{2}{3}$. | 19. $876\frac{4}{5}$. | 23. $83\frac{5}{9}$. | 27. $412\frac{18}{79}$. |
| 16. $67\frac{4}{9}$. | 20. $86\frac{2}{9}$. | 24. $413\frac{4}{13}$. | |

**NOTE.**—For additional examples, see drill-card exercises.

## *90.* TO CHANGE IMPROPER FRACTIONS TO INTEGRAL OR MIXED NUMBERS.

**PROBLEM.** — How many melons in 15 halves?

**FORMULA.** $\frac{15}{2} = 7\frac{1}{2}$.

**ANALYSIS.** — Since in 2 halves there is 1 melon, in 15 halves there are as many melons as 2 halves are contained times in 15 halves; or 7 with 1 half remaining: hence $\frac{15}{2} = 7\frac{1}{2}$.

**RULE.** — *Divide the numerator by the denominator.*

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

Write corresponding integral or mixed numbers for each of the following improper fractions.

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) | (6.) | (7.) |
|---|---|---|---|---|---|---|---|
| $\frac{43}{7} = 6\frac{1}{7}$ | $\frac{46}{7}$ | $\frac{47}{8}$ | $\frac{56}{3}$ | $\frac{49}{21}$ | $\frac{46}{37}$ | $\frac{59}{72}$ | $\frac{43}{6}$ |
| $\frac{27}{3} = 9$ | $\frac{37}{2}$ | $\frac{38}{6}$ | $\frac{72}{3}$ | $\frac{73}{16}$ | $\frac{97}{81}$ | $\frac{41}{36}$ | $\frac{94}{6}$ |
| $\frac{63}{4} = 15\frac{3}{4}$ | $\frac{46}{9}$ | $\frac{97}{6}$ | $\frac{86}{7}$ | $\frac{42}{13}$ | $\frac{143}{67}$ | $\frac{138}{71}$ | $\frac{83}{21}$ |
| $\frac{47}{4} = 11\frac{3}{4}$ | $\frac{86}{8}$ | $\frac{97}{9}$ | $\frac{96}{11}$ | $\frac{72}{16}$ | $\frac{48}{31}$ | $\frac{68}{3}$ | $\frac{47}{3}$ |
| $\frac{68}{5} = 13\frac{3}{5}$ | $\frac{47}{3}$ | $\frac{57}{3}$ | $\frac{57}{9}$ | $\frac{83}{18}$ | $\frac{67}{32}$ | $\frac{48}{21}$ | $\frac{168}{82}$ |
| $\frac{47}{13} = 3\frac{8}{13}$ | $\frac{67}{9}$ | $\frac{67}{2}$ | $\frac{68}{3}$ | $\frac{89}{17}$ | $\frac{47}{36}$ | $\frac{67}{28}$ | $\frac{46}{3}$ |

### WRITTEN EXAMPLES.

Change the following fractions to integral or mixed numbers.

28. $\frac{683}{4}$, $\frac{213}{4}$.
29. $\frac{678}{4}$, $\frac{897}{4}$.
30. $\frac{896}{5}$, $\frac{871}{5}$.
31. $\frac{680}{5}$, $\frac{413}{5}$.
32. $\frac{6713}{6}$, $\frac{312}{4}$.
33. $\frac{4913}{7}$, $\frac{317}{5}$.
34. $\frac{9836}{8}$.
35. $\frac{4163}{8}$.
36. $\frac{7888}{11}$.
37. $\frac{83764}{10}$.
38. $\frac{4873}{16}$.
39. $\frac{2839}{22}$.
40. $\frac{61381}{37}$.
41. $\frac{2134}{67}$.
42. $\frac{4134}{882}$.
43. $\frac{6713}{42}$.
44. $\frac{789}{681}$.
45. $\frac{567}{567}$.
46. $\frac{697}{897}$.
47. $\frac{413}{413}$.
48. $\frac{8643}{241}$.
49. $\frac{6783}{874}$.
50. $\frac{637}{83}$.
51. $\frac{6913}{92}$.

For additional examples, see drill-card exercises.

## ORAL REVIEW.

1. How many thirds in 1? In 2? In 4?

**Analysis.**—Since there are 3 thirds in 1, in 4 there are 4 times 3 thirds, or $\frac{12}{3}$.

2. How many quarters in 2 melons? How many fourths? How many thirds? Sixths? Elevenths?

3. How many quarters in 2 yards? In $2\frac{1}{4}$ yards? In $2\frac{3}{4}$ yards? In $6\frac{3}{4}$ yards?

4. How many units in $\frac{3}{3}$? In $\frac{5}{5}$? In $\frac{7}{7}$? In $\frac{6}{3}$? In $\frac{12}{3}$? In $\frac{18}{3}$? In $\frac{10}{5}$? In $\frac{21}{7}$?

5. How many apples in $\frac{8}{4}$? In $\frac{16}{4}$? In $\frac{5}{4}$? In $\frac{4}{4}$? In $\frac{7}{4}$?

6. How many ones in $\frac{8}{8}$? In $\frac{7}{7}$? In $\frac{14}{7}$? In $\frac{18}{9}$?

7. How many times 1 are $\frac{13}{3}$?

**Analysis.**—Since 3 thirds equal 1, 13 thirds will equal as many times 1 as 3 thirds are contained times in 13 thirds, or $4\frac{1}{3}$.

8. How many yards in $\frac{14}{7}$ of a yard? In $\frac{21}{8}$? In $\frac{16}{7}$?

9. How many fifths in 3 apples? In $6\frac{1}{5}$ apples? In $3\frac{4}{5}$ oranges? In $8\frac{1}{5}$?

10. How many halves in 1 apple? How many quarters in $\frac{1}{2}$ of an apple? How many eighths in $\frac{1}{4}$ of an apple?

11. In $\frac{1}{2}$ of an apple, how many eighths? How many fourths? How many sixteenths? Eighteenths?

12. In $3\frac{3}{4}$ how many quarters? Eighths? Twelfths? Sixteenths? Twentieths? Twenty-fourths?

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**Review.**—What is subtraction? (31) Name the terms of subtraction. (32, 33, 34.) Describe the sign of subtraction. (35.) Show its use. Give the axioms used in subtraction (36.) Analyze a short example in subtraction. (37.) By a different method. (38) What is the use of a parenthesis? (39.)

## DRILL CARD EXERCISES. — TABLE IV.

**Note.** — The teacher should select such portions of these drill exercises as are adapted to the wants of the class.

### REDUCTION OF FRACTIONS.

**Exercise I.** — Write a series of ten fractions, each equivalent to the fractions in column **V** (see card); thus, A. $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$, &c. B. $\frac{1}{5}=\frac{2}{10}=\frac{3}{15}=\frac{4}{20}$, &c. C. $\frac{1}{8}=\frac{2}{16}=\frac{3}{24}$, &c.

Ex. II. — In the same way write ten fractions, each equivalent to the fractions in column **W**.

Ex. III. — Write ten fractions, each equivalent to the fractions in column **X**.

Ex. IV. — Prefix to each fraction the integral number opposite expressed in column **1**, and change the resulting mixed number to an improper fraction; thus,

| Column V. | Column W. | Column X. |
|---|---|---|
| A. $1\frac{1}{2}=$ what? | A. $1\frac{1}{3}=$ what? | A. $1\frac{1}{4}=$ what? |
| B. $6\frac{1}{5}=$ what? | B. $6\frac{1}{6}=$ what? | B. $6\frac{1}{7}=$ what? |
| C. $8\frac{1}{8}=$ what? | C. $8\frac{1}{9}=$ what? | C. $8\frac{1}{10}=$ what? |
| D. $9\frac{1}{11}=$ what? | D. $9\frac{1}{12}=$ what? | D. $9\frac{1}{13}=$ what? |
| &c. | &c. | &c. |

Ex. V. — Prefix to each fraction the integral numbers opposite expressed in column **2**, and change the resulting mixed numbers to improper fractions; thus,

| Column V. | Column W. | Column X. |
|---|---|---|
| A. $38\frac{1}{2}=$ what? | A. $38\frac{1}{3}=$ what? | A. $38\frac{1}{4}=$ what? |
| B. $59\frac{1}{5}=$ what? | B. $59\frac{1}{6}=$ what? | B. $59\frac{1}{7}=$ what? |

Ex. VI. — Prefix the integral number expressed in column **3** to each of the fractions as before, and change to improper fractions.

Ex. VII. — In the place of each numerator in each of the columns, substitute one five times as large, and change to integral or mixed numbers when possible; thus,

| Column V. | Column W. | Column X. |
|---|---|---|
| A. $\frac{5}{2}=$ what? | A. $\frac{5}{3}=$ what? | A. $\frac{5}{4}=$ what? |
| B. $\frac{5}{5}=$ what? | B. $\frac{5}{6}=$ what? | B. $\frac{5}{7}=$ what? |
| C. $\frac{5}{8}=$ what? | C. $\frac{5}{9}=$ what? | C. $\frac{5}{10}=$ what? |

**Note.** — The fractions should always be given in the lowest terms.

Ex. VIII. — In the place of each numerator in each of the columns, substitute one ten times as large, and reduce as before.

Ex. IX. — In the place of each numerator, in each of the columns, substitute one twenty-five times as large, and reduce as before.

## FELTER'S
## ARITHMETICAL DRILL CARDS.

| | TABLE IV. | | | | | | TABLE V. | |
|---|---|---|---|---|---|---|---|---|
| | FRACTIONS. | | | | | | UNITS.<br>Dec. Point.<br>10ths.<br>100ths.<br>1000ths.<br>10000ths.<br>100000ths.<br>1000000ths. | |
| | 3 | 2 | 1 | V | W | X | 5.834709 | |
| A | 472 | 38 | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{3}{1}$ | A |
| B | 604 | 59 | 6 | $\frac{1}{5}$ | $\frac{1}{6}$ | $\frac{1}{7}$ | $\frac{50}{1}$ | B |
| C | 103 | 16 | 8 | $\frac{1}{8}$ | $\frac{1}{9}$ | $\frac{1}{10}$ | $\frac{9}{10}$ | C |
| D | 504 | 20 | 9 | $\frac{1}{11}$ | $\frac{1}{12}$ | $\frac{1}{13}$ | $\frac{20}{10}$ | D |
| E | 300 | 47 | 4 | $\frac{2}{3}$ | $\frac{2}{5}$ | $\frac{2}{7}$ | $\frac{63}{10}$ | E |
| F | 412 | 32 | 1 | $\frac{2}{9}$ | $\frac{2}{11}$ | $\frac{2}{13}$ | $\frac{25}{100}$ | F |
| G | 755 | 93 | 3 | $\frac{3}{4}$ | $\frac{3}{5}$ | $\frac{3}{7}$ | $\frac{300}{100}$ | G |
| H | 411 | 51 | 7 | $\frac{3}{8}$ | $\frac{3}{10}$ | $\frac{3}{11}$ | $\frac{208}{100}$ | H |
| I | 640 | 80 | 6 | $\frac{4}{5}$ | $\frac{4}{7}$ | $\frac{4}{9}$ | $\frac{732}{1000}$ | I |
| K | 832 | 21 | 8 | $\frac{4}{11}$ | $\frac{4}{13}$ | $\frac{4}{15}$ | $\frac{8000}{1000}$ | K |
| L | 253 | 73 | 5 | $\frac{5}{6}$ | $\frac{5}{7}$ | $\frac{5}{8}$ | $\frac{8021}{1000}$ | L |
| M | 414 | 86 | 2 | $\frac{5}{9}$ | $\frac{5}{11}$ | $\frac{5}{12}$ | $\frac{2143}{10000}$ | M |
| N | 674 | 13 | 1 | $\frac{6}{7}$ | $\frac{6}{11}$ | $\frac{6}{13}$ | $\frac{105}{10}$ | N |
| O | 305 | 14 | 8 | $\frac{7}{8}$ | $\frac{7}{9}$ | $\frac{7}{10}$ | $\frac{6134}{100}$ | O |
| P | 501 | 73 | 3 | $\frac{7}{11}$ | $\frac{7}{12}$ | $\frac{7}{13}$ | $\frac{43}{1000}$ | P |
| Q | 244 | 89 | 7 | $\frac{8}{9}$ | $\frac{8}{11}$ | $\frac{8}{13}$ | $\frac{1009}{100}$ | Q |
| R | 106 | 67 | 1 | $\frac{9}{10}$ | $\frac{9}{11}$ | $\frac{9}{13}$ | $\frac{5}{100}$ | R |
| S | 243 | 45 | 6 | $\frac{10}{11}$ | $\frac{10}{13}$ | $\frac{10}{17}$ | $\frac{2}{100000}$ | S |
| T | 182 | 38 | 2 | $\frac{11}{12}$ | $\frac{11}{13}$ | $\frac{11}{14}$ | $\frac{70}{1000000}$ | T |
| U | 100 | 72 | 3 | $\frac{12}{13}$ | $\frac{12}{17}$ | $\frac{12}{19}$ | $\frac{82000}{10}$ | U |

**N.B.—For answers *to the problems* on the above drill card, see KEY.**

## MISCELLANEOUS EXAMPLES.

### ORAL AND WRITTEN.

52. *W.* Change $121\frac{3}{5}$ to fifths.

53. *W.* Change $27\frac{1}{3}$ to thirds.

54. *W.* Reduce $43\frac{2}{7}$ to sevenths.

55. Change $\frac{9}{4}$ to an equivalent mixed number. *W.* Change $\frac{112}{4}$ to an integer.

56. How many units in $\frac{9}{3}$? *W.* In $\frac{637}{4}$? In $\frac{243}{8}$?

57. To how many whole apples are $\frac{12}{4}$ equal? Are $\frac{36}{3}$ equal? *W.* Are $\frac{413}{5}$ equal?

58. How many oranges will be required to give 10 boys $\frac{1}{2}$ of an orange each? To give 12 boys $\frac{1}{3}$ of an orange each? *W.* To give 112 boys $\frac{1}{13}$ of an orange each?

59. Arthur can pick $\frac{1}{3}$ of a peck of berries in a day: how many thirds of a peck can he pick in 6 days? *W.* In 36 days? How many pecks?

60. If a man spend $\frac{1}{4}$ of a dollar in one day, how many fourths of a dollar will he spend in 13 days? *W.* In 68 days? How many dollars?

61. Harry had $\$6\frac{1}{2}$ to give some poor boys: to how many could he give a half-dollar each? A quarter of a dollar each? *W.* A tenth of a dollar each? A twenty-fourth of a dollar each?

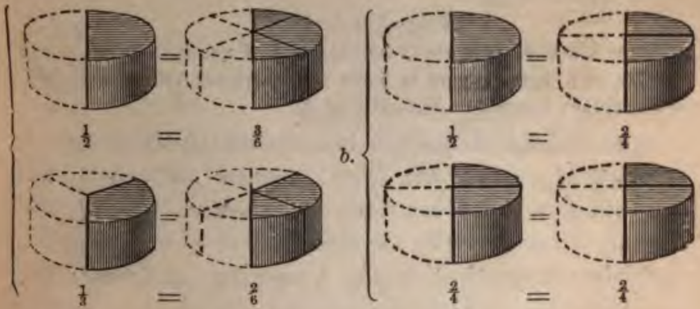
62. A boy has $5\frac{3}{5}$ apples: to how many schoolmates can he give a fifth of an apple each? A tenth of an apple? *W.* A seventy-fifth of an apple?

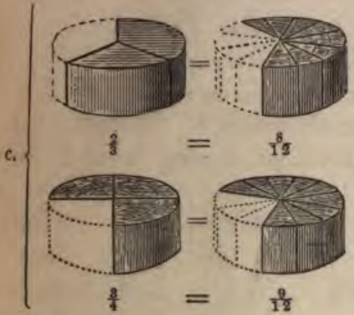
63. A man had $3\frac{4}{7}$ barrels of flour. He sold each seventh of a barrel for 1 dollar: how much did he receive for the flour? *W.* If he received 87 cents for *each seventh?* If he received $1.12 for each seventh?

## 91. TO CHANGE FRACTIONS TO EQUIVALENT FRACTIONS HAVING A COMMON DENOMINATOR.

**92. *Common Denominator.*** — When fractions are of the same denomination, they are said to have a common denominator.

ILLUSTRATION.





It will be seen by the illustration ($a$), that, if $\frac{1}{2}$ of a block of wood is divided into 3 equal parts, the pieces will be *sixths* of the block, and that, if $\frac{1}{3}$ of a block is divided into 2 equal parts, the pieces will also be sixths: hence, the pieces being of the same size, the fractions representing them will have a *common denominator.*

In the same way ($b$) $\frac{1}{2}$ and $\frac{2}{4}$ of a block may be reduced to pieces of the same *size*, or *fourths.* ($c$) $\frac{2}{3}$ and $\frac{3}{4}$ may each be changed to *twelfths* by dividing the quarters each into 3 equal parts, and the *thirds* each into 4 equal parts.

PROBLEM. — Change $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

PROCESS.

$\frac{2}{3} = \frac{8}{12}$

$\frac{3}{4} = \frac{9}{12}$

ANALYSIS.* — Since the least common multiple of the denominators of the fractions (*c*) $\frac{2}{3}$ and $\frac{3}{4}$ is 12, then *twelfths* will be the least number of equal parts into which $\frac{2}{3}$ and $\frac{3}{4}$ can be divided, and have all the pieces of equal size.

ANAL. STEPS. — 1. *Change $\frac{2}{3}$ to twelfths.*

Since in one unit there are 12 *twelfths*, in $\frac{1}{3}$ of a unit there are 1 third of 12 *twelfths*, or 4 *twelfths*; and in $\frac{2}{3}$ of a unit there are 2 times 4 *twelfths*, or 8 *twelfths*.

2. *Change $\frac{3}{4}$ to twelfths.*

Since in 1 unit there are 12 *twelfths*, in $\frac{1}{4}$ of a unit there are $\frac{1}{4}$ of 12 *twelfths*, or 3 *twelfths*; and in $\frac{3}{4}$ of a unit there are 3 times 3 *twelfths*, or 9 *twelfths*: hence $\frac{2}{3} = \frac{8}{12}$, and $\frac{3}{4} = \frac{9}{12}$.

From the above analyses is deduced the following: —

RULE I. — *Find the least common multiple of the denominators for a common denominator. Divide the common denominator by the denominator of each fraction, multiply the quotient by the numerator, and write the result over the common denominator, or,*

RULE II. — *Multiply the terms of each fraction by any number that will make the denominators common.*

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

Change the fractions in each series of the following numbers to equivalent fractions having a common denominator.

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|---|
| $\frac{3}{4}, \frac{2}{3} = \frac{9}{12}, \frac{8}{12}$. | $\frac{6}{7}, \frac{4}{8}$ | $\frac{2}{3}, \frac{4}{7}$ | $\frac{4}{7}, \frac{2}{3}$ | $\frac{4}{5}, \frac{3}{7}$ | $\frac{4}{13}, \frac{1}{2}$ |
| $\frac{4}{7}, \frac{2}{3} = \frac{12}{21}, \frac{14}{21}$. | $\frac{2}{7}, \frac{13}{14}$ | $\frac{3}{6}, \frac{2}{7}$ | $\frac{4}{11}, \frac{13}{22}$ | $\frac{3}{8}, \frac{4}{7}$ | $\frac{5}{7}, \frac{3}{14}$ |
| $\frac{5}{7}, \frac{3}{4} = \frac{20}{28}, \frac{21}{28}$. | $\frac{4}{7}, \frac{20}{21}$ | $\frac{4}{5}, \frac{3}{4}$ | $\frac{5}{11}, \frac{6}{7}$ | $\frac{2}{6}, \frac{4}{5}$ | $\frac{4}{5}, \frac{1}{15}$ |
| $\frac{3}{4}, \frac{1}{8} = \frac{6}{8}, \frac{1}{8}$. | $\frac{2}{7}, \frac{25}{21}$ | $\frac{2}{5}, \frac{3}{7}$ | $\frac{4}{17}, \frac{11}{34}$ | $\frac{1}{4}, \frac{3}{8}$ | $\frac{2}{8}, \frac{3}{4}$ |
| $\frac{5}{6}, \frac{1}{3} = \frac{5}{6}, \frac{2}{6}$. | $\frac{4}{7}, \frac{3}{4}$ | $\frac{4}{5}, \frac{2}{7}$ | $\frac{1}{6}, \frac{6}{11}$ | $\frac{5}{7}, \frac{5}{14}$ | $\frac{5}{7}, \frac{6}{7}$ |
| $\frac{4}{7}, \frac{4}{5} = \frac{20}{35}, \frac{28}{35}$. | $\frac{5}{9}, \frac{13}{18}$ | $\frac{6}{7}, \frac{3}{8}$ | $\frac{4}{5}, \frac{3}{7}$ | $\frac{5}{8}, \frac{3}{24}$ | $\frac{2}{3}, \frac{3}{4}$ |

NOTE. — When one denominator can be divided exactly by the other, it is necessary to multiply only one of the fractions; thus, $\frac{2}{3}, \frac{1}{6} = \frac{4}{6}, \frac{1}{6}$.

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* *The teacher should refer to the illustration at every step of the analysis.*

### WRITTEN EXERCISES.

**Problem.**—Change $3\frac{5}{8}$, $2\frac{1}{6}$, $\frac{3}{4}$, $4\frac{5}{12}$, to equivalent fractions having a common denominator.

**PROCESS.**

$3\frac{5}{8} = 3\frac{15}{24}$

$2\frac{1}{6} = 2\frac{4}{24}$

$\frac{3}{4} = \frac{18}{24}$

$4\frac{5}{12} = 4\frac{10}{24}$

**Analysis.**—Require the pupil to write an analysis of this problem in full, and present it to the teacher as a class-exercise.

64. $\frac{3}{7}$, $\frac{5}{8}$, $\frac{6}{12}$, $\frac{5}{14}$, $\frac{3}{4}$.
65. $\frac{6}{7}$, $\frac{3}{14}$, $\frac{7}{21}$, $\frac{11}{12}$, $\frac{11}{16}$.
66. $\frac{8}{9}$, $\frac{3}{27}$, $\frac{6}{7}$, $\frac{5}{28}$, $\frac{3}{21}$.
67. $\frac{2}{11}$, $\frac{6}{121}$, $\frac{11}{14}$, $\frac{3}{28}$.
68. $4\frac{3}{7}$, $\frac{6}{8}$, $\frac{9}{14}$, $11\frac{6}{7}$.
69. $3\frac{5}{8}$, $3\frac{4}{7}$, $5\frac{3}{8}$, $9\frac{4}{14}$.
70. $4\frac{3}{7}$, $5\frac{4}{28}$, $\frac{9}{12}$, $\frac{4}{15}$.
71. $4\frac{3}{4}$, $3\frac{11}{12}$, $\frac{6}{84}$, $\frac{8}{9}$.
72. $9\frac{4}{5}$, $3\frac{14}{28}$, $4\frac{8}{40}$, $\frac{3}{8}$.
73. $4\frac{3}{10}$, $8\frac{5}{14}$, $\frac{6}{7}$, $9\frac{2}{5}$.
74. $3\frac{6}{7}$, $5\frac{8}{9}$, $\frac{3}{27}$, $\frac{6}{28}$.
75. $3\frac{4}{7}$, $5\frac{9}{12}$, $\frac{1}{24}$, $3\frac{1}{28}$.
76. $\frac{6}{7}$, $5\frac{3}{21}$, $6\frac{6}{11}$, $3\frac{4}{9}$.
77. $\frac{8}{9}$, $6\frac{4}{7}$, $5\frac{3}{7}$, $6\frac{8}{14}$.

## ADDITION OF FRACTIONS.

**93.** Since things can be added only to like things, it is evident, that, before fractions can be added or subtracted, they must be reduced to equivalent fractions having a common denominator.

**Problem.** — What is the sum of $5\frac{3}{4}$, $\frac{5}{6}$, and $4\frac{3}{8}$?

**Formula.** $5\frac{3}{4} + \frac{5}{6} + 4\frac{3}{8} = 5\frac{18}{24} + \frac{20}{24} + 4\frac{9}{24} = 10\frac{23}{24}$.

**PROCESS.**

$5\frac{3}{4} = 5\frac{18}{24}$

$\frac{5}{6} = \frac{20}{24}$

$4\frac{3}{8} = 4\frac{9}{24}$

$10\frac{23}{24}$

**Analysis.** — (1.) The least common denominator of $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{3}{8}$, is *twenty-fourths.*

(2.) $5\frac{3}{4} = 5\frac{18}{24}$; $\frac{5}{6} = \frac{20}{24}$; and $4\frac{3}{8} = 4\frac{9}{24}$.

(3.) The sum of 18 *twenty-fourths*, 20 *twenty-fourths*, and 9 *twenty-fourths*, is 47 *twenty-fourths*, equal to 1 unit and $\frac{23}{24}$; which, added to the sum of 4 and 5, equals $10\frac{23}{24}$.

**Rule.** — *Change the fractions to equal fractions having a common denominator, and add the numerators.*

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**Review.**—What is *multiplication?* (40.) Define the terms. (41, 42, 43, 44.) *Which are factors?* (44.) *Describe the sign of multiplication.* (45.) Show its use. (45.) *Repeat the axiom.* (46.) Give the principles. (47.) Analyze a short example in *multiplication.* (48.)

NOTE.—1. When adding mixed numbers, find the sum of the fractions, and add it to the sum of the integers.

2. Answers should always be given in the lowest terms, or reduced to integral or mixed numbers when possible.

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

Prepare the following like the model, and present on slates as a class-exercise.

| MODEL. | (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|---|
| $\frac{1}{4}+\frac{1}{8}=\frac{3}{8}$ | $\frac{1}{4}+\frac{1}{2}$ | $\frac{3}{4}+\frac{1}{8}$ | $4\frac{1}{7}+\frac{5}{28}$ | $8\frac{1}{3}+\frac{2}{15}$ |
| $\frac{1}{6}+\frac{7}{6}=1\frac{1}{3}$ | $\frac{1}{3}+\frac{1}{6}$ | $\frac{7}{8}+\frac{1}{2}$ | $6\frac{1}{5}+4\frac{3}{5}$ | $4\frac{9}{11}+\frac{9}{22}$ |
| $\frac{4}{5}+\frac{1}{10}=\frac{9}{10}$ | $\frac{3}{4}+\frac{7}{8}$ | $\frac{4}{7}+\frac{1}{14}$ | $2\frac{1}{4}+1\frac{1}{8}$ | $6\frac{4}{5}+\frac{2}{35}$ |
| $3\frac{3}{8}+1\frac{1}{8}=4\frac{1}{2}$ | $\frac{1}{11}+\frac{2}{22}$ | $\frac{3}{5}+\frac{2}{15}$ | $7\frac{5}{7}+4\frac{3}{14}$ | $9\frac{6}{7}+\frac{4}{49}$ |
| $2\frac{5}{6}+\frac{1}{2}=3\frac{1}{3}$ | $\frac{5}{10}+\frac{1}{5}$ | $\frac{4}{7}+\frac{3}{14}$ | $1\frac{2}{3}+2\frac{1}{15}$ | $9\frac{4}{11}+10\frac{1}{21}$ |

### DRILL CARD EXERCISES.—TABLE IV.

#### ADDITION OF FRACTIONS.

N.B.—For brief business methods of adding fractions, see Key to Drill Cards.

EXERCISE X.—Find the sum of **V** and **W**; as, A. $\frac{1}{2}+\frac{1}{3}$, &c.

Ex. XI.—Find the sum of **W** and **X**; as, A. $\frac{1}{3}+\frac{1}{4}$; B. $\frac{1}{6}+\frac{1}{7}$, &c.

Ex. XII.—Find the sum of **V** and **X**; as, A. $\frac{1}{2}+\frac{1}{4}$; B. $\frac{1}{5}+\frac{1}{7}$, &c.

Ex. XIII.—Find the sum of **V**, **W**, and **X**; as, A. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$, &c.

Ex. XIV.—Prefix 1 to **V**, and find the sum of two numbers; thus, A + B, $1\frac{1}{2}+6\frac{1}{5}$; B + C, $6\frac{1}{5}+8\frac{1}{3}$; C + D, $8\frac{1}{3}+9\frac{1}{11}$; D + E, $9\frac{1}{11}+4\frac{2}{3}$, &c.

Ex. XV.—Prefix 1 to **W**, and find the sum of two numbers; thus A + B, $1\frac{1}{3}+6\frac{1}{6}$; B + C, $6\frac{1}{6}+8\frac{1}{9}$, &c.

Ex. XVI.—Prefix 1 to **X**, and find the sum of two numbers as before.

Ex. XVII.—Prefix 2 to **V**, and find the sum of three numbers; thus, A + B + C, $38\frac{1}{2}+59\frac{1}{5}+16\frac{1}{3}$; B + C + D, $59\frac{1}{5}+16\frac{1}{3}+20\frac{1}{11}$, &c.

Ex. XVIII.—Prefix 2 to **W**, and find the sum of three numbers as before.

Ex. XIX.—Prefix 3 to **V**, 2 to **W**, and 1 to **X**, then find the sum of the resulting mixed numbers; thus, A. $472\frac{1}{2}+38\frac{1}{3}+1\frac{1}{4}$; B. $604\frac{1}{5}+59\frac{1}{6}+6\frac{1}{7}$, &c.

### ORAL EXERCISES.

1. If a boy earns $\frac{2}{5}$ of a dollar at one time, $\frac{3}{5}$ of a dollar at another time, and $\$\frac{4}{7}$ at another, how many dollars does he earn in all?

**FORMULA.** $\$\frac{2}{5}+\$\frac{3}{5}+\$\frac{4}{7}=\$1\frac{4}{7}$.

**ANALYSIS.** $\$\frac{2}{5}$ and $\$\frac{3}{5}$ equal $\$\frac{5}{5}$ *or*, 1 dollar; and 1 dollar and $\$\frac{4}{7}$ equal $\$1\frac{4}{7}$.

**SUGGESTION.**—In adding more than two fractions, it is often more convenient to add *two* of them first, reduce, and add each of the others to the sum of the preceding addition.

2. If a boy spends $\frac{1}{6}$ of a dollar at one time, $\$1\frac{5}{6}$ at another time, and $\$\frac{1}{3}$ at another, how much does he spend in all?

3. There are 7 quarters of an apple in one pile, and 3 halves in another pile: how many apples in both?

4. A girl has 2 apples in one hand, and 3 quarter-apples in the other; her brother has 1 apple in one hand, and 1 half of an apple in the other: how many apples have both of the children?

5. Josie gave her mother $2\frac{1}{4}$ melons, and her father $2\frac{1}{3}$ melons: to which did she give the most? How much did she give to both?

6. Lillie gave to Laura $2\frac{1}{2}$ oranges, to Charlie $2\frac{3}{4}$ oranges, and to Delia $2\frac{3}{8}$ oranges: to which did she give the least? To which the most? How much did she give away in all?

7. A boy bought a pair of skates for $\$2\frac{3}{4}$, a knife for $\$\frac{7}{12}$, and a book for $\$1\frac{2}{3}$: how much did he give for all?

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**REVIEW.**—Give method of multiplying by 10, 100, &c. (49.) Give the table of United States currency. (50.) What is understood by division? (51.) How many terms in division? *(52, 53, 54, 55)* Define each. Give an example illustrating each. *Describe the sign of division.* (56.)

### WRITTEN EXERCISES.

78. A man bought 4 pieces of calico. The first contained 37$\frac{5}{9}$ yards; the second, 41$\frac{5}{7}$ yards; the third, 27$\frac{3}{8}$ yards; and the fourth, 38$\frac{1}{12}$: how many yards did he buy?

79. If a pound of sugar costs 12$\frac{5}{9}$ cents, a pound of tea \$1.21$\frac{1}{3}$, and a pound of coffee 22$\frac{4}{9}$ cents, what will all cost?

80. A rubber cost 12$\frac{1}{2}$ cents, a slate 15$\frac{3}{8}$ cents, a pencil $\frac{1}{12}$ of a cent, and a sponge 5 cents: how much did they all cost?

81. A farmer raised 326$\frac{7}{8}$ bushels of potatoes, 126$\frac{3}{4}$ bushels of turnips, and 128$\frac{1}{3}$ bushels of beets: how many bushels did he raise in all?

82. Henry has 24$\frac{2}{3}$ dollars, Edward has 13$\frac{1}{4}$ more than Henry: how many has Edward? How many have both?

83. One-third of a mill is valued at \$416$\frac{3}{4}$: what is the value of the whole mill?

84. A farm was divided into 5 parts; four of the parts contained 237$\frac{4}{7}$ acres, one part contained 75$\frac{2}{3}$ acres: how many acres in the farm?

85. \$347$\frac{2}{3}$ is \$127$\frac{3}{4}$ less than the value of my horses and carriage: what are they worth?

86. I sold a house for \$1341$\frac{3}{5}$, which was \$137$\frac{1}{4}$ less than it cost me: did I make, or lose? and how much?

87. A boy sold a sled for \$1.86$\frac{3}{7}$, which was 59$\frac{5}{14}$ cents less than it cost him: did he make, or lose? and how much?

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**REVIEW.**—Describe the fractional sign. (57.) How many signs are there in *division?* What is their difference? What is the difference between short *division and long division?* (58, 59.)

88. I bought a mowing-machine for \$$280\frac{1}{3}$, and sold it so as to gain \$$23\frac{3}{5}$: for how much did I sell it?

89. A man bought $\frac{5}{8}$ of a share in an oil-well at one time, $\frac{3}{5}$ of a share at another time, and $3\frac{2}{5}$ shares at another time: how many shares did he buy?

90. How many times a number is $\frac{1}{2}$ of it, $\frac{1}{3}$ of it, $\frac{1}{4}$ of it, and $\frac{1}{5}$ of it?

91. How many units in 4, $3\frac{3}{7}$, $2\frac{4}{5}$, and $\frac{1}{2}$?

92. A boy had $6\frac{1}{2}$ apples; he gave his father $2\frac{1}{2}$ apples, his mother $\frac{2}{3}$ of one apple, and to his sister $\frac{7}{9}$ of another: how many apples did he give away?

93. Five times $16\frac{1}{3} + \frac{2}{3}$ of $16\frac{1}{3} + \frac{5}{9}$ of $16\frac{1}{3}$ are how many times $16\frac{1}{3}$?

94. $\frac{3}{8}$ of a pear $+ \frac{46}{3}$ of a pear + 3 pears $+ 3\frac{2}{7}$ pears, make how many pears?

95. What is the sum of $\frac{5}{9}$, $\frac{41}{35}$, $7\frac{2}{3}$, $\frac{5}{14}$?

96. Three times a number plus $\frac{2}{3}$ of it, plus $4\frac{6}{7}$ times the number, plus $\frac{1}{9}$ of it, are how many times the number?

97. 34 times 28, plus $\frac{2}{3}$ of 28, plus $2\frac{5}{11}$ times 28, plus $\frac{7}{12}$ of 28, are how many times 28?

### SUBTRACTION OF FRACTIONS.

### *94.* TO SUBTRACT A FRACTIONAL NUMBER FROM AN INTEGER OR FROM ANOTHER FRACTIONAL NUMBER.

**PROBLEM.**—(*a.*) From 16 take $5\frac{2}{3}$. (*b.*) From $6\frac{3}{8}$ take $4\frac{5}{8}$. (*c.*) From $\frac{4}{5}$ take $\frac{2}{3}$.

PROCESSES.

| (*a.*) | (*b.*) | (*c.*) | | *c.*) |
|---|---|---|---|---|
| From 16 | $6\frac{3}{8}$ | $\frac{4}{5}=\frac{12}{15}$ | FORMULA. | $\frac{12}{15}-\frac{10}{15}=\frac{2}{15}$ |
| Take $5\frac{2}{3}$ | $4\frac{5}{8}$ | $\frac{2}{3}=\frac{10}{15}$ | | |
| Rem. $10\frac{1}{3}$ | $1\frac{3}{4}$ | $\frac{2}{15}$ | | |

**ANALYSIS.** — (*a.*) Since there are no thirds in the minuend, change one unit to thirds; $\frac{3}{3}$ less $\frac{2}{3}$ equals $\frac{1}{3}$; 15 units less 5 units equals 10 units.

(*b.*) Since $\frac{5}{6}$ cannot be taken from $\frac{3}{6}$, change one of the 6 units to sixths, which, with $\frac{3}{6}$, makes $\frac{9}{6}$; $\frac{9}{6} - \frac{5}{6} = \frac{4}{6} = \frac{2}{3}$; and 5 less 4 = 1.

(*c.*) Change $\frac{4}{5}$ and $\frac{2}{3}$ to a common denominator; as, $\frac{12}{15}$ and $\frac{10}{15}$. $\frac{12}{15} - \frac{10}{15} = \frac{2}{15}$.

**RULE.** — *Change the fractions to equal fractions having a common denominator, and find the difference of the numerators.*

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|---|
| $\frac{2}{3}-\frac{1}{2}=\frac{1}{6}$ | $\frac{8}{9}-\frac{7}{12}$ | $\frac{2}{3}-\frac{1}{2}$ | $3\frac{1}{3}-1\frac{2}{7}$ | $\frac{3}{5}+\frac{2}{3}$ | $4\frac{2}{11}-\frac{2}{22}$ |
| $\frac{3}{4}-\frac{2}{3}=\frac{1}{12}$ | $\frac{9}{10}-\frac{4}{5}$ | $\frac{3}{4}-\frac{2}{3}$ | $1\frac{4}{5}-\frac{3}{7}$ | $\frac{7}{9}-\frac{5}{7}$ | $2\frac{4}{7}+1\frac{1}{3}$ |
| $\frac{4}{5}-\frac{3}{4}=\frac{1}{20}$ | $\frac{10}{11}-\frac{1}{22}$ | $\frac{5}{6}-\frac{7}{12}$ | $4\frac{3}{7}-1\frac{1}{6}$ | $\frac{3}{5}+\frac{8}{9}$ | $5\frac{7}{9}-2\frac{7}{10}$ |
| $4\frac{5}{6}-\frac{4}{5}=4\frac{1}{30}$ | $\frac{11}{12}-\frac{5}{9}$ | $\frac{6}{7}-\frac{7}{14}$ | $8\frac{5}{7}-5\frac{2}{9}$ | $\frac{3}{7}-\frac{3}{8}$ | $5\frac{7}{9}+2\frac{7}{10}$ |
| $3\frac{7}{8}-2\frac{4}{7}=1\frac{17}{56}$ | $\frac{12}{13}-\frac{2}{3}$ | $\frac{7}{8}-\frac{4}{5}$ | $1\frac{2}{7}-1\frac{2}{9}$ | $\frac{3}{7}+\frac{3}{8}$ | $5\frac{3}{7}+\frac{2}{3}$ |

## DRILL CARD EXERCISES. — TABLE IV.

### SUBTRACTION OF FRACTIONS.

N.B. — For brief business methods of subtracting fractions, see Key to Drill Cards.

EXERCISE I. — From **V** subtract **W**; thus, A. $\frac{1}{2}-\frac{1}{3}$; B. $\frac{1}{5}-\frac{1}{6}$; C. $\frac{1}{3}-\frac{1}{9}$, &c.

Ex. II. — From **W** subtract **X**; thus, A. $\frac{1}{3}-\frac{1}{4}$; B. $\frac{1}{6}-\frac{1}{7}$, &c.

Ex. III. — From **V** subtract **X**; thus, A. $\frac{1}{2}-\frac{1}{4}$; B. $\frac{1}{5}-\frac{1}{7}$, &c.

Ex. IV. — From **1** subtract **V**; thus, A. $1-\frac{1}{2}$; B. $6-\frac{1}{5}$, &c.

Ex. V. — From **2** subtract **W**; thus, A. $38-\frac{1}{3}$; B. $59-\frac{1}{6}$, &c.

Ex. VI. — From **3** subtract **X**; thus, A. $472-\frac{1}{4}$; B. $604-\frac{1}{7}$, &c.

Ex. VII. — Prefix column **1** to **V**, and subtract each mixed number from $23\frac{5}{11}$; thus, A. $23\frac{5}{11}-1\frac{1}{2}$, &c.

Ex. VIII. — Prefix column **2** to **W**, and subtract each mixed number from $140\frac{1}{13}$.

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REVIEW. — Give the axiom used in division. (60.) Name the principles. (61.) Give an example illustrating each. Analyze a short example in division. (62.) *Illustrate the method of dividing by 10, 100, 1,000, &c.* (63.) Analyze a short example in long division. (64.)

Ex. IX. — Prefix column 3 to **X**, and subtract each mixed number from $920\frac{3}{5}$.

Ex. X. — Find the difference (column **V**) between A and B, $\frac{1}{2} - \frac{1}{5}$; B and C, $\frac{1}{5} - \frac{1}{8}$, &c.

Ex. XI. — Find the difference (column **W**) between A and B, B and C, &c.

Ex. XII. — Find the difference (column **X**) between A and B, B and C, &c.

Ex. XIII. — $2 - \mathbf{V} + \mathbf{W} + 1 - \mathbf{X} =$ what? as, A. $38 - \frac{1}{2} + \frac{1}{3} + 1 - \frac{1}{4} =$ what? B. $59 - \frac{1}{5} + \frac{1}{6} + 6 - \frac{1}{7} =$ what?

### ORAL EXERCISES.

1. What is meant by sixths? What by sevenths? What by ninths? By elevenths? By fifteenths?

2. What is the difference between halves and quarters? Thirds and fourths? Sixths and sevenths? Elevenths and twentieths?

3. Which is the greater, — a half, or a quarter? A third, or a fourth? A fifth, or a sixth? A ninth, or an eleventh?

4. Which is of the most value, $\frac{2}{3}$, or $\frac{3}{4}$? $\frac{2}{2}$, or $\frac{1}{2}$? $\frac{5}{8}$, or $\frac{5}{13}$? $\frac{2}{7}$, or $\frac{2}{9}$? $\frac{5}{8}$, or $\frac{5}{11}$? $\frac{3}{4}$, or $\frac{1}{4}$? $\frac{6}{7}$, or $\frac{3}{7}$? $\frac{2}{7}$, or $\frac{2}{9}$?

5. Jennie, having $\frac{3}{4}$ of an apple, gave $\frac{1}{4}$ to her sister: how much had she left?

6. What is the difference between $4\frac{3}{7}$ and $2\frac{4}{7}$?

7. If I pay \$$\frac{3}{4}$ from \$$3\frac{1}{4}$, how much remains?

8. If I sell $\frac{3}{4}$ of an acre of land from a lot which contains $\frac{11}{12}$ of an acre, what remains?

9. Ten acres less $\frac{1}{2}$ of an acre equals how much? Less $\frac{1}{3}$ of an acre? Less $\frac{3}{4}$ of an acre? Less $4\frac{2}{3}$ acres? Less $4\frac{4}{5}$ acres?

10. From $\frac{7}{8}$ take $\frac{3}{10}$; take $\frac{3}{4}$; take $\frac{8}{9}$; take $\frac{5}{7}$.

11. How much is $\frac{1}{3} + \frac{5}{6} - \frac{1}{4}$? $\frac{6}{7} + \frac{1}{2} - \frac{3}{14}$? $\frac{8}{5} - \frac{1}{3} + \frac{7}{15}$?

12. How *much is* $\frac{1}{2}$ plus $\frac{1}{3} - \frac{1}{4} + \frac{1}{8}$? $\frac{1}{2} + \frac{1}{10} - \frac{1}{3} + \frac{1}{15}$?

13. What is the difference between $4\frac{3}{7}$ and $2\frac{2}{9}$?

14. What must be added to $4\frac{3}{7}$ to make $8\frac{7}{9}$?

15. A boy bought a sled for \$$2\frac{1}{7}$; he then sold it for \$$\frac{1}{9}$ less than he gave for it: how much did he receive for it?

16. Kate had 25 cents; she bought apples for $3\frac{1}{3}$ cents, nuts for $5\frac{5}{6}$ cents, and an orange for $2\frac{5}{6}$ cents: how much money had she left?

17. Anna bought 5 oranges for $5\frac{1}{2}$ cents, 3 apples for $4\frac{1}{2}$ cents, $1\frac{3}{4}$ pounds of almonds for $16\frac{1}{2}$ cents: how much did she give for all?

### WRITTEN EXERCISES.

98. From $\frac{11}{16}$ take $\frac{3}{16}$.
99. From $\frac{21}{22}$ take $\frac{5}{11}$.
100. From $\frac{4}{9}$ take $\frac{2}{11}$.
101. $\frac{4}{7} - \frac{5}{11} =$ what?
102. $\frac{4}{9} - \frac{5}{15} =$ what?
103. $\frac{61}{17}$ less $\frac{17}{19} =$ what?
104. $\frac{5}{11}$ minus $\frac{5}{12} =$ what?
105. $3\frac{1}{4} - \frac{4}{13} =$what?
106. $27 - \frac{61}{12}$ plus $\frac{8}{8} =$ what?
107. $24\frac{1}{2} + \frac{1}{2}$ less $\frac{3}{92} =$ what?
108. $18\frac{3}{8}$ minus $1\frac{2}{7} + \frac{1}{2} =$ what?
109. $4\frac{3}{7} - \frac{2}{11} + 4\frac{3}{7} =$ what?
110. $5 + \frac{41}{7} - \frac{21}{7} + 8 =$ what?
111. $4 + \frac{8}{8} - \overline{\frac{9}{9} + 3\frac{1}{2}} =$ what?
112. $(8\frac{1}{2} - \frac{3}{4}) + \frac{2}{3} + \frac{1}{2} =$ what?
113. $(11\frac{1}{2} + \frac{2}{3}) - (\frac{2}{3} + \frac{4}{5}) =$ what?

### MISCELLANEOUS PROBLEMS FOR REVIEW.

114. A man bought a cow for \$$39\frac{3}{13}$, and sold it for \$40: *what* was the profit?

FORMULA. $\$40 - \$39\frac{3}{13} =$ what?

NOTE. — The pupil should be required to write a formula showing the relation of the numbers in each problem.

115. A grocer bought $371\frac{4}{7}$ pounds of butter; he then sold $147\frac{5}{8}$ pounds: how many pounds had he left?

116. A man bought at one time $34\frac{3}{4}$ barrels flour, at another time $47\frac{3}{15}$, and at another $44\frac{5}{12}$ barrels: how many barrels did he buy?

117. From a ten-dollar bill I spent at one time $\$1.25\frac{3}{5}$, at another $\$1.64\frac{2}{5}$, and at another $\$3.37\frac{1}{2}$: how much had I left?

118. $3\frac{4}{7}$ pounds of coffee cost $2\frac{2}{3}$ dollars, $4\frac{3}{8}$ pounds of tea cost $5\frac{5}{7}$ dollars: how much did both cost?

119. I had $347\frac{3}{8}$ pounds of tea, $325\frac{5}{8}$ pounds of coffee, and $123\frac{3}{7}$ pounds of rice; I sold $121\frac{5}{8}$ pounds of each how many pounds of each had I remaining?

120. The minuend is $314\frac{2}{7}$, the subtrahend is $116\frac{4}{35}$ what is the difference?

121. The difference is $57\frac{3}{7}$, the minuend is $411\frac{8}{35}$: what is the subtrahend?

122. The subtrahend is $63\frac{5}{8}$, the difference is $51\frac{1}{2}$: what is the minuend?

123. Henry had $47\frac{3}{4}$ dollars, and Robert as many, lacking $\$8\frac{4}{15}$: how many dollars had Robert?

124. George bought a sled for $\$3\frac{3}{5}$, and sold it so as to gain $\$1\frac{7}{11}$: for how much did he sell it?

125. Mary bought a portfolio for $\$2.34\frac{5}{7}$; but she gave it to her cousin for $25\frac{2}{3}$ cents less than it cost her: for how much did she sell it?

126. The price paid for a book is $\$5.32\frac{1}{3}$, the gain was $\$1.12\frac{1}{2}$: what was the selling-price?

127. The *price paid for* a book is $\$3.24\frac{4}{7}$; the loss *was 87½ cents: what* was the selling-price?

128. The selling-price of a house was \$3474$\frac{2}{3}$; the cost was \$5234$\frac{5}{8}$: what was the gain, or loss?

129. The selling-price of a horse was \$47$\frac{5}{7}$; the gain was \$25$\frac{3}{5}$: what was the cost?

130. The selling-price of a mowing-machine was \$135$\frac{4}{7}$; the loss was \$25$\frac{3}{8}$: what was the cost?

131. The selling-price of a barrel of flour was \$5.38$\frac{1}{3}$; the cost was \$5.08$\frac{3}{4}$: what was the gain, or loss?

## MULTIPLICATION OF FRACTIONS.

### **95.** TO MULTIPLY A FRACTION OR MIXED NUMBER BY AN INTEGER.

PROBLEM.—If I should give $\frac{2}{6}$ of a cake of maple-sugar to each of 7 boys, how many sixths of the cake would all the boys have?

FORMULA. $\frac{2}{6} \times 7 = \frac{14}{6} = 2\frac{1}{3}$.

ANALYSIS.—If 1 boy receives 2 sixths of a cake, 7 boys will receive 7 times 2 sixths, or 14 sixths: 14 sixths equals $2\frac{2}{6}$, or $2\frac{1}{3}$ cakes.

PRINCIPLE.—*Multiplying the numerator of a fraction multiplies the value of the fraction.*

The number of parts is increased, while their size remains the same.

#### ORAL EXERCISES.

1. If there are 3 halves of an apple in one pile, how many halves in 4 such piles? How many whole *apples?*

2. How many dollars in 5 times $\frac{3}{4}$ of a dollar? In 4 times $\frac{1}{2}$ of a dollar? In 5 times $\frac{2}{3}$ of a dollar?

3. At $\$\frac{2}{3}$ a pound, what will 4 pounds of coffee cost?

**Analysis.** — Since 1 pound cost $\$\frac{2}{3}$, 4 pounds will cost 4 times $\$\frac{2}{3}$, or $\$\frac{8}{3}$, equal to $\$2\frac{2}{3}$.

4. At $\$\frac{3}{4}$ a bushel, what will 3 bushels of apples cost? 5 bushels? 6 bushels? 3 bushels? 10 bushels?

5. How many times 1 are 4 times $\frac{5}{7}$? 6 times $\frac{3}{4}$? 9 times $\frac{5}{9}$? 7 times $\frac{3}{4}$?

6. What will 5 lemons cost at $4\frac{1}{2}$ cents each?

**Analysis.** — Since 1 lemon costs $4\frac{1}{2}$ cents, 5 lemons will cost 5 times $4\frac{1}{2}$ cents. 5 times $\frac{1}{2}$ cent $= \frac{5}{2}$, or $2\frac{1}{2}$ cents; 5 times 4 cents $= 20$ cents; 20 cents $+ 2\frac{1}{2}$ cents $= 22\frac{1}{2}$ cents.

7. What will 8 pounds of sugar cost at $12\frac{1}{2}$ cents per pound? At $8\frac{2}{3}$ cents? At $9\frac{2}{5}$ cents? At $10\frac{1}{2}$ cents?

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

| MODEL. | (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|---|
| $\frac{3}{4} \times 2 = \frac{6}{4}$* $= 1\frac{1}{2}$ | $\frac{2}{3} \times 4$ | $\frac{4}{7} \times 6$ | $1\frac{2}{3} \times 3$ | $4\frac{1}{4} \times 3$ |
| $\frac{5}{8} \times 4 = \frac{20}{8} = 2\frac{1}{2}$ | $\frac{5}{8} \times 7$ | $\frac{8}{11} \times 4$ | $4\frac{1}{2} \times 7$ | $7\frac{4}{7} \times 4$ |
| $\frac{3}{4} \times 5 = \frac{15}{4} = 3\frac{3}{4}$ | $\frac{5}{9} \times 3$ | $\frac{7}{9} \times 3$ | $3\frac{1}{4} \times 6$ | $2\frac{1}{4} \times 6$ |
| $3\frac{2}{3} \times 3 = 9\frac{6}{3} = 11$ | $\frac{7}{5} \times 5$ | $\frac{4}{11} \times 9$ | $7\frac{1}{3} \times 8$ | $5\frac{3}{4} \times 7$ |
| $4\frac{3}{7} \times 2 = 8\frac{6}{7} = 8\frac{6}{7}$ | $\frac{3}{8} \times 9$ | $\frac{7}{8} \times 3$ | $5\frac{1}{2} \times 3$ | $5\frac{5}{7} \times 3$ |

### ORAL AND WRITTEN EXERCISES.

**Problem.** — Multiply $\$251\frac{2}{5}$ by 3. By 5. By 7.

**PROCESS.**

$$\begin{array}{r} \$251\frac{2}{5} \\ 3 \\ \hline \$754\frac{1}{5} \end{array}$$

**Analysis.** — (1.) For convenience multiply the integer and fraction separately.

(2.) 3 times $\$\frac{2}{5} = \$\frac{6}{5} = \$1\frac{1}{5}$.

(3.) 3 times $\$251 = \$753$. $\$753 + \$1\frac{1}{5}$ equals $\$754\frac{1}{5}$.

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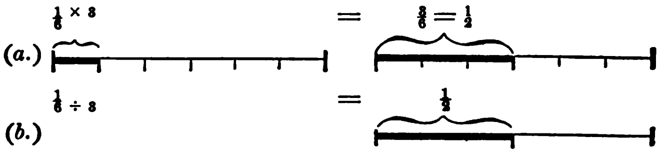
* This intermediate step should be omitted as soon as the pupil is able to perform the work *readily in the mind.*

1. *W.* Multiply $341\frac{2}{3}$ by 6. By 8. By 9. By 11.
2. *W.* Multiply $302\frac{1}{4}$ by 5. By 11. By 4. By 8.
3. *W.* Multiply $404\frac{3}{5}$ by 4. By 12. By 8. By 10.

**PROBLEM.**—What part of a line is 3 times $\frac{1}{6}$ of a line?

ILLUSTRATION.



**ANALYSIS.**—3 times $\frac{1}{6}$ of a line equals $\frac{3}{6}$, or $\frac{1}{2}$, of a line.

**PRINCIPLE.**—*Dividing the denominator of a fraction multiplies the value of the fraction.*

The number of parts remains the same, while their size is increased.

**RULE.**—*To multiply a fraction by an integer, multiply the numerator, or divide the denominator, by the integer.*

**NOTE.**—When the multiplicand is a mixed number, multiply the integer and fractions separately, and add the products.

4. *W.* What is the product of $453\frac{2}{3}$ multiplied by 3? By 5? By 8? By 13? By 11? By 29?

5. If 1 pound of tea costs $\$\frac{7}{8}$, what will 7 pounds cost? 4 pounds? 9 pounds? *W.* 341 pounds? 246 pounds?

6. If coffee is worth $37\frac{1}{2}$ cents per pound, what are 2 pounds worth? *W.* 7 pounds? 25 pounds? 87 pounds?

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**REVIEW.**—Recite the table of United States money. (65.) Give the table of aliquot parts. (66.) Of what are coins made? (67.) What is paper money? (68.) What is an invoice, or bill? (69.) Who is the debtor? Who the creditor? What is an account?

7. If a barrel of flour is worth \$$9\frac{3}{7}$, how much are 2 barrels worth? 3 barrels? *W.* 11 barrels? 223 barrels?

8. $2\frac{2}{3}$ loaves of bread will last 1 man 2 days: at the same rate how much will 3 men require? 5 men? *W.* 68 men? 125 men?

## *96.* TO MULTIPLY AN INTEGER BY A FRACTION, OR TO TAKE THE FRACTIONAL PART OF A NUMBER.

### ORAL EXERCISES.

1. What is $\frac{1}{2}$ of an apple? ANS. — *One of the two equal parts into which an apple is divided.*

2. What is $\frac{1}{4}$ of an orange? $\frac{1}{7}$ of a pound? $\frac{1}{5}$ of an inch?

3. What is $\frac{2}{3}$ of a lemon? ANS. — *Two of the three equal parts into which a lemon is divided.* What is $\frac{3}{4}$ of a gallon? $\frac{5}{7}$ of an ounce?

4. Multiply \$2 by $\frac{3}{4}$, or, what is $\frac{3}{4}$ of \$2?

**NOTE.** — The word "of" is used instead of "times" when the multiplier is a fraction. We say "$\frac{3}{4}$ of 2," instead of "$\frac{3}{4}$ times 2."

**FORMULA.** $\$2 \times \frac{3}{4} = \$2 \times 3 \div 4$.

**ANALYSIS.** — (1.) $\frac{3}{4}$ of 1 dollar $= \frac{1}{4}$ of 3 times 1 dollar; or, $\$1 \times 3 \div 4$.

(2.) $\frac{3}{4}$ of 2 dollars $= \frac{1}{4}$ of 3 times 2 dollars; or, $\$2 \times 3 \div 4 = \$1\frac{1}{2}$. Hence the

**RULE.** — *To multiply by a fraction, multiply by the numerator, and divide by the denominator of the fraction.*

**NOTE.** — Whenever it is possible, the equal factors of the dividend and divisor should be canceled.

5. Multiply \$7 by $\frac{5}{6}$, by $\frac{2}{3}$, by $\frac{5}{7}$.

6. Multiply 8 by $\frac{4}{5}$, by $\frac{2}{3}$, by $\frac{7}{9}$.

7. Multiply 9 by $\frac{5}{6}$, by $\frac{2}{3}$, by $\frac{6}{7}$.

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**REVIEW.** — Define a composite number. (70.) What is a prime number. (71.) What is a factor, or divisor? (72.) What is a greatest common divisor? (74.) What is a common multiple? (75.)

5. Multiply 8 by $\frac{3}{4}$. 7 by $\frac{5}{14}$. 9 by $\frac{7}{18}$. 12 by $\frac{5}{8}$.

**By Cancellation.** FORMULA. $8 \times \frac{3}{4} = 8 \times 3 \div 4 = \overset{2}{\cancel{8}} \times 3 \div 4 = 6$

6. What is $\frac{6}{7}$ of 14? $\frac{5}{8}$ of 16? $\frac{2}{3}$ of 21? $\frac{5}{11}$ of 22?

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|---|
| $4 \times \frac{3}{4} = 3$ | $6 \times \frac{5}{6}$ | $5 \times \frac{4}{7}$ | $9 \times \frac{2}{7}$ | $4 \times \frac{3}{4}$ | $12 \times \frac{4}{5}$ |
| $3 \times \frac{2}{5} = 1\frac{1}{5}$ | $5 \times \frac{4}{7}$ | $11 \times \frac{3}{7}$ | $1 \times \frac{3}{4}$ | $1 \times \frac{2}{7}$ | $14 \times \frac{2}{7}$ |
| $4 \times \frac{4}{5} = 3\frac{1}{5}$ | $4 \times \frac{8}{9}$ | $2 \times \frac{5}{8}$ | $3 \times \frac{6}{7}$ | $3 \times \frac{5}{6}$ | $16 \times \frac{1}{4}$ |
| $5 \times \frac{2}{7} = 1\frac{3}{7}$ | $3 \times \frac{3}{7}$ | $8 \times \frac{3}{6}$ | $4 \times \frac{5}{9}$ | $7 \times \frac{4}{7}$ | $15 \times \frac{3}{5}$ |
| $6 \times \frac{3}{7} = 2\frac{4}{7}$ | $8 \times \frac{6}{7}$ | $9 \times \frac{5}{9}$ | $7 \times \frac{2}{7}$ | $1 \times \frac{5}{9}$ | $21 \times \frac{3}{7}$ |

### ORAL AND WRITTEN EXERCISES.

9. If 1 bushel of potatoes is worth \$1.25, what are 2 bushels worth? *W.* $\frac{1}{2}$ of a bushel? $\frac{5}{8}$ of a bushel? $2\frac{5}{6}$ bushels?

10. If a bushel of wheat costs \$2.10, what must be given for 2 bushels? *W.* For $\frac{2}{5}$ of a bushel? For $5\frac{3}{7}$ bushels?

11. *W.* An acre of land is valued at \$314.27: what are 2 acres of such land worth? $\frac{3}{4}$ of an acre? $5\frac{7}{8}$ acres?

12. *W.* If I pay \$4.37 for a cord of wood, what must I pay for 3 cords of such wood? For $4\frac{3}{5}$ cords? For $7\frac{5}{9}$ cords? For $\frac{6}{11}$ of a cord?

13. What will $\frac{5}{9}$ of a pound of indigo cost, at \$4 per pound? *W.* At \$3.40? At \$8.96?

14. *W.* What will $1\frac{2}{3}$ bushels clover-seed cost, at 8 dollars per bushel? At \$9.70? At \$11.23?

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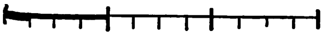
***REVIEW.*** — *When is a fraction said to be proper?* (82.) Illustrate by an object. *Describe an improper fraction.* (83.) Illustrate by an object. *What is a mixed number? (84.)*

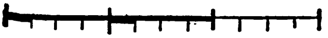
## 97. TO MULTIPLY A FRACTION BY A FRACTION OR MIXED NUMBER.

**PROBLEM.**—Multiply $\frac{2}{3}$ by $\frac{3}{4}$; or, what is $\frac{3}{4}$ of $\frac{2}{3}$?

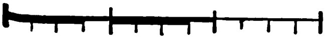
ILLUSTRATIONS.



$\frac{1}{4}$ of $\frac{1}{3}=\frac{1}{12}$.



$\frac{1}{4}$ of $\frac{2}{3}=\frac{2}{12}$.



$\frac{3}{4}$ of $\frac{2}{3}=\frac{6}{12}=\frac{1}{2}$.

**ANALYSIS.**—1. If $\frac{1}{3}$ is divided into 4 equal parts, each part is $\frac{1}{4}$ of $\frac{1}{3}$: as there are 3 thirds in a unit, each of which may be divided into 4 such parts, the unit contains 12 such parts, and each part is $\frac{1}{12}$.

2. Since $\frac{1}{4}$ of $\frac{1}{3}$ is $\frac{1}{12}$, $\frac{1}{4}$ of $\frac{2}{3}=\frac{2}{12}$.

3. Since $\frac{1}{4}$ of $\frac{2}{3}$ is $\frac{2}{12}$, $\frac{3}{4}$ of $\frac{2}{3}=\frac{6}{12}$.

**RULE.**—*Multiply the numerators together for a new numerator, and the denominators for a new denominator.*

**NOTE.**—Mixed numbers may be changed to improper fractions when more convenient, or when the operation can be shortened by cancellation. Fractions with the word "of" between them are sometimes called "*compound fractions.*" The word "of" is equivalent to the word "times," or the sign of multiplication; thus, $\frac{3}{4}$ of $\frac{2}{3}=\frac{3}{4}$ times $\frac{2}{3}$; or, $\frac{3}{4}\times\frac{2}{3}$.

### ORAL EXERCISES.

1. Multiply $\frac{3}{8}$ by $\frac{7}{12}$; or, what is $\frac{7}{12}$ of $\frac{3}{8}$?

**FORMULA.** $\frac{\cancel{3}}{8}\times\frac{7}{\underset{4}{\cancel{12}}}=\frac{7}{32}$.

2. What is $\frac{5}{7}$ of $\frac{14}{15}$? $\frac{5}{8}$ of $\frac{10}{11}$? $\frac{8}{9}$ of $\frac{18}{19}$? $\frac{6}{7}$ of $\frac{14}{15}$?

3. Multiply $5\frac{1}{2}$ by $\frac{4}{5}$; or, what is $\frac{4}{5}$ of $5\frac{1}{2}$?

**FORMULA.** $5\frac{1}{2}=\frac{11}{\cancel{2}}\times\frac{\overset{2}{\cancel{4}}}{5}=\frac{22}{5}=4\frac{2}{5}$.

4. If $\frac{1}{2}$ of a pie be cut into 2 equal pieces, what part of the whole pie is 1 piece? If cut into 3 equal pieces?

5. A girl, having $\frac{1}{2}$ of an orange, gave $\frac{1}{3}$ of it to her brother: what part of the orange did she give him.

**ANALYSIS.**—$\frac{1}{2}$ of an orange is equal to $\frac{3}{6}$: $\frac{1}{3}$ of $\frac{3}{6}$ of an orange is $\frac{1}{6}$. Hence she gave him $\frac{1}{6}$ *of an orange.*

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

| MODEL.* | (1.) | (2.) | (3.) |
|---|---|---|---|
| $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ | $\frac{2}{3} \times \frac{4}{5}$ | $\frac{6}{7} \times \frac{5}{8}$ | $\frac{8}{9} \times \frac{5}{6}$ |
| $\frac{4}{7} \times \frac{3}{8} = \frac{3}{14}$ | $\frac{6}{7} \times \frac{3}{8}$ | $\frac{3}{9} \times \frac{6}{7}$ | $\frac{7}{11} \times \frac{1}{14}$ |
| $\frac{8}{9} \times \frac{5}{7} = \frac{40}{63}$ | $\frac{5}{9} \times \frac{7}{10}$ | $\frac{5}{8} \times \frac{9}{10}$ | $\frac{9}{10} \times \frac{5}{18}$ |
| $2\frac{1}{3} \times \frac{4}{7} = \frac{7}{3} \times \frac{4}{7} = 1\frac{1}{3}$ | $\frac{4}{11} \times \frac{3}{8}$ | $\frac{3}{4} \times \frac{5}{8}$ | $\frac{8}{9} \times \frac{6}{4}$ |
| $2\frac{4}{5} \times 1\frac{2}{8} = \frac{14}{5} \times \frac{10}{8} = 3\frac{1}{2}$ | $\frac{5}{13} \times \frac{9}{10}$ | $\frac{6}{7} \times \frac{11}{12}$ | $\frac{5}{8} \times \frac{3}{10}$ |

### DRILL CARD EXERCISES.—TABLE IV.

#### MULTIPLICATION OF FRACTIONS.

**Exercise I.** — Multiply **V** by 5. By 6. By 7.
**II.** — Multiply **W** by 8. By 9. By 10.
**III.** — Multiply **X** by 11. By 12. By 13.
**IV.** — Prefix **1** to **V**, and multiply by 2. By 3. By 4.
**V.** — Prefix **2** to **W**, and multiply by 4. By 5. By 6.
**VI.** — Prefix **3** to **X**, and multiply by 7. By 8. By 9.
**VII.** — What is **V** of **1**? **W** of **2**? **X** of **3**?
**VIII.** — What is **V** of **W**? **V** of **X**? **W** of **X**?
**IX.** — Prefix **1** to **V**, and multiply by $\frac{11}{8}$. By $\frac{9}{2}$. By $\frac{11}{1}$.
**X.** — Prefix **2** to **W**, and multiply by $\frac{11}{13}$. By $\frac{10}{11}$. By $\frac{21}{22}$.
**XI.** — Prefix **3** to **X**, and multiply by $2\frac{6}{7}$. By $8\frac{2}{9}$. By $4\frac{3}{4}$.
**XII.** $1 - \mathbf{V} + \mathbf{W} + 2 \times \mathbf{X} =$ what?

#### ORAL AND WRITTEN EXERCISES.

15. If a pound of rice is worth $\$\frac{1}{9}$, what is $\frac{5}{7}$ of a pound worth? *W.* $6\frac{1}{2}$ pounds? $9\frac{2}{3}$ pounds?

16. *W.* How much grain can be put into $7\frac{1}{2}$ bags, if each bag holds $2\frac{1}{8}$ bushels? $2\frac{2}{7}$ bushels?

17. *W.* What will $4\frac{1}{3}$ pounds of sugar cost, at $8\frac{3}{4}$ cents per pound? At $9\frac{4}{5}$ cents? At $11\frac{5}{8}$ cents?

18. What will 114 pounds of tea cost, at $1 per pound? At $\$\frac{1}{2}$ per pound? *W.* At $\$\frac{2}{5}$? At $\$\frac{9}{10}$? At $\$1\frac{3}{10}$?

19. What will 228 bushels of corn cost, at 1 dollar a bushel? At $\$1\frac{1}{2}$ per bushel? At $\$1\frac{1}{8}$?

---

* Change mixed numbers to improper fractions, and cancel when possible.

### WRITTEN EXERCISES.

20. What will $\frac{3}{4}$ of a pound of sugar cost at $19\frac{1}{3}$ cents per pound?

**PROCESS.**

$$\begin{array}{r} 19\frac{1}{3}\text{ cents.} \\ 3 \\ \hline 4)58 \\ \hline 14\frac{1}{2}\text{ cents.} \end{array}$$

**BY CANCELLATION.**

$$\frac{\overset{29}{\cancel{58}}}{\cancel{3}} \times \frac{\cancel{3}}{\underset{2}{\cancel{4}}} = 14\tfrac{1}{2}.$$

FORMULA. $19\frac{1}{3}$ cents $\times \frac{3}{4} = 14\frac{1}{2}$ cents.

ANALYSIS.—If 1 pound costs $19\frac{1}{3}$ cents, $\frac{3}{4}$ of a pound will cost $\frac{3}{4}$ of $19\frac{1}{3}$ cents (or $\frac{1}{4}$ of 3 times $19\frac{1}{3}$ cents),* which is $14\frac{1}{2}$ cents.

21. What will $\frac{3}{4}$ of a bushel of corn weigh, if 1 bushel weighs 56 pounds?

22. If a box of starch weighs $28\frac{2}{3}$ pounds, how many pounds in 3 boxes? In $\frac{2}{3}$ of a box? In $13\frac{2}{3}$ boxes?

23. How many yards in $\frac{3}{5}$ of $176\frac{1}{3}$ yards? In $\frac{4}{7}$ of $343\frac{5}{9}$ yards? In $\frac{5}{9}$ of $113\frac{4}{5}$ yards?

24. If 1 yard of lace is worth $88\frac{1}{4}$ cents, what are $16\frac{2}{3}$ yards worth?

**PROCESS.**

$$\begin{array}{rl} \$.88\frac{1}{4} & \\ 16\frac{2}{3} & \\ \hline 3)1.76\frac{1}{2} & \\ \hline .58\frac{5}{6} & \text{val. } \frac{2}{3}\text{ yd.} \\ 5.32 & \rbrace\text{ val. 16 yds.} \\ 8.8 & \\ \hline \$14.70\frac{5}{6} & \text{val. } 16\frac{2}{3}\text{ yds.} \end{array}$$

FORMULA. $\$.88\frac{1}{4} \times 16\frac{2}{3} = \$14.70\frac{5}{6}$, value $16\frac{2}{3}$ yards.

ANAL. STEPS.—1. *Find value $\frac{2}{3}$ yard.*
2. *Find value 16 yards.*

ANALYSIS.—(1.) $\frac{2}{3}$ of $\$88\frac{1}{4} = \$58\frac{5}{6}$.
(2.) 16 times $\$.88\frac{1}{4} = \$14.12$; the sum of $\$.58\frac{5}{6}$ and $\$14.12 = \$14.70\frac{5}{6}$.

25. If in 1 pane of glass there are $20\frac{1}{4}$ square inches, how many square inches in $4\frac{3}{4}$ such panes?

26. Find the weight of $3\frac{1}{7}$ boxes of raisins, if 1 box weighs $67\frac{1}{3}$ pounds?

27. If there are $90\frac{1}{4}$ quarts of milk in a can, how many quarts in $\frac{2}{9}$ of the can?

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* Either expression may be used at the option of the teacher.

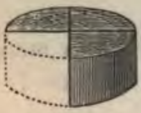
## DIVISION OF FRACTIONS.

### *98.* TO DIVIDE A FRACTION OR MIXED NUMBER BY AN INTEGER.

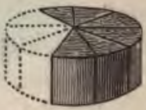
PROBLEM. — (*a.*) I wish to divide $\frac{3}{4}$ of a cake equally among 3 boys: what should each boy receive?

(*b.*) I wish to divide $\frac{2}{3}$ of a cake in the same way: what should each boy receive?

ILLUSTRATION.



$\frac{3}{4} \div 3 = \frac{1}{4}$.



$\frac{2}{3} \div 3 = \frac{2}{9}$.

ANALYSIS. — (*a.*) If 3 boys are to receive $\frac{3}{4}$ of a cake, each boy should receive one-third of $\frac{3}{4}$, or $\frac{1}{4}$ of a cake.

(*b.*) Since 2 thirds cannot be divided into 3 equal parts without a remainder, change the $\frac{2}{3}$ to $\frac{6}{9}$ by cutting each third into 3 equal parts.

If 3 boys are to receive $\frac{6}{9}$ of a cake, each boy should receive one-third of $\frac{6}{9}$, or $\frac{2}{9}$ of the cake.

PRINCIPLE. — (*a.*) I. *Dividing the numerator of a fraction divides the value of the fraction.*

The *number* of parts are decreased, while their *size* remains the same.

(*b.*) II. *Multiplying the denominator of a fraction divides the value of the fraction.*

The *number* of parts remain the same, while the *size* is decreased.

NOTE TO THE TEACHER. — The teacher, in explaining the above principles, should refer to the corresponding illustrations, remembering that children can readily learn to *recite* explanations, and yet be utterly ignorant of their meaning.

#### ORAL EXERCISES.

1. What is 1 third of 3 apples? Of 6 apples? Of 12 lemons? Of 3 fifths of a pound? Of 6 eighths of an ounce? Of 12 sevenths of a gallon?

2. Divide 6 apples and 3 fifths of an apple into 3 equal parts. What is 1 third of $6\frac{3}{5}$ apples? What is 1 fifth of $10\frac{15}{16}$ oranges? Of $15\frac{5}{7}$ melons?

3. Divide 1 orange into 3 equal parts. Divide $\frac{1}{2}$ of an orange into 3 equal parts. In the same way divide $\frac{1}{3}$ of an orange. Divide $\frac{2}{3}$ of an orange. Divide $\frac{5}{8}$ of of an orange.

**Problem.** — Divide $347\frac{2}{3}$ by 5.

PROCESS.

$$5)\underline{347\frac{2}{3}}$$
$$69\frac{8}{15}$$

**Analysis.** — (1.) One-fifth of 347 is 69, with a remainder of 2 units.

(2.) 2 units are equal to $\frac{6}{3}$, which, added to $\frac{2}{3}$, equal $\frac{8}{3}$. One-fifth of $\frac{8}{3}$ is $\frac{8}{15}$, which annexed to $69 = 69\frac{8}{15}$.

**Rule.** — *To divide a fraction by an integer, divide the numerator, or multiply the denominator, by the integer.*

**Note.** — When the dividend is a mixed number, and less than the divisor, change it to an improper fraction, otherwise divide the integral number, and add the remainder, if any, to the fraction; then divide the fraction.

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

| MODEL. | (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|---|
| $\frac{2}{3} \div 4 = \frac{1}{6}$ | $\frac{3}{7} \div 5$ | $\frac{3}{5} \div 7$ | $\frac{5}{9} \div 10$ | $1\frac{1}{2} \div 3$ |
| $\frac{5}{8} \div 2 = \frac{5}{16}$ | $\frac{8}{9} \div 4$ | $\frac{9}{11} \div 5$ | $\frac{10}{11} \div 5$ | $2\frac{1}{4} \div 4$ |
| $\frac{6}{7} \div 5 = \frac{6}{35}$ | $\frac{5}{11} \div 7$ | $\frac{6}{7} \div 4$ | $\frac{8}{9} \div 16$ | $5\frac{3}{4} \div 6$ |
| $\frac{8}{9} \div 2 = \frac{4}{9}$ | $\frac{8}{11} \div 8$ | $\frac{3}{8} \div 3$ | $\frac{5}{12} \div 10$ | $7\frac{1}{2} \div 5$ |

### ORAL AND WRITTEN EXERCISES.

**28.** *W.* Divide $572\frac{4}{7}$ by 3. By 8. By 12. By 14.

**29.** *W.* Divide $210\frac{7}{9}$ by 3. By 5. By 10. By 21.

**30.** *W.* Divide $607\frac{5}{8}$ by 2. By 7. By 41. By 20.

**31.** If I gave $\frac{3}{7}$ of a dime for 11 boxes of matches, what did 1 box cost? *W.* If I gave $3\frac{5}{8}$ dimes?

**Analysis.** — If 11 boxes cost $\frac{3}{7}$ of a dime, 1 box costs one-eleventh of $\frac{3}{7}$ of a dime, or, $\frac{3}{77}$ of a dime.

**32.** What will be the cost of 1 box of pencils, if 8 boxes cost $\$\frac{7}{8}$? Cost $\$\frac{5}{9}$? *W.* Cost $\$2\frac{3}{5}$? $\$1\frac{13}{15}$?

33. What will a lead pencil cost, if 9 cost $\$\frac{4}{5}$? $\$\frac{6}{7}$? *W.* $\$\frac{1}{11}$? $18\frac{7}{11}$ cents? $38\frac{4}{5}$ cents?

34. If 2 oranges cost $10\frac{1}{2}$ cents, what will 4 cost?

**FORMULA.** $10\frac{1}{2}$ cents $\div \cancel{2} \times \overset{2}{\cancel{4}} = 21$ cents.

**ANAL. STEPS.**—1. *Find the cost of* 1 *orange.*
2. *Find the cost of* 4 *oranges.*

35. If 5 lemons cost $10\frac{2}{3}$ cents, what will be the cost of 3? Of 7? Of 5? *W.* Of 19? Of 211?

36. If 3 pounds of butter cost $37\frac{1}{2}$ cents, what will 2 pounds cost? *W.* 30 pounds? 119 pounds?

37. If 5 pounds of rice cost $31\frac{1}{2}$ cents, what will 2 pounds cost? 5 lbs.? *W.* 23 lbs.? 69 lbs.?

38. If 4 apples cost $\frac{5}{6}$ of a shilling, what will 6 apples cost? *W.* 47 apples? 129 apples?

39. If 3 pounds of sugar cost $21\frac{1}{3}$ cents, what will 2 pounds cost? *W.* What will $5\frac{2}{3}$ pounds cost? $11\frac{5}{7}$ pounds? $120\frac{4}{5}$ pounds?

40. If 9 pounds of ginger are worth $50\frac{1}{2}$ cents, what is the value of 5 pounds? *W.* $11\frac{2}{3}$ pounds? $37\frac{5}{7}$ pounds?

41. *W.* $112\frac{3}{4}$ yards of silk will make 9 dresses: how many yards are required for one of the dresses?

42. *W.* 9 pounds of sugar cost $\$1.37\frac{1}{2}$: what is the cost of 1 pound? 5 pounds? $6\frac{2}{3}$ pounds?

43. *W.* If 7 fish weigh $57\frac{1}{2}$ pounds, how much will 9 of equal size weigh?

44. *W.* What will $3\frac{3}{4}$ yards of lace cost, if 12 yards cost $\$13.82\frac{1}{8}$? If 7 yards cost $\$8.13\frac{1}{4}$?

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**REVIEW.**—What is a least common multiple? (76.) What is meant by cancellation? (77.) Upon what principle is cancellation based? What is a fraction? (79.) Illustrate by use of an object. What does the denominator of a fraction show? (80.) Which term of a fraction is the denominator? What is the use of the denominator of a fraction? Illustrate the meaning of $\frac{3}{8}$ by an object. $\frac{1}{4}$.

### *99.* TO DIVIDE AN INTEGER OR MIXED NUMBER BY A FRACTION.

**PROBLEM.**—(*a.*) 48 are how many times $\frac{5}{7}$? (*b.*) $40\frac{5}{6}$ are how many times $\frac{8}{9}$?

PROCESSES.

| (*a.*) | (*b.*) |
|---|---|
| 48 | $40\frac{5}{6}$ |
| 7 | 9 |
| 5)336 | 8)$360\frac{45}{6}$ |
| $67\frac{1}{5}$ | $45(\frac{45}{48})\frac{15}{16}$ |

**ANALYSIS.**—(*a.*) Since 1 unit $= \frac{7}{7}$, 48 units $=$ 48 times $\frac{7}{7}$, or $\frac{336}{7}$: 5 sevenths are contained in 336 sevenths, $67\frac{1}{5}$ times: hence $48 \div \frac{5}{7} = 67\frac{1}{5}$.

(*b.*) Since 1 unit $= \frac{9}{9}$, $40\frac{5}{6}$ units $= 40\frac{5}{6}$ times $\frac{9}{9}$, or $360\frac{45}{6}$ *ninths;* and 8 *ninths* are contained in $360\frac{45}{6}$ *ninths* $45\frac{15}{16}$ times: hence $40\frac{5}{6} \div \frac{8}{9} = 45\frac{15}{16}$.

**NOTE FOR THE TEACHER.**—It will be observed that the above processes are but abbreviated methods of reducing the dividends and divisors to equivalent fractions having a common denominator, after which the division is performed.

**RULE.**—*To divide by a fraction, divide by the numerator, and multiply the quotient by the denominator, of the fraction.*

It is usually more convenient to multiply before dividing. When the divisor is a mixed number, change to an improper fraction. Do not change the dividend to an improper fraction, unless it is smaller than the divisor.

#### WRITTEN EXERCISES.

45. Divide 49 by $\frac{3}{7}$. By $\frac{6}{11}$. By $\frac{4}{11}$. By $\frac{9}{13}$.
46. Divide 186 by $1\frac{3}{4}$. By $2\frac{7}{11}$. By $8\frac{3}{4}$. By $9\frac{3}{5}$.
47. Divide $67\frac{3}{4}$ by $2\frac{3}{4}$. By $4\frac{5}{12}$. By $9\frac{6}{7}$. By $3\frac{4}{7}$.
48. Divide $483\frac{3}{13}$ by $3\frac{5}{7}$. By $9\frac{4}{7}$. By $11\frac{6}{7}$. By $2\frac{9}{13}$.

## **100.** TO DIVIDE A FRACTION BY A FRACTION OR MIXED NUMBER.

### ORAL EXERCISES.

1. How many halves in 1? One is how many times a half? How many quarters in 1? One is how many times a fourth? A fifth? A ninth? A seventh?

2. How many fourths in 2? 2 are how many times $\frac{1}{4}$? 3 are how many times $\frac{1}{5}$? $\frac{2}{3}$? $\frac{3}{6}$? $\frac{1}{7}$?

3. One is how many times $\frac{3}{4}$? $\frac{5}{6}$? $\frac{2}{7}$? $\frac{3}{5}$?

FORMULA. $1 = \frac{4}{4} \div \frac{3}{4} = 1\frac{1}{3}$.

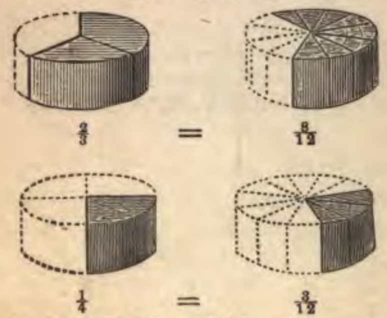
ANALYSIS. — One equals 4 fourths; and 3 fourths are contained in 4 fourths $1\frac{1}{3}$ times.

4. How many quarters in a half? How many times can a quarter be taken from a half?

5. How many sixths in an apple? How many sixths in a third of an apple? In $\frac{1}{5}$? In $\frac{3}{5}$?

PROBLEM. — Divide $\frac{2}{3}$ by $\frac{1}{4}$; or, $\frac{2}{3}$ are how many times $\frac{1}{4}$?

ILLUSTRATION.



$\frac{2}{3}$ = $\frac{8}{12}$
$\frac{1}{4}$ = $\frac{3}{12}$

ANALYSIS. $\frac{2}{3} = \frac{8}{12}$; $\frac{1}{4} = \frac{3}{12}$. 3 *twelfths* are contained in 8 *twelfths* $2\frac{2}{3}$ times: hence $\frac{2}{3} \div \frac{1}{4} = 2\frac{2}{3}$.

FORMULA. $\frac{2}{3} \div \frac{1}{4} = \frac{8}{12} \div \frac{3}{12} = 2\frac{2}{3}$.

NOTE FOR THE TEACHER. $\frac{2}{3}$ and $\frac{1}{4}$ must be changed to the same denomination before dividing. In practice, the numerator of each fraction must be multiplied by the denominator of the other fraction. This is really done by inverting the divisor, and multiplying.

RULE. — *Change the integral or mixed numbers to improper fractions, invert the divisor, and multiply or cancel the terms as in the multiplication of fractions.*

### ORAL, SLATE, AND BLACKBOARD EXERCISES.

| MODEL. | (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|---|
| $\frac{1}{1} \div \frac{3}{4} = \frac{4}{3} = 1\frac{1}{3}$ | $\frac{5}{6} \div \frac{3}{7}$ | $1 \div \frac{3}{4}$ | $6 \div \frac{3}{8}$ | $\frac{2}{3} \div \frac{4}{7}$ |
| $1 \div \frac{3}{4} = \frac{4}{3} = 1\frac{1}{3}$ | $\frac{8}{9} \div \frac{4}{11}$ | $1 \div \frac{3}{11}$ | $3 \div \frac{4}{7}$ | $\frac{5}{8} \div \frac{5}{9}$ |
| $\frac{6}{1} \div \frac{5}{6} = \frac{36}{5} = 7\frac{1}{5}$ | $\frac{2}{7} \div \frac{3}{4}$ | $1 \div \frac{5}{8}$ | $5 \div \frac{8}{9}$ | $\frac{3}{5} \div \frac{4}{5}$ |
| $6 \div \frac{5}{6} = \frac{36}{5} = 7\frac{1}{5}$ | $\frac{8}{9} \div \frac{6}{7}$ | $1 \div \frac{4}{7}$ | $9 \div \frac{4}{7}$ | $\frac{3}{5} \div \frac{4}{5}$ |
| $\frac{5}{7} \div \frac{2}{3} = \frac{15}{14} = 1\frac{1}{14}$ | $\frac{4}{5} \div \frac{6}{7}$ | $1 \div \frac{5}{13}$ | $11 \div \frac{2}{3}$ | $\frac{6}{7} \div \frac{2}{5}$ |
| $\frac{4}{3} \div \frac{3}{4} = \frac{16}{9} = 1\frac{7}{9}$ | $\frac{3}{4} \div \frac{8}{9}$ | $1 \div \frac{7}{11}$ | $4 \div \frac{7}{9}$ | $\frac{8}{9} \div \frac{3}{7}$ |

### ORAL AND WRITTEN EXERCISES.

**49.** Among how many persons can \$$\frac{8}{9}$ be distributed, if each receives $\frac{1}{5}$ of a dollar? \$$\frac{1}{8}$? \$$\frac{1}{20}$? \$$\frac{1}{50}$?

**50.** Among how many persons can \$$7\frac{2}{3}$ be distributed, if each receives \$$\frac{1}{2}$? \$$\frac{1}{3}$? \$$\frac{1}{4}$? \$$\frac{1}{10}$? *W.* \$$\frac{3}{5}$? \$$\frac{5}{9}$? \$$\frac{7}{11}$?

**51.** Among how many persons can \$$\frac{39}{40}$ be distributed, if each receives \$$\frac{1}{2}$? \$$\frac{1}{3}$? \$$\frac{2}{3}$? \$$\frac{3}{4}$? *W.* \$$\frac{5}{11}$? \$$\frac{7}{12}$? \$$\frac{13}{17}$?

**52.** How many times is $\frac{2}{3}$ of a dollar contained in \$1? In \$2? In \$5? *W.* In \$$7\frac{2}{3}$? In \$$13\frac{5}{7}$? In \$$123\frac{4}{11}$?

**53.** At $\frac{2}{3}$ of a cent apiece, how many oranges can be bought for 2 cents? For 5 cents? For $8\frac{2}{3}$ cents? *W.* For $437\frac{2}{3}$ cents? For \$6.82? For \$$4.53\frac{5}{8}$?

**54.** If there are 100 cents in a dollar, how many cents in \$$\frac{1}{2}$? \$$\frac{2}{3}$? \$$\frac{7}{10}$? \$$\frac{9}{10}$? *W.* \$$\frac{5}{11}$? \$$\frac{7}{9}$? \$$\frac{8}{17}$? \$$\frac{1}{13}$?

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**REVIEW. — What is the use of the numerator of a fraction? (81.) Illustrate by the use of an object. What does the numerator show in $\frac{3}{8}$ of an apple? What is meant by a proper fraction? (82.) When is a fraction said to be improper? (83.) *Which of the following fractions are proper, and which are improper, two-thirds, three-thirds,* seven-fifths, five-sevenths?**

PROCESS.

100 cents.
5
11)500
45+ cents.*

FORMULA. $\$1.00 \times \frac{5}{11} = 45+$ cents.

ANALYSIS. — Since there are 100 cents in $1, in $\frac{5}{11}$ of a dollar, there are $\frac{5}{11}$ of 100 cents (or $\frac{1}{11}$ of 5 times 100 cents), which is 45+ cents.

55. *W.* How many cents and mills in $\frac{8}{9}$ of a dollar? In $\$\frac{4}{5}$? In $\$\frac{6}{7}$? In $\$\frac{12}{13}$? In $\$\frac{6}{21}$?

FORMULA. $1000 \text{ mills} \times \frac{8}{9} = \$.888+$.

ANALYSIS. — Since there are 1000 mills in $1, in $\frac{8}{9}$ of a dollar there are $\frac{8}{9}$ of 1000 mills (or $\frac{1}{9}$ of 8 times 1000 mills), which is 888 mills = $.888.

NOTE. — For additional examples see drill cards.

### DRILL CARD EXERCISES. — TABLE IV.

#### DIVISION OF FRACTIONS.

Exercise I. — Divide **V** by 5. By 6. By 7.
II. — Divide **W** by 8. By 9. By 10.
III. — Divide **X** by 11. By 12. By 13.
IV. — Prefix 1 to **V**, and divide by 1. By 2. By 3.
V. — Prefix 2 to **W**, and divide by 4. By 5. By 6.
VI. — Prefix 3 to **X**, and divide by 7. By 8. By 9.
VII. — 28 is how many times **V**? **W**? **X**?
VIII. — $200\frac{3}{5}$ is how many times **V**? **W**? **X**?
IX. — Prefix 1 to **V**, and divide by $\frac{11}{8}$. By $\frac{9}{2}$. By $\frac{11}{1}$.
X. — Prefix 2 to **W**, and divide by $\frac{11}{13}$. By $\frac{10}{11}$. By $\frac{21}{22}$.
XI. — Prefix 3 to **X**, and divide by $3\frac{5}{7}$. By $8\frac{8}{11}$. By $4\frac{5}{7}$.
XII. — $2 - \mathbf{V} \div 1 + \mathbf{W} + 3 \times \mathbf{X} =$ what?

#### UNITED STATES CURRENCY.

XIII. — Change $**V** to cents and mills.
XIV. — Change $**W** to cents and mills.
XV. — Change $**X** to cents, mills, and tenths.

#### PRACTICAL PROBLEMS, ORAL AND WRITTEN.

NOTES. — (1.) For each of these problems, the pupil should prepare a clear and logical analysis.

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* *The sign* + affixed to answers denotes a remainder of too little value to be *noticed in business* transactions.

(2.) This analysis should be expressed in words, and also in a well-arranged arithmetical formula, which may be a visible plan of the solution.

(3.) The formula should be reduced by the simplest and shortest methods, cancelling when there is opportunity.

(4.) To secure *accuracy*, the teacher should require many analyses in writing, with their corresponding formulas: both should be carefully criticised with reference to simplicity of method, clearness of statement, and neatness of execution.

(5.) To secure *readiness*, the oral analyses of similar questions should be required, the formulas being written upon the blackboard.

56. *W.* 7 equal bins contain $421\frac{1}{5}$ bushels of salt: how much in each bin?

PROCESS.

$7)421\frac{1}{5}$

$60\frac{6}{35}$ bushels.

FORMULA. $421\frac{1}{5} \div 7 = 60\frac{6}{35}$ bushels, contents 1 bin.

ANALYSIS.—If 7 bins contain $421\frac{1}{5}$ bushels of salt, 1 bin contains 1 seventh of $421\frac{1}{5}$, or $60\frac{6}{35}$ bushels.

57. *W.* If 5 equal piles of coal contain $1941\frac{2}{3}$ tons, how many tons in each pile?

58. *W.* I filled $123\frac{4}{7}$ barrels of apples from 10 loads of apples: how many barrels in each load?

59. The aggregate weight of two men is $300\frac{2}{16}$ pounds: what is the average weight of each? What is the average weight of each, if the aggregate weight of 5 is $830\frac{2}{7}$ pounds? Of 12 is $1680\frac{3}{5}$ pounds?

60. If 87 cows cost $\$3870\frac{1}{5}$, what is their average cost?

61. What is the value of $\frac{5}{9}$ of a dollar?

PROCESS.

$\$1.000$

$5$

$9)5.000$

$.555+$

FORMULA. $\$1.000 \times \frac{5}{9} = \$555+$, value in cents and mills.

ANALYSIS.—Since in $\$1$ there are 1000 mills, in $\frac{5}{9}$ of a dollar there are $\frac{5}{9}$ of 1,000 mills (or $\frac{1}{9}$ of 5 times 1000 mills), *which is 555* mills $= \$.555+$.

62. If 3 yards of calico cost $38\frac{1}{2}$ cents, what does 1 yard cost? *W.* 5 yards? $6\frac{3}{7}$ yards?

**FORMULA.** $.385 $\div$ 3 = cost 1 yard.
$.385 $\div 3 \times 6\frac{3}{7}$ = cost of $6\frac{3}{7}$ yards.

63. A tract of 17 acres of land cost $34: what was the price per acre? *W.* If it cost \$$263\frac{5}{7}$? If it cost \$$215\frac{4}{9}$?

### WRITTEN EXAMPLES.

64. If 7 firkins of butter weigh $680\frac{3}{4}$ pounds, what will $\frac{3}{4}$ of a firkin weigh? What will it be worth at $30\frac{2}{3}$ cents per pound?

**FORMULA.** $680\frac{3}{4}$ lbs. $\div$ 7 = weight of 1 firkin.
$680\frac{3}{4}$ lbs. $\div 7 \times \frac{3}{4}$ = weight of $\frac{3}{4}$ of a firkin.
\$$.30\frac{2}{3} \times (680\frac{3}{4} \div 7 \times \frac{3}{4})$ = cost of $\frac{3}{4}$ of a firkin, at $30\frac{2}{3}$ cents.

65. Paid 25 laborers \$$206\frac{1}{4}$ for 6 days' or 1 week's work: what were the wages of each per day?

66. 16 car-loads of wood, of $3\frac{5}{8}$ cords each, cost \$$297\frac{3}{4}$: what was the cost of 1 cord?

67. If 12 yards of silk are worth \$$16.46\frac{2}{3}$, what is the value of 1 yard? $\frac{1}{7}$ of a yard? $\frac{3}{11}$ of a yard? $2\frac{5}{6}$ yards?

68. What cost $6\frac{2}{3}$ barrels of flour, if $4\frac{1}{3}$ barrels cost \$$17.37\frac{1}{2}$?

**PROCESS.**

$$\begin{array}{r} 13)\$17.375 \\ \hline 1.336+ \\ 20 \\ \hline \$26.720+ \end{array} = \text{cost } 6\tfrac{2}{3} \text{ lbs.}$$

**ANAL. STEPS.** — (1.) *Find cost $\frac{1}{3}$ barrel.*
(2.) *Find cost $\frac{20}{3}$ barrels.*

**FORMULA.** $17.375 $\div 13 \times 20 = 26.72$.

BY CANCELLATION.

$$\$17.375 \div \frac{13}{\cancel{3}} \times \frac{20}{\cancel{3}^{*}} = \$26.720+.$$

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* *SUGGESTION.* — When, in dividing one fraction by another, the denominators are alike, they may be disregarded, since they would be cancelled in the process.

69. If 17 baskets of strawberries cost $110\frac{1}{2}$ cents, what will $2\frac{2}{3}$ baskets cost?

70. What cost $1\frac{1}{2}$ bushels of wheat, if $46\frac{1}{3}$ bushels cost \$$82.30\frac{1}{4}$?

71. If $25\frac{2}{5}$ yards of flannel are worth \$$12.87\frac{2}{3}$, what is the value of $\frac{1}{2}$ a yard?

72. What is the cost of $\frac{1}{8}$ of a pound of tea, if $7\frac{2}{7}$ pounds cost \$$8\frac{2}{5}$?

73. If $36\frac{2}{3}$ acres of wheat yield 635 bushels, what is the yield of $5\frac{4}{5}$ acres?

74. What cost 56 lemons, at \$$1.12\frac{1}{2}$ a dozen?

75. Bought a barrel of sugar at $14\frac{1}{8}$ cents a pound. The gross weight was $256\frac{1}{2}$ pounds; weight of barrel, $13\frac{3}{4}$ pounds: what did I pay for the sugar?

76. Paid \$$34.11\frac{1}{8}$ for a barrel of sugar. Gross weight, $247\frac{3}{4}$ pounds; weight of barrel, $12\frac{1}{2}$ pounds: what was the cost of 100 pounds?

77. Bought 3 crocks of butter, weighing $25\frac{7}{16}$, $29\frac{3}{4}$, and $27\frac{1}{8}$ pounds; the empty crocks weigh $5\frac{3}{16}$, $5\frac{3}{4}$, and $5\frac{7}{8}$ pounds: what did the butter cost at $23\frac{3}{4}$ cents a pound?

78. Sold butter of first and second crocks at 26 cents what did I receive?

79. Sold butter in third crock at 28 cents: what did I receive?

80. What did I make on the whole lot of butter?

81. If a man can saw $13\frac{1}{3}$ cords of wood in $\frac{1}{6}$ of a month, how many cords can he saw in $\frac{3}{4}$ of a month?

**FORMULA.** $13\frac{1}{3} \div \frac{1}{6} \times \frac{3}{4}$; or, $\frac{13\frac{1}{3}}{\frac{1}{6}} \times \frac{3}{4} = \frac{\overset{10}{\cancel{40}} \times 6 \times \cancel{3}}{\cancel{3} \times \cancel{4}} = 60.$

**ANAL. STEPS.**—(1.) *Find what he can saw in 1 month.*
(2.) *Find what he can saw in $\frac{3}{4}$ of a month.*

82. If a bird can fly $10\frac{3}{4}$ miles in $\frac{1}{3}$ of an hour, how far can it fly in $2\frac{1}{2}$ hours?

83. If $\frac{2}{5}$ yard of velvet costs $74\frac{1}{4}$ cents, how much must be given for $13\frac{2}{5}$ yards?

84. If $\frac{3}{5}$ of a steeple casts a shadow $83\frac{2}{3}$ feet long, how long is the shadow cast by $\frac{5}{9}$ of it?

## MISCELLANEOUS PROBLEMS. — REVIEW.

**Note.** — Should these problems be considered too difficult, the class may omit them until Decimals and Denominate Numbers have been finished.

**N.B.** — The final results should always be given in dollars and cents, instead of fractions of a dollar.

85. How many yards of muslin can be bought for 57 cents, at $9\frac{1}{2}$ cents per yard?

**Formula.** 57 cents $\div \frac{19}{2}$ cents $= 6$ (yards).

**Analysis.** — If 1 yard costs $9\frac{1}{2}$ cents ($\frac{19}{2}$), as many yards can be bought for 57 cents as $9\frac{1}{2}$ cents are contained times in 57 cents, or 6 yards.

86. How many yards of tape can be bought for 11 cents, at $2\frac{3}{5}$ cents per yard? *W.* At $1\frac{5}{8}$ cents? At $9\frac{3}{5}$ cents?

87. How many yards of silk, at $97\frac{2}{3}$ cents per yard, can be bought for $97\frac{2}{3}$ cents? For 2 times $97\frac{2}{3}$ cents? For $2\frac{1}{2}$ times $97\frac{2}{3}$ cents? *W.* For \$1? For \$5.25? For \$6.75?

88. At $\frac{1}{10}$ of a dollar per quart, how many quarts of vinegar can be bought for $\$\frac{1}{3}$? For $\$\frac{5}{9}$? *W.* For $\$16\frac{3}{11}$? For $\$27\frac{5}{8}$?

**Formula.** $\$\frac{1}{3} \div \$\frac{1}{10} = 3\frac{1}{3}$ (quarts).

**Analysis.** — If $\$\frac{1}{10}$ will buy 1 quart, $\frac{1}{3}$ of a dollar will buy as *many quarts as* $\$\frac{1}{10}$ is contained times in $\$\frac{1}{3}$, or $3\frac{1}{3}$ quarts.

89. At \$$2\frac{1}{4}$ per yard, how many yards of carpet can be purchased for \$$5\frac{1}{2}$? For \$9? *W.* For \$$17\frac{5}{11}$? For \$$186\frac{8}{9}$?

90. If 3 oranges cost $10\frac{1}{2}$ cents, what will 4 cost?

**Anal. Steps.** — (1.) *Find cost of 1 orange.*
(2.) *Find cost of 4 oranges.*

91. What will 8 lemons cost, if 5 cost $10\frac{2}{3}$ cents? *W.* If 9 cost $29\frac{5}{8}$ cents? If 27 cost $75\frac{4}{7}$ cents? If 47 cost \$$2.25\frac{1}{2}$?

92. What will $7\frac{2}{3}$ pounds of butter cost, if 5 pounds cost \$$1.25\frac{2}{3}$? If 8 pounds cost \$$2.87\frac{1}{2}$? If 95 pounds cost \$$27.49\frac{3}{4}$?

93. If 5 pounds of rice cost $31\frac{1}{2}$ cents, what will 2 pounds cost? *W.* 8 pounds? $11\frac{1}{2}$ pounds? $113\frac{5}{7}$ pounds?

94. If 4 apples cost $\frac{5}{6}$ of a shilling, what will 1 apple cost? 6 apples? *W.* 37 apples? 387 apples?

95. What will 3 apples cost, if 1 apple costs $\frac{5}{6}$ of a cent? If 2 apples cost $1\frac{2}{3}$ cents? If $\frac{1}{2}$ of an apple costs $\frac{7}{8}$ of a cent? *W.* If $\frac{2}{3}$ costs $\frac{13}{14}$ of a cent? If $\frac{7}{8}$ costs $1\frac{3}{4}$ of a cent?

96. What will $\frac{1}{3}$ of a pound of sugar cost, if 1 pound costs $16\frac{2}{3}$ cents? *W.* If 8 pounds cost \$$1.25\frac{1}{2}$? If $7\frac{1}{2}$ pounds cost $99\frac{3}{8}$ cents? If $21\frac{3}{4}$ pounds cost \$$12.10\frac{3}{4}$?

97. Change $\dfrac{\frac{2}{3}}{\frac{2}{5}}$ to a simple fraction.

**Suggestion.** — Since the value of a fraction is the quotient of the numerator divided by the denominator, the above expression $= \frac{2}{3} \div \frac{2}{5}$.

**Note.** — Fractions having a fractional quantity for either numerator or denominator, or both, are sometimes called "*complex fractions.*" They are simply the fractional expression for the division of fractions. $2\frac{2}{5} \div \frac{7}{8}$ *may be written* $\dfrac{2\frac{2}{5}}{\frac{7}{8}}$.

98. Change $\frac{\frac{7}{8}}{\frac{11}{12}}$ to a simple fraction.

99. Find the value of $\frac{1\frac{4}{15}}{2\frac{1}{55}}$.

100. Reduce $\frac{3\frac{7}{8}}{5}$ to its simple fractional expression.

101. Change $\frac{7}{\frac{11}{13}}$ to a simple fraction.

102. Reduce $\frac{\frac{5}{8}}{\frac{3}{7}} \times 1\frac{7}{20}$.

**Formula.** $\frac{5}{8} \div \frac{3}{7} \times 1\frac{7}{20} = \frac{5}{8} \times \frac{7}{3} \times \frac{27}{20} = \frac{63}{32} = 1\frac{31}{32}$. Ended May

## PROBLEMS CONTAINING NOT MORE THAN THREE ELEMENTARY QUESTIONS.

103. If $2\frac{1}{5}$ chests of tea cost $\$63\frac{5}{7}$, what will $6\frac{2}{3}$ chests cost?

**Anal. Steps.** — (1.) *Find the cost of* 1 *chest:* $63\frac{5}{7} \div 2\frac{1}{5} = 28\frac{74}{77}$ = cost of 1 chest.

(2.) *Find the cost of* $6\frac{2}{3}$ *chests:* $28\frac{74}{77} \times 6\frac{2}{3} = 193\frac{17}{231}$; or, $\$193.073+$, cost of $6\frac{2}{3}$ chests.

**Formula.** $63\frac{5}{7} \div 2\frac{1}{5} \times 6\frac{2}{3} = \frac{446}{7} \times \frac{5}{11} \times \frac{20}{3} = \$193\frac{17}{231}$.

104. If $2\frac{1}{2}$ bushels of apples cost 7 shillings, what will $\frac{1}{7}$ of a bushel cost? *W.* $\frac{5}{9}$ of a bushel. $6\frac{5}{8}$ bushels? $143\frac{5}{7}$ bushels?

105. If a peck of corn is worth $15\frac{3}{4}$ cents, what are 3 pecks worth? 4 pecks, or a bushel? *W.* $2\frac{1}{2}$ bushels? $38\frac{4}{7}$ bushels?

106. If 3 pecks of rye are worth $\$\frac{7}{8}$, what is 1 peck worth? *W.* What are 4 pecks, or 1 bushel, worth? 3 bushels? $29\frac{2}{3}$ bushels.

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**Review.** — Describe a mixed number. (84.) Illustrate by objects. Give Principles I. and II. (85.) How are fractions written? How is the value of a fraction affected by multiplying both terms by the same number? (86.) When are fractions said to be expressed in the lowest terms? (87.)

107. If 2 pecks of barley are worth \$$\frac{8}{11}$, what is 1 bushel worth? *W.* $5\frac{1}{2}$ bushels? $27\frac{3}{4}$ bushels? $346\frac{7}{8}$ bushels?

108. If 4 pounds of sugar cost 50 cents, what will 2 pounds cost? *W.* 100 pounds, or 1 cwt.? 5 cwt.? $7\frac{3}{9}$ cwt.?

109. If a piece of meat weighs 100 pounds, or 1 cwt., what will $\frac{1}{2}$ of it weigh? *W.* $\frac{3}{4}$ of it? $\frac{5}{9}$ of it? $\frac{6}{7}$ of it?

110. A barrel of flour weighs 196 pounds: how much will $\frac{1}{2}$ of a barrel weigh? *W.* $\frac{5}{8}$ of a barrel? 16 barrels? $23\frac{1}{3}$ barrels?

111. A bushel of wheat weighs 60 pounds: what will 2 bushels weigh? *W.* $\frac{2}{3}$ of a bushel? $46\frac{3}{5}$ bushels?

112. 1 rod of wire fence costs 98 cents, what will be the cost of 2 rods? *W.* Of $89\frac{1}{2}$ rods? Of 320 rods, or 1 mile? Of 2 miles? Of $63\frac{1}{2}$ miles?

113. 1 acre of land is worth \$$30\frac{2}{3}$ · at the same rate, what are 2 acres worth? *W.* $\frac{2}{3}$ of an acre? $3\frac{4}{5}$ acres? 640 acres, or 1 section? $5\frac{1}{2}$ sections?

114. 1 cubic foot of stone is worth $2\frac{1}{2}$ cents, what are 3 cubic feet worth? *W.* $16\frac{2}{3}$ cubic feet? $24\frac{3}{4}$ cubic feet, or 1 perch? 128 cubic feet, or 1 cord? $3\frac{2}{3}$ cords?

115. A man's age is $47\frac{1}{2}$ years; his son is $\frac{1}{2}$ as old: how old is his son? *W.* His wife is $\frac{5}{8}$ as old: how old is she?

116. *W.* A merchant has \$300.25 in the bank, and $\frac{3}{5}$ as much in his safe: how much has he in his safe? How much has he in both?

**Review.** — Give the analysis for reducing an improper fraction to a mixed number. (90.) What is meant by a common denominator? (92.) Can fractions having different denominators be added or subtracted? (93.) Why?

117. 20 is 5 times a certain number: what is $\frac{1}{2}$ of it? *W.* 200 is $6\frac{2}{3}$ times a certain number: what is $\frac{3}{5}$ of it? $101\frac{1}{4}$ is $\frac{6}{7}$ of a certain number: what is $\frac{3}{5}$ of it? $671\frac{2}{3}$ is $\frac{5}{7}$ of a certain number: what is $3\frac{3}{4}$ times the number?

118. 60 is $\frac{1}{2}$ of a certain number: what is $\frac{1}{3}$ of it? *W.* 60 is $\frac{3}{4}$ of a certain number: what is $\frac{5}{8}$ of it? 60 is $3\frac{1}{2}$ times a certain number: what is $\frac{4}{5}$ of it?

119. Matthew had $\frac{5}{8}$ of a bushel of peaches. He sold $\frac{3}{4}$ of them: how many did he sell? *W.* How many had he left? Had he sold $\frac{2}{5}$ of a bushel, how many would he have had left?

120. *W.* Helen paid $\$\frac{5}{6}$ for a geography, and $\$\frac{3}{4}$ for a reader: how much more did she give for the one than the other? How much for both? How much had she left of a $\$10$ bill?

121. Alexander has $\$9\frac{7}{8}$: how much more must I give him to make $\$10$? *W.* To make $\$13\frac{1}{2}$? To make $\$37\frac{5}{7}$?

122. Robert earns $\$7\frac{3}{4}$ a week; Joseph earns $\$1\frac{1}{4}$ less than Robert: how much does Joseph earn? *W.* How much do both earn? How much do they lack of earning $\$20$?

123. Ruth will be 21 years old in $3\frac{5}{12}$ years: how old is she now? Maude is $18\frac{7}{12}$ years old: what is the sum of their ages?

124. A boy sold $\frac{3}{4}$ of his nuts: what part had he remaining?

ANALYSIS.—If a boy sold $\frac{3}{4}$ of his nuts, he must have remaining the difference between $\frac{3}{4}$ and $\frac{4}{4}$ (the whole) of his nuts, or $\frac{1}{4}$.

125. If he sold what he had left for 25 cents, for how

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*REVIEW.—Give* the analysis of a short example in addition of fractions. (93) *Give the analysis of* a short example in subtraction of fractions. (94).

much could he have sold the whole at the same rate? How much would that lack of $5?

126. *W.* A gambler lost in play $\frac{1}{4}$ and $\frac{2}{3}$ of his money: what part of the whole did he lose? What part had he left? If the amount left was $125.25, how much had he at first?

127. *W.* Bought $7\frac{2}{3}$ yards of cloth at one time, and $4\frac{5}{7}$ yards at another: what is the cost of the whole at $\$2\frac{3}{7}$ per yard?

128. *W.* Bought 37 yards of muslin; kept $9\frac{5}{8}$ yards; sold the remainder at $\$1\frac{3}{5}$: how much did I get for it?

129. *W.* A boy spent $\$1\frac{5}{8}$ for a book, $\$2\frac{3}{4}$ for $\frac{3}{5}$ of a ream of paper, and had $\$1\frac{2}{3}$ left: how much had he at first?

130. John wishes to pay $\$\frac{18}{25}$ for a pair of skates; he has $\frac{3}{5}$ of a dollar: how much more does he need? *W.* How much would he need to buy 3 pairs of such skates? One dozen pairs?

131. By mistake a man paid $\$\frac{1}{2}$, which was $\frac{1}{4}$ of a dollar too much: how much ought he to have paid?

132. *W.* By mistake a man paid $\$14\frac{7}{12}$, which was $5\frac{1}{10}$ dollars too much: how much ought he to have paid?

133. *W.* A boy by mistake subtracted $\frac{7}{8}$ instead $\frac{8}{7}$: was his answer too large, or too small, and how much? What would have been the condition of his answer, had he subtracted $\frac{4}{5}$? $2\frac{1}{7}$? $\frac{1}{11}$?

134. *W.* A man bought eggs for $\$3\frac{8}{9}$, and sold them for $\$3\frac{9}{10}$: did he make, or lose, and how much?

135. *W.* A man bought $13\frac{2}{3}$ dozen of eggs at $\frac{3}{10}$ of a

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**Review.** — Tell how fractions are added. (93.) Tell how they are subtracted. (94.) *Tell how fractions are* multiplied by an integer. (95.) Tell how a number *is multiplied by a fraction.* (96.) How do you take the fractional part of a *number.*

dollar a dozen, and sold them at $\$\frac{3}{11}$: did he make, or lose, in the transaction, and how much?

136. *W.* If $\$2\frac{5}{8}$ will pay a woman for working $2\frac{1}{2}$ days, how much will pay for $5\frac{1}{2}$ days' work? For 6 days', or 1 week's work? For 2 weeks'? For $5\frac{5}{6}$ weeks'?

137. If 2 barrels of beef are worth $\$14\frac{4}{9}$, what is 1 barrel worth? *W.* What are 10 barrels? $12\frac{2}{3}$ barrels?

138. *W.* If $5\frac{1}{2}$ pounds of sugar cost $\$\frac{3}{5}$, what will $3\frac{2}{3}$ pounds cost? What will $\frac{5}{8}$ of a pound cost? $\frac{7}{13}$ of a pound? 250 pounds?

### PROBLEMS CONTAINING NOT MORE THAN FOUR ELEMENTARY QUESTIONS.

**139. If 100 oranges cost $\$3.20\frac{2}{3}$, what will be the cost of $5\frac{1}{2}$ boxes, each containing 250 oranges?**

**FORMULA.** $\$3.20\frac{2}{3} \div 100 \times 250 \times 5\frac{1}{2} = \$3.20\frac{2}{3} \times \frac{1}{\underset{2}{\cancel{100}}} \times \frac{\overset{5}{\cancel{250}}}{1} \times \frac{11}{2} = \$44.09+$.

**PROCESS.**

$2)\$3.20\frac{2}{3}$

$1.60\frac{1}{3}$

$5$

$8.01\frac{2}{3}$

$11$

$2)88.18\frac{1}{3}$

$\$44.09\frac{1}{6}$;

or, $\$44.09+$

**ANAL. STEPS.** — (1.) *Find cost of 1 orange.*

(2.) *Find cost of 250 oranges, or 1 box.*

(3.) *Find cost of $5\frac{1}{2}$ boxes.*

**SUGGESTION.** — Cancel the common factor 50.

140. If 2 pounds of indigo cost $\$5\frac{2}{3}$, how much will 1 pound cost? *W.* $\frac{5}{9}$ of a pound? $13\frac{5}{7}$ pounds?

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**REVIEW.** — How is a fraction multiplied by a fraction or mixed number? (97.) How is a fraction divided by an integer? (98.) How is a mixed number divided by a fraction? (99.) Give the rule for dividing a fraction by a fraction. (100.)

141. If I buy 8 eggs for 25 cents, and sell 7 eggs for 25 cents, will I make, or lose, and how much on each egg? *W.* How much on each dozen? How much on 25 dozen? How much on a barrel containing $80\frac{5}{12}$ dozen? On 5 barrels of the same size?

142. *W.* $7\frac{1}{2}$ pounds of coffee cost \$$1.87\frac{1}{2}$: what will a bag cost weighing $63\frac{2}{3}$ pounds? What will 5 bags of the same size cost?

143. *W.* 7 pounds of cheese are worth $87\frac{1}{2}$ cents: what is a cheese worth weighing $45\frac{1}{2}$ pounds? How much is that a ton of 2,000 pounds?

144. (*a.*) 6 is two times what number? (*b.*) $\frac{2}{3}$ of what number?

**FORMULA.** (*a.*) $6 \div 2 = 3$. (*b.*) $6 \div \frac{2}{3} = 9$.

**ANALYSIS.**—(*b*). If 6 is $\frac{2}{3}$ times a certain number, the number must contain as many units as $\frac{2}{3}$ is contained times in 6, or 9 (times or units).

145. 25 bushels of apples are twice a certain quantity: how many bushels in the quantity? *W.* 138 bushels are 3 times what quantity? How many bushels in the quantity, if 43 bushels are 5 times the quantity? If they are $\frac{1}{2}$ of the quantity? $\frac{2}{3}$? $\frac{7}{11}$?

146. What is the monthly salary of a clerk, if \$60 is 3 times his salary? If it is $\frac{1}{2}$ of it? If it is $\frac{3}{4}$ of it? *W.* If it is $\frac{7}{11}$ of it?

147. 7 is $\frac{3}{4}$ of what number? $\frac{1}{2}$ is $\frac{1}{2}$ of what number? $\frac{2}{3}$ is $\frac{2}{3}$ of what number? *W.* $6\frac{5}{11}$ is $\frac{5}{8}$ of what number? $\frac{8}{9}$ is 3 times what number? $4\frac{5}{7}$ is $8\frac{2}{7}$ times what number?

148. What is the value of my horse, if \$40 is twice its value? Is $\frac{1}{2}$ of its value? *W.* Is $\frac{7}{9}$ of its value? Is $\frac{11}{23}$ of its value?

149. I have $320\frac{1}{5}$ acres of land: how much will be left after selling $\frac{3}{5}$ of it?

150. I had $34.26, and lost $\frac{2}{7}$ of it: how much was left?

151. Five hundred dollars is $\frac{2}{3}$ of the money I had in a bank: what was the amount?

152. Five hundred dollars is $\frac{3}{7}$ of the money I have in my safe: what is the amount of money in the safe?

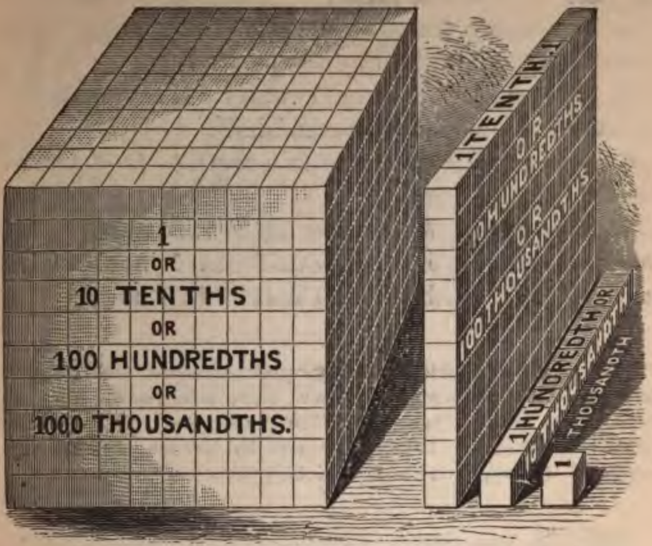
153. Twenty-five dollars is $2\frac{3}{10}$ times the money I gave to a poor man: how much did he receive?

154. A man has $4\frac{6}{7}$ bushels of potatoes, which is $\frac{5}{9}$ of the quantity that he planted: how many did he plant?

## ANALYSIS SECTION VIII.

### Fractions.

1. ILLUSTRATION.
2. DEFINITIONS.
3. TERMS.
   - *How written.*
4. REDUCTION.
   - a. *To Higher Terms.*
   - b. *To Lowest Terms.*
     - Illustration of each.
     - Analysis.
     - Principle.
     - Rule.
   - c. *Of Integers or Mixed Numbers to Improper Fractions.*
   - d. *Of Improper Fractions to Integral or Mixed Numbers.*
   - e. *To Common Denominators.*
     - Illustration of each.
     - Analysis.
     - Rule.
5. ADDITION OF FRACTIONS.
   - a. *Illustration.*
   - b. *Principle.*
   - c. *Analysis.*
   - d. *Rule.*
6. SUBTRACTION OF FRACTIONS.
   - a. *Illustration.*
   - b. *Analysis.*
   - c. *Rule.*
7. MULTIPLICATION OF FRACTIONS.
   - a. *Fractions by Integers.*
   - b. *Mixed Numbers by Integers.*
     - Illustration of each.
     - Analysis.
     - Principle.
     - Rule.
   - c. *Integers by Fractions.*
   - d. *Fractions by Fractions.*
     - Illustration of each.
     - Analysis.
     - Rule.
8. DIVISION OF FRACTIONS.
   - a. *Fractions by Integers.*
   - b. *Mixed Numbers by Integers.*
     - Illustration of each.
     - Analysis.
     - Principle.
     - Rule.
   - c. *Integers by Fractions.*
   - d. *Mixed Numbers by Fractions*
     - Illustration of each.
     - Analysis.
     - Rule.
9. CONCRETE PROBLEMS.
   - a. *Formula.*
   - b. *Analysis.*



## SECTION IX.

### DECIMAL FRACTIONS.

#### ILLUSTRATIONS.

1. If a block of wood is divided into ten equal parts, what is *one* of the parts called? What are two of the parts called? 5 of the parts? 7 of the parts?

2. If 1 *tenth* of a block of wood is divided into ten equal parts, what is one of the parts called? 3 of the parts? 6 of the parts.

3. If 1 *hundredth* of a block of wood is divided into

ten equal parts, what is one of the parts called? 4 of the parts? 8 of the parts?

4. How many *tenths* in a unit? In 2 units? In 5 units? In 20 units? In 45 units?

5. How many *tenths* in 1 unit? How many hundredths? How many thousandths?

6. How many *hundredths* in a tenth? In 3 tenths? In 1 unit? In 5 units?

7. How many *thousandths* in 1 hundredth? In 3 hundredths? In 5 hundredths? In 1 tenth? In 6 tenths? In 8 tenths? In 1 unit? In 7 units?

When a unit is divided into ten equal parts, and each fractional part into ten other equal parts, &c., these subdivisions are called respectively, *tenths*, *hundredths*, *thousandths*, &c. As each subdivision is one-tenth as large as the part divided, the parts are called *Decimal Fractions*, from *decem*, which signifies ten.

***101. A Decimal Fraction, or Decimal,*** is a fraction whose denominator is ten, or some power of ten, as, $\frac{1}{10}$, $\frac{11}{100}$, $\frac{43}{1000}$.

***102. A Mixed Decimal*** consists of an integer and a decimal; as, $4\frac{3}{10}$, $8\frac{59}{1000}$.

***103. Decimals*** may be written like a common fraction, with the denominator expressed; as, $\frac{3}{10}$, $\frac{5}{100}$, $\frac{25}{1000}$; or with the denominator indicated by placing a period before the decimal at the right of units; as, .3, .05, .025.

***104. U. S. Currency*** is written in like manner; dollars occupying the place of units, and the cents and mills being considered as hundredths and thousandths, as seen by the following

### NUMERATION TABLE.

| INTEGERS. | | | | | | | DECIMALS. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Millions. | Hundreds of Thousands. | Tens of Thousands. | Thousands. | Hundreds. | Tens. | ***Units.*** | *Tenths.* | *Hundredths.* | *Thousandths.* | *Ten-Thousandths.* | *Hundred-Thousandths.* | *Millionths.* |
| 1 | 1 | 1 | 1 | 1 | 1 | 1. | 1 | 1 | 1 | 1 | 1 | 1 |

### U. S. CURRENCY NUMERATION TABLE.

| DOLS. | | | | CTS. | | M. |
|---|---|---|---|---|---|---|
| Thousands. | Hundreds. | Tens. | ***Units.*** | *Tenths.* | *Hundredths.* | *Thousandths.* |
| 1 | 1 | 1 | 1. | 1 | 1 | 1 |

NOTE. — It will be observed that figures in places equidistant at the right or left of *units* have corresponding names; as, *tens* correspond to *tenths*, *hundreds* to *hundredths*, *thousands* to *thousandths*, &c.

Decimals are written and read as integral numbers; the denomination being shown by the place of the right-hand figure. At the right of units, the

| | | | | |
|---|---|---|---|---|
| 1st place | shows | *tenths*; as, | .3 | $= \frac{3}{10}$. |
| 2d " | " | *hundredths*; as, | .03 | $= \frac{3}{100}$. |
| 3d " | " | *thousandths*; as, | .003 | $= \frac{3}{1000}$. |
| 4th " | " | *ten-thousandths*; as, | .0003 | $= \frac{3}{10000}$. |
| 5th " | " | *hundred-thousandths*; as, | .00003 | $= \frac{3}{100000}$. |
| 6th " | " | *millionths*; as, | .000003 | $= \frac{3}{1000000}$. |

NOTE. — The above table should be so committed to memory, that the pupils can write any fraction *instantly*.

### EXERCISES IN READING DECIMAL NUMBERS.

The integral part of a mixed decimal number is read as a simple number: the fractional part is read as a simple number of the denomination of the right-hand figure. Thus, 432.327 is read, four hundred and thirty-two, and three hundred and twenty-seven thousandths.

Read the following numbers: *—

| (1.) | (2.) | (3.) | (4.) | (5.) | (6.) | (7.) | (8.) |
|---|---|---|---|---|---|---|---|
| .2 | 4. | 3.14 | 2.1 | 3.14 | 4.70 | 51.03 | 1.634 |
| 41.1 | 6.3 | 2.21 | 4.6 | .03 | 20.01 | 5. | 2.504 |
| 3.4 | 14.2 | .47 | .5 | 5. | .03 | 70.71 | 8.101 |
| 7.1 | .8 | .19 | 8.23 | .6 | 25.1 | 50.05 | 5.001 |
| 2.8 | .7 | 5.27 | 5.1 | 8.10 | 1.83 | 2.02 | 20.020 |
| 1.5 | 11.8 | 4.51 | .27 | .58 | 10.10 | 41.41 | 5.005 |

| (9.) | (10.) | (11.) | (12.) | (13.) |
|---|---|---|---|---|
| 30.201 | 4.037 | 30.020 | 1.413 | 1.305 |
| 75.005 | .067 | 5.005 | 1.27 | .4 |
| 93.505 | .004 | 50.005 | 50.037 | 6.103 |
| 40.076 | .4 | 80.08 | 2.143 | 4.004 |
| 30.300 | 30. | 5.011 | .087 | .6 |
| 100.001 | 8.010 | 500.501 | .002 | 8.008 |

NOTE. — Also read the above as dollars, cents, and mills.

| (14.) | (15.) | (16.) | (17.) |
|---|---|---|---|
| 413.4021 | 403.413 | 50000.001 | 413.407012 |
| 41.0341 | 4.4021 | 5000.0001 | 4001.300010 |
| 5000.001 | 30.204101 | 500.00001 | 4.301 |
| 41.0213 | 3.012401 | 50.0001 | 400.300901 |
| 4.4021 | $2.401\frac{2}{3}$ | 5.00001 | 30.410801 |
| 412.4134 | $3.1241\frac{4}{5}$ | .500001 | .600001 |
| 21.4012 | $4.3101\frac{1}{2}$ | .050001 | 4.300101 |
| 412.4302 | $400.4001\frac{3}{7}$ | .00501 | 5.800001 |

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* The following exercises should be read aloud by the pupils at recitation. The class should be still further practised, if necessary, by reading similar exercises from the blackboard, until they can read quickly and accurately.

The teacher should then, with the books closed, dictate the same exercises for the class to copy. When they have finished, let them exchange slates, and let *the teacher read the lesson* from the book, requiring each pupil to note his *neighbor's errors by* drawing a line under the numbers incorrectly written.

## 105. NOTATION AND ADDITION OF DECIMALS.

### WRITTEN EXERCISES.

| Write and add, | PROCESS. |
|---|---|
| 5 and 2 *tenths*, | 5.2 |
| 3 and 6 *tenths*, | 3.6 |
| 41 and 1 *tenth*, | 41.1 |
| 25 and 14 *hundredths*, | 25.14 |
| 1 and 7 *thousandths*. | 1.007 |
| | 76.047 |

**ANALYSIS.** — The analysis and rule the same as in simple numbers.

1. Add,
   4 and 3 *tenths*,
   5 and 4 *tenths*,
   7,
   1 and 3 *tenths*,
   6 and 7 *tenths*.

2. Add,
   6 and 21 *hundredths*,
   14 and 6 *hundredths*,
   5 and 7 *hundredths*.
   8 and 23 *hundredths*,
   5 and 74 *hundredths*.

3. Add,
   5 and 4 *tenths*,
   6 and 3 *hundredths*,
   5 *hundredths*,
   5 *tenths*,
   46 and 1 *hundredth*.

4. Add,
   6 and 5 *tenths*,
   47,
   40 and 7 *tenths*,
   47 *hundredths*,
   2 and 5 *tenths*.

5. Add,
   5 and 21 *hundredths*,
   8 and 8 *hundredths*,
   21 and 8 *tenths*,
   7 and 3 *tenths*,
   46 and 2 *hundredths*.

6. Add,
   41 and 1 *thousandth*,
   5 and 31 *thousandths*,
   89 *thousandths*,
   47 and 371 *thousandths*.

7. Add,
   26 and 4 *tenths*,
   4 and 5 *hundredths*,
   41 and 5 *thousandths*,
   6 and 32 *thousandths*,
   4 and 32 *hundredths*.

8. Add,
   25 and 16 *hundredths*,
   17 and 5 *thousandths*,
   1 and 27 *hundredths*,
   327 *thousandths*,

---

**REVIEW.** — What is the difference between a composite and a prime number? (70, 71.) Can a prime number have a factor or divisor? Can a composite number have a factor or divisor? (72.) Give an example. What is meant by the greatest common divisor? (74.)

**9.** Add,

3 *ten-thousandths,*
41 *ten-thousandths,*
321 *ten-thousandths,*
6002 *ten-thousandths,*

**10.** Add,

5 and 3 *tenths,*
5 and 3 *ten-thousandths,*
6 and 3 *thousandths,*
600 and 3 *thousandths,*
6 and 33 *hundredths.*

**11.** Add,

4 and 46 *hundred-thousandths,*
2 and 11 *hundred-thousandths,*
59 *hundred-thousandths,*
70 *hundred-thousandths.*

**12.** Add,

27 and 3 *tenths,*
27 and 3 *hundredths,*
27 and 3 *ten-thousandths,*
27 and 3 *hundred-thousandths.*

**13.** Add,

1 and 37 *hundredths,*
5 and 27 *hundred-thousandths,*
1 and 11 *thousandths,*
1 and 11 *ten-thousandths,*
3 and 5 *tenths.*

**14.** Add,

6 and 3 *tenths,*
11 and 47 *hundredths,*
5 *hundred-thousandths,*
500 *thousandths,*

NOTE. — The teacher should dictate examples similar to the above, until the pupils are ready and accurate in notation.

### *106.* SUBTRACTION OF DECIMALS.

From 97 *hundredths* take 37 *thousandths.*

PROCESS.

$$\begin{array}{r} .97 \\ .037 \\ \hline .933 \end{array}$$

ANALYSIS AND RULE. — The same as in simple numbers.

15. From 63 and 4 *hundredths* take 4 and 3 *tenths.*

16. From 25 take 5 and 3 *hundredths.*

17. From 547 *ten-thousandths* take 447 *hundred-thousandths.*

18. From 495 *thousandths* take 459 *millionths.*

19. From 1 take 4701 *millionths.*

20. Take 5 and 3 *thousandths* from 25.

21. Take 33 *thousandths* from 33 *hundredths.*

22. Take 3 *thousandths* from 3000.

23. Take 101 *millionths* from 101000000.

24. Take 79 *ten-thousandths* from 79000.

### MISCELLANEOUS WRITTEN EXAMPLES.

25. What is the difference between 59 and .59? Between 3.7 and .37? Between 37 and .037?

26. John is 28.4 years old; Edward is 30.94 years old: what is the sum of their ages? What is the difference of their ages?

27. A man received $37.43 for corn, and $274.07 for wheat: how much did he receive for both? How much more did he receive for the wheat than for the corn?

28. From a cask containing 32.5 gallons of wine, 20.47 gallons were drawn: how much remained? How many gallons would three such casks hold?

29. Two men build 118 rods of stone fence; one built 63.50 rods: how much did the other build? How much more did one build than the other?

30. Two men earned $195.73; one earned $86.80; how much did the other earn? How much more did one earn than the other?

31. What is the sum of $\frac{2}{10}$, $\frac{7}{10}$, $\frac{25}{100}$, $\frac{6}{100}$, $\frac{37}{100}$, and $\frac{2}{100}$?

32. $2.4 + \frac{1}{100} + \frac{3}{100} + \frac{2}{10} - \frac{83}{100} =$ how many?

33. $5 - \frac{3}{10} + \frac{4}{10} + 6 - \frac{46}{100} + 3 =$ how many?

34. $4.1 + \frac{6}{10} + \frac{120}{1000} + \frac{4}{100} + \frac{5}{100} =$ how many?

35. $5.02 + \frac{3}{100} - \frac{6}{10} + \frac{41}{100} - \frac{1}{1000} =$ how many?

36. $6.3 + \frac{4}{100} + 5.4 - \frac{6}{1000} + \frac{1}{10} =$ how many?

---

**Review.** — How can a composite number be resolved into prime factors? (73.) What is meant by a common multiple? (75.) What by a least common multiple? (76.) What is the use of cancellation? (77.) Upon what principle does it depend? (78.) Give an illustration. Give an illustration of what is meant by a fraction. (79.) Define a fraction. (79.) What does the denominator of a fraction show? (80.) The numerator of a fraction? (81.) What is a proper fraction? (82.) An improper fraction? (83.) *A mixed number?* (84.) Give the principles. (85.) How are *fractions written?*

# FELTER'S
# ARITHMETICAL DRILL CARDS.

| TABLE IV. | | | | | | | TABLE V. | |
|---|---|---|---|---|---|---|---|---|
| FRACTIONS. | | | | | | | UNITS. Dec. Point. 10ths. 100ths. 1000ths. 10000ths. 100000ths. 1000000ths.<br>5.834709 | |
| | 3 | 2 | 1 | V | W | X | | |
| A | 472 | 38 | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{3}{1}$ | A |
| B | 604 | 59 | 6 | $\frac{1}{5}$ | $\frac{1}{6}$ | $\frac{1}{7}$ | $\frac{50}{1}$ | B |
| C | 103 | 16 | 8 | $\frac{1}{8}$ | $\frac{1}{9}$ | $\frac{1}{10}$ | $\frac{9}{10}$ | C |
| D | 504 | 20 | 9 | $\frac{1}{11}$ | $\frac{1}{12}$ | $\frac{1}{13}$ | $\frac{20}{10}$ | D |
| E | 300 | 47 | 4 | $\frac{2}{3}$ | $\frac{2}{5}$ | $\frac{2}{7}$ | $\frac{63}{10}$ | E |
| F | 412 | 32 | 1 | $\frac{2}{9}$ | $\frac{2}{11}$ | $\frac{2}{13}$ | $\frac{25}{100}$ | F |
| G | 755 | 93 | 3 | $\frac{3}{4}$ | $\frac{3}{5}$ | $\frac{3}{7}$ | $\frac{300}{100}$ | G |
| H | 411 | 51 | 7 | $\frac{3}{8}$ | $\frac{3}{10}$ | $\frac{3}{11}$ | $\frac{208}{100}$ | H |
| I | 640 | 80 | 6 | $\frac{4}{5}$ | $\frac{4}{7}$ | $\frac{4}{9}$ | $\frac{732}{1000}$ | I |
| K | 832 | 21 | 8 | $\frac{4}{11}$ | $\frac{4}{13}$ | $\frac{4}{15}$ | $\frac{8000}{1000}$ | K |
| L | 253 | 73 | 5 | $\frac{5}{6}$ | $\frac{5}{7}$ | $\frac{5}{8}$ | $\frac{8021}{1000}$ | L |
| M | 414 | 86 | 2 | $\frac{5}{9}$ | $\frac{5}{11}$ | $\frac{5}{12}$ | $\frac{2143}{10000}$ | M |
| N | 674 | 13 | 1 | $\frac{6}{7}$ | $\frac{6}{11}$ | $\frac{6}{13}$ | $\frac{105}{10}$ | N |
| O | 305 | 14 | 8 | $\frac{7}{8}$ | $\frac{7}{9}$ | $\frac{7}{10}$ | $\frac{6134}{100}$ | O |
| P | 501 | 73 | 3 | $\frac{7}{11}$ | $\frac{7}{12}$ | $\frac{7}{13}$ | $\frac{43}{1000}$ | P |
| Q | 244 | 89 | 7 | $\frac{8}{9}$ | $\frac{8}{11}$ | $\frac{8}{13}$ | $\frac{1009}{100}$ | Q |
| R | 106 | 67 | 1 | $\frac{9}{10}$ | $\frac{9}{11}$ | $\frac{9}{13}$ | $\frac{5}{100}$ | R |
| S | 243 | 45 | 6 | $\frac{10}{11}$ | $\frac{10}{13}$ | $\frac{10}{17}$ | $\frac{2}{100000}$ | S |
| T | 182 | 38 | 2 | $\frac{11}{12}$ | $\frac{11}{13}$ | $\frac{11}{14}$ | $\frac{70}{1000000}$ | T |
| U | 100 | 72 | 3 | $\frac{12}{13}$ | $\frac{12}{17}$ | $\frac{12}{19}$ | $\frac{82000}{10}$ | U |

N.B. — For answers to the problems on the above drill card, see KEY.

## DRILL CARD EXERCISES. — TABLE V.

### ADDITION AND SUBTRACTION.

### ADDITION.

Exercise I. — 3 numbers.
II. — 4 "
III. — 5 "
IV. — 6 "
Exercise V. — 7 numbers.
VI. — 8 "
VII. — 9 "
VIII. — 10 "

### SUBTRACTION.

Take each of the decimals in Table V. from the following numbers as minuends: —

Exercise 1. — 93.416 minuend.
II. — 95.021 "
III. — 100.0003 "
IV. — 301 "
Exercise V. — 702.5016 minuend.
VI. — 450.021 "
VII. — 700 "

Ex. VIII. — Find the difference between 54.6 and A, B, C, &c.
IX. — Find the difference between 2.01613 and A, B, C, &c.
X. — Find the difference between .20153 and A, B, C, &c.
XI. — Find the difference between .000913 and A, B, C, &c.
XII. — Find the difference between .10101 and A, B, C, &c.
XIII. — Find the difference between A and B, B and C, C and D, &c.

### MISCELLANEOUS WRITTEN EXAMPLES.

37. $\frac{8}{1000} + 4.3 + \frac{9}{100} - \frac{6}{1000} + \frac{1}{10} =$ how many?

38. $2.4 + 3.4 - \frac{6}{100} + \frac{3}{1000} - \frac{5}{1000} =$ how many?

39. $\frac{5}{100} + 6.03 - \frac{2}{1000} + \frac{1}{10} - .02 =$ how many?

40. $\frac{2}{10} - .02 + 4.3 - \frac{6}{1000} + 3 =$ how many?

41. $\frac{81}{1000} - .02 + .5 - \frac{4}{1000} + 6.01 =$ how many?

42. $8.3 - .002 + 5.004 - .43 =$ how many?

43. $\frac{5}{10} + \frac{8}{1000} - \frac{3}{10} + 5.3 - \frac{6}{1000} =$ how many?

44. What is the sum of $\frac{4}{10} + \frac{8}{1000} + \frac{2}{10000} + \frac{1}{100000} + \frac{2}{1000}$ and $\frac{5}{10}$?

45. What is the sum of $\frac{5}{10} + .0002 + \frac{15}{10000} + \frac{6}{100000} + \frac{43}{1000}$ and 3?

---

**Review.** — What is meant by reducing a fraction to its lowest terms? (87.) Upon what principle does it depend? (88.)

## MULTIPLICATION OF DECIMALS.

### ORAL EXERCISES.

1. How many are 3 times 8 tenths? 12 times 8 tenths? 9 times 11 hundredths? 7 times 6 thousandths?

FORMULA. $.8 \times 3 = 2.4$.

ANALYSIS.—3 times 8 tenths are 24 tenths, equal to 2 units and 4 tenths.

2. How many are 4 times 8 cents? 7 times 11 mills? 9 times 5 tenths? 6 times 12 hundredths?

3. How many are 3 times 1 tenth? 2 hundredths? 12 hundredths? 5 thousandths? 25 thousandths? 6 ten-thousandths? 4 millionths? 12 hundred-thousandths?

4. What is $\frac{1}{10}$ of 3? $\frac{2}{10}$ of 3? .4 of 3? .7 of 3? .9 of 6?

FORMULA. $3 \times \frac{1}{10} = 3 \times .1 = \frac{3}{10}$ or .3. $3 \times \frac{2}{10} = 3 \times .2 = \frac{6}{10}$ or .6.

ANALYSIS.—Since 1 tenth of 3 equals $\frac{3}{10}$ of 1, or .3, $\frac{2}{10}$ of 3 equals 2 times .3, or .6.

5. What is $\frac{7}{10}$ of 8? .7 of 9? .9 of 6? .4 of 12?

6. What is $\frac{1}{100}$ of 3? $\frac{2}{100}$ of 3? .05 of 7? .08 of 9?

7. What is $\frac{1}{1000}$ of 3? $\frac{2}{1000}$ of 3? .006 of 5? .012 of 8?

8. What is $\frac{2}{10}$ of 1? $\frac{2}{10}$ of $\frac{1}{10}$? $\frac{2}{10}$ of $\frac{6}{10}$? $\frac{7}{10}$ of $\frac{8}{10}$?

FORMULA. $\frac{2}{10}$ of $1 = \frac{2}{10}$, or .2 $\frac{2}{10}$ of $.1 = \frac{2}{10}$ of $\frac{1}{10} = \frac{2}{100}$, or .02.

ANALYSIS.—Since 2 tenths of 1 = 2 tenths, 2 tenths of 1 tenth = 2 hundredths = .02.

---

REVIEW.—How are integral numbers changed to improper fractions? (89.) How are mixed numbers changed to improper fractions? How are improper fractions reduced to integral or mixed numbers? (90.) Illustrate what is meant by a common denominator of two or more fractions. (92.) What axioms are used in the addition and subtraction of fractions? (93.) Give the analysis of a short example. Give the analysis of an example in the subtraction of fractions. (94.)

9. What is $\frac{2}{10}$ of 3? Of .3? Of .7? Of .6? Of .03? Of .08? Of .09?

10. What is .5 of 8? Of .6? Of .9? Of .03? Of .08?

11. What is $\frac{3}{100}$ of 1? Of 6? Of .2? Of .8? Of .08?

**Problem.** — Multiply 2.43 by 3.4.

**Formula.** $\frac{243}{100} \times \frac{34}{10} = \frac{8262}{1000}$, or 8.262.

**PROCESS.**

$$\begin{array}{r} 2.43 \\ 3.4 \\ \hline 972 \\ 729\phantom{0} \\ \hline 8.262 \end{array}$$

**Analysis.** — By the rule for multiplication of fractions, the numerators must be multiplied together for a new numerator, and the denominators for a new denominator.

As the denominator of a decimal fraction is always a power of ten, the denominator of the result will be expressed decimally by the sum of the factors of ten in each denominator: hence the following

***107.*** Rule. — *Multiply as in simple numbers, and point off from the right of the product as many figures for decimals as there are decimal places in both factors.*

### DRILL CARD EXERCISES. — TABLE V.

#### MULTIPLICATION.

**Note.** — Use only such portions of these exercises as the class may need.

Multiply each decimal in Table V. by the following multipliers: —

| | | | | | |
|---|---|---|---|---|---|
| Exercise I. | — 2 | multiplier. | Exer. XI. | — 2.002 | multiplier. |
| II. | — 25 | " | XII. | — 20.01 | " |
| III. | — .2 | " | XIII. | — 700.07 | " |
| IV. | — 4.2 | " | XIV. | — 709.09 | " |
| V. | — .08 | " | XV. | — 5000 | " |
| VI. | — 3.07 | " | XVI. | — 200.002 | " |
| VII. | — .29 | " | XVII. | — 3.0004 | " |
| VIII. | — .006 | " | XVIII. | — .03002 | " |
| IX. | — .407 | " | XIX. | — 30.09 | " |
| X. | — 6.001 | " | XX. | — .0006 | " |

XXI. — A × B, B × C, C × D, D × E, E × F, F × G, &c.
XXII. — A × C, B × D, C × E, D × F, E × G, &c.

### ORAL AND WRITTEN EXERCISES.

46. How many are 3 times 1 tenth? 8 tenths? 6 hundredths? 25 hundredths? *W.* 4.3? 25.03?

47. How many are 9 times 2 tenths? 11 hundredths? 7 thousandths? *W.* 328 thousandths? 674 hundredths?

48. How many are 8 times 9 hundredths? 7 ten-thousandths? 11 hundred-thousandths? 9 millionths? *W.* 473 millionths? 203 hundred-thousandths?

49. What is 6 tenths of 3? Of 8 tenths? Of 11 hundredths? Of 12 thousandths? *W.* Of 27 thousandths? Of 103 hundred-thousandths?

50. What is 8 hundredths of 7? Of .7? Of .09? Of .011? *W.* Of 6.03? Of .4034?

51. *W.* Multiply 30.003 by 8. By .08. By .12.

52. What is 2 thousandths of 6? Of .007? Of 9 millionths? *W.* Of 34.67? Of 4.0003? Of 7043 hundred-thousandths?

53. What will 7 barrels of sugar cost, at $20 a barrel? At $30? *W.* At $28.25? At $32.37?

54. What is the value of 30 pounds of sugar, at 6 cents per pound? At 8 cents? *W.* At 8.2 cents? At $8\frac{1}{2}$ cents? At $7\frac{5}{7}$ cents?

55. At $30 a barrel, what is the value of 2 barrels of sugar? Of 3 tenths of a barrel? Of .6? Of .03? *W.* Of 2.8 barrels? Of .025?

56. At $$2\frac{1}{2}$ a box, what is the cost of 2 boxes of raisins? Of .3 of a box? *W.* Of 2.37? Of .81?

---

**Review.**—How are fractions multiplied by integers? (95.) Illustrate by an example. How are integers multiplied by fractions? (96.) Illustrate by an example. How are fractions multiplied by fractions? (97.) Illustrate by an example. How are fractions divided by integers? (98.)

57. *W.* A man bought 19.5 barrels of cider, at $2.20 a barrel; he sold it all for $28: did he make, or lose, and how much?

58. *W.* A man bought .3 of a barrel of cider, at $$2\frac{3}{4}$ a barrel; he sold what he purchased for $$2\frac{2}{3}$: did he make, or lose, by the transaction?

59. *W.* A boy bought .75 of a bushel of nuts, at $2 a bushel; he sold them again for $2: did he make, or lose, and how much?

60. *W.* Another boy bought 1.75 bushels of nuts, at $2 a bushel; he sold them again for $2: did he make, or lose, and how much?

61. *W.* A girl had a ten dollar bill; she bought .38 of a box of raisins, at $$2\frac{7}{9}$ a box: how much money had she left?

62. $4.3 \times \frac{5}{8} + 3.47 - .08 =$ what?

63. $43.24 - \overline{2.4 \times 6\frac{1}{2}} + 40.74 =$ what?

64. $(2.01 - .003) \times (1 + \frac{3}{4}) =$ what?

65. $\overline{7 \times \frac{1}{5}} - (3 \times .3) + 12\frac{1}{2} =$ what?

66. $(\overline{6 \times .3} - \overline{.6 \times 3}) \times 8.401 =$ what?

67. $(\overline{\frac{4}{5} \times .7} - \overline{.8 \times .7}) \times 4.2 \times 3 =$ what?

## DIVISION OF DECIMALS.

### ORAL EXERCISES.

1. How many tenths in a unit? In 2 units? In 5 units?

2. In 1 unit how many tenths? How many hundredths? How many thousandths? How many ten-thousandths?

3. How many tenths in 1 half of a unit? In 1 fourth? In 1 fifth?

---

**REVIEW.**—How are integers divided by fractions? (99.) Illustrate by an example. *How are fractions divided by fractions?* (100.) Illustrate by an example.

4. In 1 fourth of a unit, how many tenths? How many hundredths? How many thousandths? How many millionths?

**FORMULA.** 10 tenths ÷ 4 = $.2\frac{1}{2}$.
100 hundreds ÷ 4 = .25.

**ANALYSIS.** — Since in 1 unit there are 10 tenths, in 1 fourth of a unit there are 1 fourth of 10 tenths, or $2\frac{1}{2}$ tenths.

5. Divide .8 by 4. .14 by 7. 1.4 by 7. 2.8 by 4.

6. What is 1 fifth of 2.5? Of 35? Of .35? Of 1.00?

7. What is 1 eighth of 24? Of 4.8? Of $4\frac{8}{10}$? Of .72?

8. What is 1 twelfth of 72? Of 7.2? Of $7\frac{2}{10}$? Of $9\frac{6}{10}$?

***108.*** PRINCIPLE. — ***Any order divided by the same order gives a quotient of units.***

Decimals, like common fractions, *must* be changed to like denominations, before one fraction can be divided by another; and the quotient must be units, or parts of a unit: hence the place of the decimal point can be determined by inspection, by reducing the divisor and dividend to like denominations, and placing an accent in the dividend over the place of the decimal point in the quotient, annexing ciphers, if necessary; thus, —

ILLUSTRATIONS.

| (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|
| 4.7)4.7′ | .007)2.000′ | 4)6.08 | .2).0′8 | 40.2).0′0804 |
| 1 | 285.71+ | 1.52 | .4 | .0002 |

REVIEW. — What is meant by a decimal fraction? (101.) What is a mixed decimal? (102.) How are decimals written? (103.) In how many ways can decimals be written? How does the notation of United States currency differ from decimals? (104.) What is the first place at the right of the units? The second? The third? The fourth? The first at the left of the units' place? The second? The third at the right? The third at the left? The fourth at the right? At the left? How are decimals added? (105.) How is subtraction of decimals performed? (106.)

**PROBLEM.** — Divide 22.848 by 3.4.

PROCESS.

3.4)22.8′48(6.72
  204
  244
  238
   68
   68

**ANALYSIS.** — (1.) For convenience, write the numbers as in simple division.

(2.) 3.4 equals 34 tenths; 22.8 equals 228 tenths; 34 tenths are contained in 228 tenths 6 times; which write in the quotient as units. The remainder of 24 tenths is equal to 240 hundredths.

(3.) 240 hundredths and 4 hundredths equal 244 hundredths; 34 tenths are contained in 244 hundredths 7 tenths times; which write in the quotient. The remainder, 6 hundredths, is equal to 60 thousandths.

(4.) 60 thousandths and 8 thousandths equal 68 thousandths; 34 tenths are contained in 68 thousandths 2 hundredths times; which write in the quotient.

To determine the decimal point by inspection: —

***109.*** **RULE.** — I. *Make the decimal places in the dividend equal to those in the divisor,* write the quotient of these figures as units. Place the decimal point, and continue dividing as in simple division.*

Some may prefer the following: —

**RULE.** — II. *Make the number of decimal places in the dividend at least equal to those in the divisor.*

*Point off as many decimals in the quotient as the decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.*

---

**REVIEW** — Any order divided by the same order gives a quotient of what order? (108.) Units divided by units give what? Give the rule for the multiplication of decimals. (107.) Give the rule for the division of decimals. (109.)

* Annex ciphers to the dividend, if necessary.

### BLACKBOARD EXERCISES.

Determine the decimal point in the quotient of each of the following: —

#### ILLUSTRATIONS.

| (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|
| 1)82 | 1).21 | 100).360 | .1)7.5′ | .0001).0930′ |
| 82 | .21 | .0036 | 75 | 930 |

| (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|
| 82 ÷ 1 | 47 ÷ .1 | .31 ÷ 1. | .34 ÷ .1 |
| 74 ÷ 10 | 63 ÷ .01 | .86 ÷ 10. | .65 ÷ .01 |
| 57 ÷ 100 | 27 ÷ .001 | .57 ÷ 100. | .72 ÷ .001 |
| 32 ÷ 1000 | 53 ÷ .0001 | .43 ÷ 1000. | .82 ÷ .0001 |

### WRITTEN EXERCISES.

The answers to the following examples should be correct to five decimal places. Annex ciphers to the dividend, if necessary.

**Note.** — The sign + annexed to the quotient shows that the division is not completed.

68. 3.41 ÷ 4
69. 20.1 ÷ .6
70. 91.4 ÷ 7
71. 13.4 ÷ .08
72. 134 ÷ .6
73. 2 ÷ 8
74. 7 ÷ .08
75. 5 ÷ .3
76. 4 ÷ .007
77. 2 ÷ .0005
78. 3 ÷ .6
79. 21 ÷ 2.1
80. .21 ÷ 21
81. .21 ÷ .21
82. .21 ÷ .021
83. 5 ÷ 3
84. 6 ÷ 9
85. 8 ÷ 4
86. .3 ÷ 3
87. 20 ÷ .02

### DRILL CARD EXERCISES. — TABLE V.

#### DIVISION.

Divide each decimal in Table V. by the following divisors: —

**Exercise I. — 2 divisor.**
**II. — .04 "**
**III. — .009 "**
**IV. — .007 "**
**V. — .0006 "**
**VI. — .0008 "**
**VII. — 20 "**
**VIII. — 25 "**
**IX. — 4.2 "**
**X. — .13 "**

**Exercise XI. — .014 divisor.**
**XII. — .0028 "**
**XIII. — 400 "**
**XIV. — 403 "**
**XV. — 12.7 "**
**XVI. — 1.87 "**
**XVII. — 8.93 "**
**XVIII. — .721 "**
**XIX. — .0718 "**
**XX. — .00207 "**

**Exercise XXI. — A ÷ B, B ÷ C, C ÷ D, D ÷ E, &c.**
**XXII. — A ÷ C, B ÷ D, C ÷ E, D ÷ F, &c.**

#### ORAL AND WRITTEN EXERCISES.

88. If a man can cut 2.3 cords of wood in 1 day, how many cords can he cut in 2 days? In 3 days? *W.* In 47 days? In 123 days?

89. If I give $1.2 for a pair of gloves, how much will 3 pairs cost? 4 pairs? *W.* 58 pairs? 137 pairs?

90. $2\frac{3}{10}$ dollars will buy 1 pair of shoes: at that rate, how many dollars will it take to purchase 3 pairs? 5 pairs? *W.* 27 pairs? 2883 pairs?

91. One barrel of flour is worth $5.34: at that rate, what are 2 barrels worth? *W.* 2.5 barrels worth? .63 of a barrel? 42.73 barrels?

92. If it takes .4 of a bushel of apples to make a gallon of cider, how many gallons can be made from

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**Review.** — What is a fraction? (79.) A denominator? (80.) The numerator of a fraction? (81) *A proper fraction?* (82) An improper fraction? (83.) A mixed number? (84.)

.8 of a bushel? 1.6 of a bushel? *W.* 45.8 bushels? 49.47 bushels?

93. If 8 bushels of apples cost $4, what will one bushel cost? 2 bushels? *W.* 5.2 bushels? 7.57 bushels?

94. *W.* If 80.2 pounds of starch can be made from 2.1 bushels of corn, how much starch can be made from 1 bushel? From 7.5 bushels?

**FORMULA.** (1.) 80.2 lbs. ÷ 2.1 = (?) lbs.
(2.) (?) lbs. × 7.5 = (?) lbs.

95. *W.* If 3 cubic feet of water weigh 187.5 lbs., how much will 9 cubic feet weigh? 37.42 cubic feet?

## *110.* TO CHANGE COMMON FRACTIONS TO DECIMAL.

### ORAL EXERCISES.

1. What is $\frac{1}{2}$ of 8? $\frac{2}{3}$ of 5? $\frac{5}{9}$ of 11? $\frac{8}{13}$ of 12?

2. How many tenths in a unit? How many hundredths? How many thousandths? How many millionths?

3. How many tenths in $\frac{3}{4}$ of a unit? or, change $\frac{3}{4}$ to tenths.

**ANALYSIS.** — $\frac{3}{4}$ of a unit = 3 ÷ 4, or one-fourth of 3 units. In one unit there are 10 tenths, and in $\frac{3}{4}$ of a unit there are $\frac{1}{4}$ of 3 times 10 tenths, or $7\frac{1}{2}$ tenths.

4. Express $\frac{3}{4}$ of 1 in an equivalent decimal.

**PROCESS.** **FORMULA.** $\frac{3}{4} = 3.00 \div 4 = .75$

$$\begin{array}{r} 4)\underline{3.00} \\ .75 \end{array}$$

**ANALYSIS.** — 3 units equal 300 hundredths: $\frac{1}{4}$ of 300 hundredths equal 75 hundredths.

Since a fraction indicates the division of the numerator by the denominator, if the numerator is reduced to the required denomination, and divided by the denominator, the result will be an equivalent decimal expression: hence the following

**RULE.** — *Add ciphers to the numerator, and divide by the denominator, as in the division of decimals.*

### BLACKBOARD EXERCISES.

Change each of the following fractions to tenths, hundredths, and thousandths.

The sign +, placed after a decimal, shows that there is a remainder; thus, $\frac{5}{7} = .714\frac{2}{7}$, or .714 +.

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) | (6.) | (7.) | (8.) | (9.) |
|---|---|---|---|---|---|---|---|---|---|
| $\frac{3}{4} = .7\frac{1}{2}$ | $\frac{8}{9}$ | $\frac{6}{7}$ | $\frac{8}{9}$ | $\frac{1}{4}$ | $\frac{2}{7}$ | $\frac{4}{9}$ | $\$\frac{6}{13}$ | $\$\frac{3}{10}$ | $\$\frac{5}{11}$ |
| $\frac{4}{5} = .8$ | $\frac{7}{8}$ | $\frac{5}{8}$ | $\frac{4}{7}$ | $\frac{1}{7}$ | $\frac{2}{13}$ | $\frac{4}{15}$ | $\$\frac{7}{10}$ | $\$\frac{4}{7}$ | $\$\frac{6}{11}$ |
| $\$\frac{6}{7} = \$.857+$ | $\frac{5}{11}$ | $\frac{3}{4}$ | $\frac{8}{11}$ | $\frac{1}{10}$ | $\frac{3}{7}$ | $\frac{5}{8}$ | $\$\frac{7}{13}$ | $\$\frac{4}{13}$ | $\$\frac{7}{9}$ |
| $\$\frac{4}{11} = \$.363+$ | $\frac{4}{9}$ | $\frac{2}{3}$ | $\frac{4}{15}$ | $\frac{1}{13}$ | $\frac{3}{11}$ | $\frac{5}{12}$ | $\$\frac{8}{13}$ | $\$\frac{5}{7}$ | $\$\frac{7}{12}$ |

### WRITTEN EXERCISES.

96. Change $\frac{2}{41}$ to a decimal of four places.

**Formula.** $2 \div 41 = .0487+$.

97. Reduce $\frac{5}{8}$ to a decimal, $\frac{6}{11}$, $\frac{4}{7}$, $\frac{9}{11}$.

Find the decimal equivalents, correct to four places, for each of the following fractions:—

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|---|
| $\$\frac{1}{2} = \$.50$ | $\$\frac{1}{6}$ | $\frac{4}{7}$ | $\frac{8}{9}$ | $\frac{2}{27}$ | $\frac{5}{137}$ |
| $\$\frac{1}{3} = .3333+$ | $\$\frac{1}{7}$ | $\frac{8}{9}$ | $\frac{7}{74}$ | $\frac{6}{57}$ | $\frac{9}{672}$ |
| $\$\frac{1}{4} = .25$ | $\$\frac{2}{7}$ | $\frac{4}{11}$ | $\frac{8}{37}$ | $\frac{2}{324}$ | $\frac{10}{681}$ |
| $\$\frac{3}{8} = .375$ | $\$\frac{3}{8}$ | $\frac{6}{7}$ | $\frac{9}{95}$ | $\frac{8}{117}$ | $\frac{2}{1001}$ |

**Note.**—Those fractions whose denominators are composed of prime factors other than 2 or 5, when changed to a decimal, repeat one figure, or a series of figures; as, $\frac{1}{3} = .333+$, $\frac{1}{7} = .142857+$, &c. These figures are sometimes marked with a dot over the recurring figure, or series of figures, and are called "repetends."

## *111.* TO CHANGE DECIMALS TO COMMON FRACTIONS.

### ORAL EXERCISES.

1. 5 tenths of an apple is what part of an apple expressed in its lowest terms?

**Formula.** $.5 = \frac{5}{10} = \frac{1}{2}$.

2. Express $\frac{4}{10}$ of an orange in its lowest terms. 8 tenths of a lemon. 25 hundredths. 50 hundredths.

3. What is the value of .4 expressed by a common fraction in its lowest terms? Of .25? Of .6? Of .28? Of .06? Of .75? Of .44? Of .16? Of .15?

4. Give the common fraction equivalent to each of the following: —

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) | (6.) | (7.) |
|---|---|---|---|---|---|---|---|
| $.04 = \frac{1}{25}$ | .6 | .04 | .38 | .15 | .06 | .012 | .085 |
| $.8 = \frac{4}{5}$ | .8 | .08 | .28 | .60 | .05 | .885 | .012 |
| $.05 = \frac{1}{20}$ | .9 | .25 | .35 | .15 | .03 | 1.755 | 5.058 |
| $2.4 = 2\frac{2}{5}$ | .5 | .30 | .75 | .12 | .2 | 6.855 | 1.025 |

5. Change each of the following number of cents and mills to the fraction of a dollar: —

| MODEL. | (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|---|
| $\$.36 = \$\frac{9}{25}$ | \$.125 | \$1.10 | \$.20 | \$.875 | \$.004 |
| $.25 = \frac{1}{4}$ | 1.25 | 6.90 | .55 | 1.375 | .008 |
| $2.12 = 2\frac{3}{25}$ | .18 | .12 | .06 | .605 | .002 |
| $1.375 = 1\frac{3}{8}$ | .40 | .15 | .88 | .005 | .104 |

### ORAL AND WRITTEN EXERCISES.

**Problem.** — Reduce .75 to an equivalent common fraction.

**Formula.** $.75 = \frac{75}{100} = \frac{3}{4}$.

98. Reduce .5 to a common fraction. Reduce .05.

99. What common fraction is equal to .25? To .75? To .55? To .125? To .185?

**Note.** — When the decimal is a repetend, as, $\frac{1}{3} = .\dot{3}$, $\frac{1}{11} = .0\dot{9}$, these repetends can be changed to a fraction by writing for a denominator as many *nines* as there are figures in the repetend; as, $.\dot{3} = \frac{3}{9} = \frac{1}{3}$, $.0\dot{9} = \frac{9}{99} = \frac{1}{11}$, $.2\dot{3} = .2\frac{3}{9} = .2\frac{1}{3}$.

**100.** Change each of the following repetends to fractions: —

| MODEL. | (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|---|
| $.\dot{3} = \frac{3}{9} = \frac{1}{3}$ | $.\dot{2}\dot{7}$ | $.\dot{7}$ | $.4\dot{3}3\dot{3}+$ | $.\dot{1}4285\dot{7}$ |
| $.\dot{6} = \frac{6}{9} = \frac{2}{3}$ | $.\dot{8}$ | $.\dot{6}\dot{3}$ | $.9\dot{6}6\dot{6}+$ | $.\dot{2}8571\dot{4}$ |
| $.4\dot{0}\dot{9} = .4\frac{9}{99} = .4\frac{1}{11}$ | $.\dot{5}$ | $.2\dot{3}$ | $.\dot{1}11\dot{1}+$ | $.413\dot{5}\dot{4}$ |
| $.6\dot{1} = .6\frac{1}{9} = .6\frac{1}{9}$ | $.\dot{4}\dot{5}$ | $.\dot{3}\dot{6}$ | $.\dot{7}77\dot{7}+$ | $.\dot{4}44\dot{4}+$ |

**101.** Reduce $.33\dot{3}$ to a common fraction. Reduce $.16\frac{2}{3}$.

**FORMULA.** $.33\dot{3} = \frac{33\frac{1}{3}}{100} = \frac{100}{3} \div 100 = \frac{100}{300} = \frac{1}{3}$.

$.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{50}{3} \div 100 = \frac{50}{300} = \frac{1}{6}$.

**ANALYSIS.** — $.33\dot{3}$, written with a denominator, equals $\frac{33\frac{1}{3}}{100}$. The numerator $33\frac{1}{3} = \frac{100}{3}$; and $\frac{100}{3}$ divided by $100 = \frac{100}{300}$, or $\frac{1}{3}$: hence the following

**RULE.** — *Write the denominator under the decimal, erase the decimal point, and reduce the fraction to its lowest terms.*

**NOTE.** — Repetends must be changed to fractions, and the fractions to improper fractions, before dividing by the denominator.

**102.** Write common fractions for each of the following decimals: —

| MODEL. | (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|---|
| $.25 = \frac{25}{100} = \frac{1}{4}$ | .875 | .15 | $.33\dot{3}$ | .24 |
| $.12\frac{1}{2} = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$ | .42 | .015 | $.6\frac{2}{3}$ | .35 |
| $.4\dot{4} = \frac{4\frac{4}{9}}{100} = \frac{40}{900} = \frac{2}{45}$ | .55 | .025 | $.7\dot{7}$ | .004 |
| $.09\dot{0}\dot{9} = \frac{9\frac{9}{99}}{100} = \frac{900}{9900} = \frac{1}{11}$ | .75 | .2 | $.88\dot{8}$ | $.\dot{4}\dot{5}$ |
|  | .45 | .8 | .24 | $.25\dot{3}$ |
|  | .755 | .28 | .63 | .74 |
|  | $.12\frac{1}{2}$ | $.37\frac{1}{2}$ | $.\dot{4}$ | .28 |

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**REVIEW.** — *Give principles.* (85.) Give rule for reducing a mixed number to an improper fraction. (89.) For reducing an improper fraction to an integral or mixed number. (90.)

What is the value of each of the following, expressed in decimals?

103. $1.13 \times 5 \div 7$.
104. $3.7 \text{ lbs.} \div 8 \times 2.5$.
105. $\overline{\$1.75 \times .3} \div 5.7$.
106. $(\$4.75 + .27) \div 90$.
107. $\overline{4.3 \text{ bbls.} \times \frac{8}{10}} + 2.5 \text{ bbls.}$
108. $7.1 \times 8.2 - \overline{2\frac{3}{5} \div 7.4}$.
109. $\overline{4.25 \div .4} - (4\frac{3}{4} + 5.2)$.
110. $(\overline{1\frac{4}{7} \times 5.3} + \overline{2\frac{3}{7} \div .0\frac{5}{7}}) \times .1\dot{3}$.
111. $(8.\dot{3} + 5.\dot{4}) \times 8\frac{3}{8} \div 7.3\frac{1}{4}$.
112. $\overline{\frac{8}{10} + 3.4} \times \overline{6 + \dot{5}0\dot{1}} \div 1.\dot{4}7\dot{1}$.

**112.** *To find the cost when price per* 100 *or* 1000 *is given.*

WRITTEN EXAMPLES.

What is the cost of 234 feet lumber, at $2.70 per hundred feet? At $29.30 per thousand?

NOTE.—When the mills in the answer equal or exceed 5, add 1 cent; when less than 5, reject them.

FORMULA. (1.) $\$2.70 \times \overline{234 \div 100} = \$6.32$.
(2.) $\$29.30 \times \overline{234 \div 1000} = 6.86$.

PROCESS.
$2.70 cost C.* feet.
2.34 No. of C. feet.
$6.32

ANALYSIS.—(1.) 234 feet equals 2.34 hundred feet.

(2.) Since 100 feet costs $2.70, 2.34 hundred feet will cost 2.34 times $2.70, or $6.318.

REVIEW.—Define common denominator. (92.) Give the rule for the addition of fractions. (93.) Give the rule for the subtraction of a fraction from an integer. (94.) A fraction from a fraction. A fraction from a mixed number. A mixed number from a mixed number. Give a rule for multiplying a fraction by an integer. (95.) A mixed number by an integer. Give the rule for multiplying an integer by a fraction. (96.) An integer by a mixed number. Give the rule for multiplying a fraction by a fraction. (97.) A mixed number by a fraction. A mixed number by a mixed number.

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* In business transactions, C. is sometimes used for *hundreds*, M. for *thousands*, and T. for *tons*, or two thousand pounds.

**Note.** — As any number can be divided by 10, 100, or 1000 by removing the decimal point to the left, the first step of the analysis may be omitted in the process.

113. What will 574 feet of boards cost, at $5.90 per C.?

114. What will 7243 feet of lumber cost, at $63.20 per M.?

115. At $8.59 per M., what will 8900 bricks cost? 2750 bricks? 300 bricks?

116. At $5.25 per C., what will 340 pounds of lead cost? 47 pounds? 9 pounds?

117. At $3.25 per M., what will 450 envelopes cost? 50 envelopes? 58,700 envelopes?

118. What is the freight on 834 pounds of plough castings, at $1.37½ per 100 pounds? At $.95⅔ per C.?

119. What will 583 lath cost, at $.75⅔ per C.?

120. What will 950 shingles cost, at $2.17½ per thousand?

## MISCELLANEOUS EXAMPLES FOR REVIEW.

### WRITTEN EXERCISES.

Find the cost of the following items expressed in dollars, cents, and mills.

@ means by the pound, yard, &c.

121 47½ pounds rice, @ 5¼ cents.

122. 73⅞ yards calico, @ 18¼ cents.

123. 89¼ quarts beans, @ 7⅔ cents.

124. 67⅝ yards lace, @ 9¾ cents.

125. 112½ yards cloth, @ $1$\frac{3}{7}$.

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**Review.** — Give the rule for dividing a fraction by an integer. (98.) A mixed number by an integer. Give the rule for dividing an integer by a fraction. (99.) An integer by a mixed number. Give the rule for dividing a fraction by a fraction. (100.) A mixed number by a fraction. A mixed number by a mixed number.

126. $413\frac{1}{2}$ bushels oats, @ $66\frac{1}{3}$ cents.
127. $6\frac{1}{2}$ bushels apples, @ $174\frac{1}{2}$ cents.
128. $5\frac{1}{2}$ quarts nuts, @ $19\frac{3}{4}$ cents.
129. 415 pounds starch, @ $4\frac{1}{7}$ cents.
130 $2\frac{3}{5}$ yards silk, @ $\$1\frac{1}{8}$.
131. 417 pounds honey, @ $18\frac{3}{4}$ cents.
132. $97\frac{3}{4}$ pounds coffee, @ $\$\frac{3}{5}$.
133. 24.2 bushels apples, @ $\$\frac{8}{9}$.
134. $97\frac{5}{8}$ yards calico, @ $11\frac{1}{2}$ cents.
135. $410\frac{7}{8}$ pounds cheese, @ $15\frac{1}{2}$ cents.
136. $73\frac{4}{5}$ tons coal, @ $\$12\frac{3}{5}$.
137. 81.3 pounds veal, @ $19\frac{3}{4}$ cents.
138. $23\frac{2}{3}$ pecks peaches, @ $37\frac{1}{2}$ cents.
139. 4210.5 pounds wool, @ $49\frac{1}{2}$ cents.
140. 742.6 pounds cotton, @ $24\frac{2}{3}$ cents.
141. 2173 pounds pork @ $\$5\frac{2}{7}$ per C.
142. $4113\frac{1}{2}$ pounds butter, @ $\$27\frac{1}{2}$ per hundred weight?
143. $751\frac{1}{2}$ pounds lard, @ \$8.70 per C.
144. 5350 shingles, @ \$2.50 per M.
145. 7400 bricks, @ $\$9\frac{1}{4}$ per M.
146. 5234 feet timber, @ \$35 per M.

147. A banker received, on Jan. 1, cash to the amount of \$30.40, and paid out \$25.70; Jan. 2, received 18 dollars, \$5, \$20.50, paid out \$7.30, \$6.95, \$3.47; Jan. 3, received \$4.20, \$923, paid out \$9.20, \$6.35: how much did he receive on the three days? How much did he pay out? Did he pay out more, or less, than he received? and how much?

148. On Jan. 4, I bought $3\frac{1}{2}$ bushels apples @ 95 cents, 6 pecks of beans @ 90 cents, and sold $4\frac{3}{4}$ bushels *potatoes @ 87 cents,* 24 cabbages @ 10 cents, 3 pecks

apples @ 28 cents; on Jan. 5, I bought 35 bushels potatoes @ 60 cents, and sold 5 cabbages @ 10 cents, 5 pecks apples @ 28 cents, 3 quarts beans @ 12 cents: how much did I buy? How much did I sell? Did I sell more, or less, than I bought? and how much?

## TEST QUESTIONS FOR EXAMINATION.

**70.** What is a composite number? Name the composite numbers less than 20. Are the following numbers prime, or composite, 27, 36, 40, 41, 71, 21, 83, 63, 55, 67, 57, 17, 77? Give the factors of each of the composite numbers. What composite number is formed by the multiplication of 2 and 3? 7 and 2? 3, 5, and 7?

**71.** What is a prime number? How does it differ from a composite number? Give an example of each. Is 6 a prime number? Why? Is 12? Why? Is 7? Why?

**72.** What is a factor, or divisor? What is a factor of 14? How many factors has 14? When is a factor said to be prime? Give the prime factors of 4, 8, 24.

**73, 74.** What is the greatest common divisor, or factor, of two or more numbers? Give two numbers that have a common factor, and name the factor. Name the common factor of 4 and 2. Of 10 and 15. Name the greatest common factor of 8 and 12. Illustrate by an example the method of finding the greatest common factor of three numbers. Give the analysis.

**75.** What is a common multiple of two or more numbers? What is a common multiple of 3 and 4? Of 2 and 7? Of 5 and 10?

**76.** What is a least common multiple? What is the least common multiple of 3 and 4? Of 6 and 9? Of 2 and 7? Give the rule for finding the least common multiple. Illustrate by an example.

**77.** What is cancellation? Illustrate by an example.

**78.** Upon what principle does it depend? Is a number changed by being both multiplied and divided by the same number? How is a multiplier known in arithmetic? ANS.—*By the sign of multiplication placed before it.* How is the divisor known? ANS.—*By the sign of division before it.* When a sign of division is placed before a parenthesis, is a part, or all, the numbers within it used as a divisor? ANS.—*All of them.* What are the numbers considered when placed below a horizontal line, as in the fractional sign of division? ANS.—*They are considered as divisors.*

**79.** What is a fraction? What is meant by $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$? Upon what does the size of the equal parts depend? Which is the larger, 4ths or 10ths? 3ds or 5ths?

**80.** What does the denominator show? To what does the denominator give a name?

**81.** What does the numerator show? In the following fractions, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{3}{8}$. $\frac{5}{9}$, which is the denominator? Which the numerator?

**82.** What is meant by a proper fraction? Give an example.

**83.** Give an example of an improper fraction. Why so called? How does a proper fraction differ from an improper fraction?

**84, 85.** What is a mixed number? What sign is understood between the integer and the fraction?

How are fractions usually written? Give an example. What other method is sometimes used? Give an example.

What is meant by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$? What by $\frac{2}{3}$, $\frac{3}{4}$? When no definite number is expressed, what is considered the unit of a fraction? ANS. — 1.

**86-88.** What is meant by reducing a fraction to lower terms? To its lowest terms? Give the analysis of an example.

Upon what principle does it depend? Is the value of the fraction changed?

**89.** What is meant by changing integral or mixed numbers to improper fractions? Give the analysis of an example.

**90.** What is meant by changing improper fractions to integral or mixed numbers? Give the analysis of an example.

**92.** What is meant by reducing fractions to a common denominator? Is the value changed?

When are fractions said to have a common denominator? Give an example. Give the analysis of an example. Give a rule for reducing fractions to like denominators. Upon what principle does it depend? What does the least common multiple of the denominators of the several fractions indicate? ANS. — *It indicates the least common denominator of those fractions.*

**93.** What axioms are used in addition and subtraction of fractions? Upon what two axioms are the operations based?

Like fractions are what? Can fractions of different units be added or subtracted? Could $\frac{3}{4}$ of a barrel be taken from $\frac{3}{4}$ of a pound? Could $\frac{2}{3}$ of a pound of flour be taken from $\frac{2}{3}$ of a bushel of potatoes? Could $\frac{5}{7}$ of a pound be taken from $\frac{8}{9}$ of a pound? Could $\frac{3}{7}$ of an apple be added to $\frac{1}{3}$ of an apple?

**94.** Give the analysis of the method of finding the difference between two fractions. Give the rule.

**95.** How may a fraction be multiplied by an integer? How may a mixed number be multiplied by an integer? Why does multiplying the numerator multiply the fraction? Why does dividing the denominator by the integer multiply the fraction? Which method should be preferred? Should a mixed number ever be changed to an improper fraction before multiplying by an integer? Illustrate by an example, and give an analysis of the process.

**96, 97.** How are integers multiplied by fractions? How are mixed numbers multiplied by fractions? How are fractions multiplied by fractions? How are fractions multiplied by mixed numbers? To what is multiplying by a fraction, as $\frac{1}{2}$, $\frac{1}{7}$, $\frac{1}{8}$, equivalent? ANS. — *To dividing by an integer equal to the denominator of each fraction.* Illustrate by an example. To what is multiplying by a fraction, as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{5}{8}$, equivalent? ANS. — *To taking the fractional part of a number; thus,* $\frac{3}{4}$ of 7, $\frac{2}{3}$ of 11, $\frac{5}{8}$ of $2\frac{1}{2}$. In the multiplication of fractions, when is the word "of" used? When is the word "times" used? What is the difference of meaning? Give a rule for multiplying by a fraction or mixed number.

**98.** Give the rule for dividing a fraction by an integer. Why does dividing e numerator divide the value of the fraction? Why does multiplying the nominator divide the value of the fraction? Give the method of dividing a ixed number by an integer When should the mixed number be changed to an mproper fraction before being divided? ANS. — *When the integer of the mixed* *number is less than the divisor.* Illustrate by an example, and give an analysis of the process.

**99, 100.** How is an integer divided by a fraction? How is a fraction divided by a fraction? How is a mixed number divided by a fraction? How are mixed numbers divided by mixed numbers? What must be done to mixed numbers before dividing? Give a rule for dividing by a fraction or mixed number. Write an analysis of the process of an example in the division of fractions.

**General Questions.** In how many ways can fractions be reduced? Give rules for each. In how many ways can a fraction be multiplied? In how many ways can a fraction be divided?

## DECIMAL FRACTIONS.

**101.** What is a decimal fraction? What is meant by a power of ten? Give an example of a decimal fraction. Of a common fraction.

**102.** What is a mixed decimal? What is placed between the integer and the decimal? How do the orders increase in the decimal scale? How do they decrease? Illustrate what is meant by a tenfold ratio.

**103.** How may decimals be written? Are decimals ever written like common fractions? Can they be added, subtracted, multiplied, and divided like common fractions? Can they be added to, or subtracted from, common fractions? Give an example.

**104-106.** How do the orders increase in U. S. Currency? How does the notation differ from decimals? To what do dimes correspond? Cents? Mills? Tenths of mills? Dollars? How is dollars' place indicated? How is units' indicated? How many hundredths make a tenth? How many tenths a unit? How many hundredths make a unit? How many cents make a dollar? How many thousandths make a unit? How many mills make a dollar? How many ten-thousandths make a unit? How many tenths of a mill make a dollar?

What is the denomination of the first decimal place? The third? The second? The sixth? The fourth? The fifth? How do the addition and subtraction of decimals differ from the addition and subtraction of integers?

**107.** Give the rule for the multiplication of decimals. Will the result be affected by alternating the factors?

**108.** A decimal of any order divided by the same order gives a quotient of what order? Units by units gives what? Tenths by tenths? Tens by tens? Hundredths by hundredths?

**109.** What is the rule for the division of decimals? How is the units' place determined? What must be done when the divisor is of an order less than that of the dividend? When the dividend is of an order less than the divisor?

**110.** What is the rule for changing common fractions to decimals? When will the decimal figure, or series of figures, repeat?

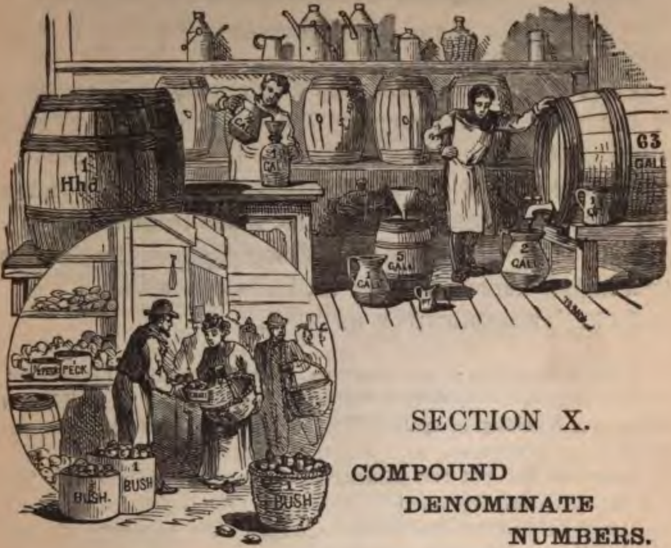
**111.** What is the rule for changing a decimal to a common fraction? When the decimal contains a fraction, what must be done?

**112.** When the price per 100 or 1000 is given, how can the cost be found? Give rule. Give the analysis.

## ANALYSIS SECTION IX.

### Decimal Fractions.

1. Illustration.
2. Definitions.
3. Notation.
4. Addition and Subtraction.
    - *Axioms.*
5. Multiplication.
    - *Rule.*
6. Division.
    - (*a.*) *Principle.*
    - (*b.*) *Rule.*
7. Reduction.
    - (*a*) *Common Fractions to Decimals.*
        - *Rule.*
    - (*b.*) *Decimals to Common Fractions.*
        - *Rule.*
8. Abstract Problems.
    - (*a*) *Analysis.*
    - (*b*) *Rule.*
9. Concrete Problems.
    - *Analysis.*



## SECTION X.

## COMPOUND DENOMINATE NUMBERS.

In buying and selling, the quantity of different articles is shown by counting, weighing, or measuring: thus some articles are bought and sold by the dozen or hundred; some, by the quart or gallon; some, by the peck or bushel, &c. The names — as foot, mile, gallon, or pound — used to designate certain quantities are called **denominations**; hence 4 yards, 9 pounds, are said to be *denominate numbers*. When a number representing a quantity is expressed in two or more denominations, it is said to be a *compound denominate number;* as, 4 yards, 2 feet, 5 inches.

Those denominations in a compound number, which express by comparison the greater amount, are said to be the *higher denominations;* those which express the less amount are said to be the *lower denominations:* thus a pound is a higher denomination than an ounce.

When numbers of a higher denomination are changed to a lower, without altering their values, the reduction is said to be *descending;* when changed from a lower to a higher, the reduction is said to be *ascending*.

**113. *A Compound Denominate Number*** is one expressing more than one denomination; as, 1 pound and 7 ounces.

**114. *The Reduction of Compound Numbers*** is the process of changing the denomination without altering the value; as, 1 gallon = 4 quarts or 8 pints.

**115. *Reduction Descending*** changes the number to a lower denomination; as, miles to inches.

**116. *Reduction Ascending*** changes the number to a higher denomination; as, inches to miles.

### **117.** MEASUREMENT OF SPACE.

*Space* is extension in any direction: it may have three dimensions, or measurements, — length, breadth, and thickness. A *Solid*, or a *Volume*, is a portion of space which has three dimensions, — length, breadth, and thickness; as, a block, or a book.

*A Surface* has but two dimensions, — length and breadth. The leaf on which this is printed is a solid: the page is the surface.

*A Line* has but one dimension, — length. The boundary of a surface is a line; as, the edge of this page.

## LINEAR MEASURE.

**118. *The Measures of Length*** are Long Measure and Surveyors' Measure. They are used for measuring distance.

### TABLES OF LINEAR MEASURE.

#### LONG MEASURE.

12 inches (*in.*) = 1 foot (*ft.*).
3 feet = 1 yard (*yd.*).
5½ yards
(11 half yds.) = 1 rod (*rd.*).
16½ feet
320 rods = 1 mile (*mi.*).

#### SURVEYORS' MEASURE.

7.92 inches = 1 link (*lk.*).
25 links = 1 rod (*rd.*).
100 links
4 rods = 1 chain (*ch.*).
80 chains = 1 mile (*mi.*).

The following denominations are sometimes used: —

4 inches = 1 hand. Used to meas. height of horses.
3 feet = 1 pace. Used for stepping off distances.
6 feet = 1 fathom. Used to measure depths at sea.
1⅐ statute miles (nearly) = 1 geographic mile, or nautical knot.
60 geographic miles = 1 degree of latitude on equator.

### TABLE OF EQUIVALENTS.

1 mile = 80 chains.
1 mile = 320 rods.
1 mile = 1760 yards.
1 mile = 5280 feet.

1 chain = 4 rods.
1 chain = 66 feet.
1 chain = 100 links.
1 yard = 36 inches.

1 meter = 39.37 inches, or 1 yard, 3⅓ inches.

NOTES. — (1.) The surveyors' chain contains 100 links, and measurements are given in chains and links; thus 12.34 = 12 chains and 34 links.

(2.) Railroad engineers use a chain containing 100 links, each 1 foot long; and they record their measurements in miles and feet.

(3.) The yard, half-yard, quarter, eighth, and sixteenth are used in measuring cloth.

## REDUCTION DESCENDING.

### ORAL EXERCISE.

1. How many inches in 3 feet?

**ANALYSIS.** — Since in 1 foot there are 12 inches, in 3 feet there are 3 times 12 inches, or 36 inches.

2. How many yards in 3 rods?

**ANALYSIS.** — Since in 1 rod there are 5½ yards, in 3 rods there are 3 times 5½ yards, or 16½ yards. 3 times 5 yards are 15 yards; 3 times 1 half-yard are 3 half-yards, equal to 1½ yard; 15 yards plus 1½ yard equals 16½ yards.

3. How many inches in 5 feet? In 4 feet?
4. How many half-yards in 1 rod? In 2 rods?

# FELTER'S

# ARITHMETICAL DRILL CARDS.

## TABLE VI.

| STEPS.* | I. | II. | III. | IV. | V. | VI. |
|---|---|---|---|---|---|---|
| Linear Meas. | *ml.* | ...... | *rd.* | *yd.* | *ft.* | *in.* |
| | *ml.* | ...... | *rd.* | ...... | *ft.* | *in.* |
| | *ml.* | *ch.* | *rd.* | ...... | *lk.* | *in.* |
| | *ml.* | *ch.* | ...... | ...... | *lk.* | *in.* |
| Square Meas. | *sq.ml.*† | *A.* | *sq.rd.* | *sq.yd.* | *sq. ft.* | *sq. in.* |
| | *sec.*† | *A.* | *sq.ch.* | *sq.rd.* | *sq. lk.* | *sq. in.* |
| | *sec.*† | *A.* | *sq.ch.* | ...... | *sq. lk.* | *sq. in.* |
| Cubic Meas. | *cd.* | *cu. ft.* | ...... | ...... | ...... | *cu. in.* |
| | *pch.* | *cu. ft.* | ...... | .... | ...... | *cu. in.* |
| | *cu. yd.* | *cu. ft.* | ...... | ...... | ...... | *cu. in.* |
| Fluid Meas. | *gal.*† | *qt.* | *pt.*† | *gi.* | ...... | *cu. in.* |
| | *gal.* | ...... | ...... | ...... | ...... | *cu. in.* |
| | *cong.*† | ...... | *O.*† | *f℥.* | *fʒ.* | *m.* |
| Dry Meas. | *bu.* | ...... | .... | ...... | ...... | *cu. in.* |
| | *bu.* | *pk.* | *qt.* | *pt.* | ...... | *cu. in.* |
| Troy Wt. | *lb.* | *oz.* | *pwt.* | ...... | *gr.* | ...... |
| | *lb.* | .... | ...... | ...... | *gr.* | ...... |
| Apoth. Wt. | *lb.* | ℥. | ʒ. | ℈. | *gr.* | ...... |
| Avoirdu. Weight. | *T.* | *cwt.* | *lb.* | *oz.* | *gr.* | ...... |
| | *T.* | ...... | *lb.* | .... | *gr.* | ...... |
| | *T.* | ...... | *lb.* | *oz.* | *gr.* | ...... |
| Time. | *yr.* | *mo.* | *da.* | *hr.* | *min.* | *sec.* |
| | *yr.* | *W.* | *da.* | *hr.* | *min.* | *sec.* |
| | *yr.* | ...... | *da.* | *hr.* | *min.* | *sec.* |
| Circular and Ang. Meas. | *cir.* | *quad.*† | *deg.* | *min.* | *sec.* | ...... |
| | *cir.* | *r. a.*† | *deg.* | *min.* | *sec.* | ...... |
| | *cir.* | *S.* | ° | ′ | ″ | ...... |
| Paper. | *B.* | *bun.* | *rm.* | *qr.* | *sheet* | ...... |
| Currency | $ | *ct.* | *m.* | ...... | ...... | mills |
| | *F.* | ...... | ...... | ...... | ...... | mills |
| | *F.* | *ct.* | *m.* | ...... | ...... | mills |
| | £ | ...... | ...... | ...... | ...... | mills |
| | £ | *S.* | *d.* | *far.* | ...... | mills |

| | V | W | X | Y | Z |
|---|---|---|---|---|---|
| A | 2 | 20 | 127 | 4019 | $\frac{1}{2}$ |
| B | 1 | 17 | 105 | 3806 | $\frac{2}{5}$ |
| C | 3 | 86 | 420 | 2914 | $\frac{3}{7}$ |
| D | 5 | 35 | 900 | 8637 | $\frac{4}{9}$ |
| E | 7 | 42 | 109 | 5189 | $\frac{5}{11}$ |
| F | 2 | 63 | 207 | 7933 | $\frac{6}{13}$ |
| G | 9 | 87 | 350 | 2380 | $\frac{7}{15}$ |
| H | 8 | 67 | 765 | 6734 | $\frac{8}{17}$ |
| I | 4 | 95 | 903 | 2981 | $\frac{9}{19}$ |
| K | 6 | 43 | 508 | 9171 | $\frac{10}{21}$ |
| L | 10 | 23 | 343 | 2986 | $\frac{11}{100}$ |
| M | 7 | 60 | 627 | 3941 | $\frac{12}{1000}$ |
| N | 11 | 25 | 895 | 7243 | $\frac{8}{10}$ |
| O | 4 | 37 | 546 | 8907 | $\frac{14}{10}$ |
| P | 2 | 80 | 387 | 6000 | $\frac{15}{100}$ |
| Q | 12 | 14 | 216 | 5900 | $\frac{46}{10}$ |
| R | 9 | 28 | 950 | 3009 | $\frac{7}{100}$ |
| S | 4 | 79 | 221 | 1957 | $\frac{98}{100}$ |
| T | 5 | 75 | 333 | 9192 | $\frac{9}{1000}$ |
| U | 3 | 40 | 575 | 8000 | $\frac{20}{100}$ |

* Indicates the number of steps in changing one denomination to another.

† *Those denominations* of the same weight or measure, in the same column, havin *different names*, and marked with a dagger, are of the same value.

## DRILL CARD EXERCISES. — TABLE VI.

Prepare the following Drill Card Exercises on slate or paper at seats, to be read as a class-exercise.

**Exercise I. — Change V miles to chains. (Twenty examples.)**
**II. — Change V miles to rods.**
**III. — Reduce V rods to yards.**

**Directions. — Copy the number opposite each letter as miles, and reduce it to chains. Arrange the exercises as follows: —**

| **Exercise I.** | **Exercise II.** | **Exercise III.** |
|---|---|---|
| **A. 2 mi. = 160 ch.** | **A. 2 mi. = 640 rds.** | **A. 2 rds. = 11 yds.** |
| **B. 1 " = 80 "** | **B. 1 " = 320 "** | **B. 1 " = 5½ "** |
| **C. 3 " = 240 "** | **C. 3 " = 960 "** | **C. 3 " = 16½ "** |
| **&c. to U.** | **&c. to U.** | **&c. to U.** |

**N. B. — Under each table, the teacher should dictate Drill Card Exercises, similar to the following, until the pupils are prompt and accurate in their work.**

| **REDUCTION DESCENDING.** | **REDUCTION ASCENDING.** |
|---|---|
| **Ex. I. — Reduce W rds. to feet.** | **Ex. IV. — Reduce X feet to rds.** |
| **II. — Reduce V ml. to yds.** | **V. — Reduce Y in. to rods.** |
| **III. — Reduce W ch. to feet.** | **VI. — Reduce X ch. to ml.** |
| **VII. — Change V sq. ml. to acres.** | **X. — Change Y sq. in. to sq. feet.** |
| **VIII. — Change W sec. to acres.** | **XI. — Change Y sq. yds. to acres.** |
| **IX. — Change V acres to sq. rods.** | **XII. — Change X sq. yds. to sq. rods.** |
| **XIII. — Reduce V cds. to cu. ft.** | **XVI. — Reduce Y cu. in to cu. feet.** |
| **XIV. — Reduce V pch. to cu. ft.** | **XVII. — Reduce X cu. ft. to cords.** |
| **XV. — Reduce V cu. yds. to cu. inches.** | **XVIII. — Reduce W cu. ft. to cu. yds.** |
| **XIX. — Change V gals to gills.** | **XXII. — Change X pts. to gals.** |
| **XX. — Change W gal. to cu. in.** | **XXIII. — Change Y cu. in. to gallons.** |
| **XXI. — Change V Cong. to m.** | **XXIV. — Change X gills to flʒ.** |

4. In 1
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FORMUL

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7. What
.72?
8. What
Of $9\frac{9}{10}$?

**108.**
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1

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REVIEW.
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mals be
decimals?
The third?
The third
left? How
(106.)

rods? *W.* In 320 rods? In 7 miles? In 3 miles and 1 rod?

7. *W.* How many ties will it take for 24 miles of track, if every 3 feet requires one tie?

8. How many yards of carpet will it take to cover a hall 18 feet long, if the width requires 2 breadths? *W.* If the hall is 156 feet long, and the width requires 3 breadths? What will the carpet cost, at $1¾ per yard?

9. *W.* From New York to Binghamton is 215 miles: how many railroad-ties will it take to lay the track, if they are placed 3 feet apart? What will they cost, at 8 cents each?

10. *W.* There is a door-yard 2 rods long and 2 rods wide: how many feet of fence will it take to go around it? If it is 4 rods long and 2 rods wide? If it is 12 rods long and 8 rods wide?

11. *W.* There is a barn-yard 6 rods square: how many feet around it? How many boards 16½ feet long will it take to fence it, if the fence is 4 boards high? What will the boards cost, at 12 cents each?

12. *W.* I have a room 24 feet long by 15 feet wide: how many feet of paper bordering will it take to go around the walls? How many yards will it make? What will it cost, at 3 cents a yard?

13. *W.* How many panels of fence, 16½ feet each, will it take to fence a lot 26 chains, 25 links long, and 12 chains and 50 links wide? How much will it cost, at $1.87 per panel?

---

**Review.**—What is the effect of repeating a letter in the Roman notation? (8.) Of placing a letter of less value before one of greater value? Of placing a letter of less value after one of greater value? Of placing a dash over a letter, or combination of letters? How does the Arabic notation differ from the Roman? (9.) How many characters are used? How is the number fifteen expressed?

14. *W.* My farm is 23 chains, 50 links wide: how many steps across it, if each step measures 3 feet?

### REDUCTION ASCENDING.

#### ORAL EXERCISES.

1. How many feet in 14 inches? In 25 inches?

FORMULA. 14 inches ÷ 12 inches = 1 (foot) + 2 inches.

ANALYSIS. — Since 12 inches = 1 foot, 14 inches = as many feet as 12 inches are contained times in 14 inches, which is once, with a remainder of 2 inches.

2. How many yards in 6 feet? In 9 feet? In 18 feet? In 24 feet?

3. How many yards in 14 feet? In 22 feet? In 13 feet? In 25 feet?

4. How many half-yards in 3 yards? In 17 yards? In 18 yards? In 7 yards?

5. How many rods in 11 half-yards? In 22 half-yards? In 33 half-yards?

6. How many rods in 648 inches?

FORMULA. (1.) 648 inches ÷ 12 inches = 54 (feet).*
(2.) 54 feet ÷ 3 feet = 18 (yards).
(3.) 2 half-yards × 18 (yards*) = 36 half-yards.
(4.) 36 half-yards ÷ 11 half-yards = 3 (rods*) + 1½ half-yards.

ANALYSIS. — (1.) Since 12 inches = 1 foot, 648 inches = as many feet as 12 inches are contained times in 648 inches; or, 54 feet.

(2.) Since 3 feet = 1 yard, 54 feet = as many yards as 3 feet are contained times in 54 feet; or, 18 yards.

(3.) Since 1 yard = 2 half-yards, 18 yards = 18 times 2 half-yards; or, 36 half-yards.

(4.) Since 11 half-yards (5½ yards) = 1 rod, 36 half-yards = as many rods as 11 half-yards are contained times in 36 half-yards; or, 3 rods and 3½ yards; or, 3 rods, 1 yard, 1 foot, 6 inches.

---

[*] The numbers having the denomination enclosed in parentheses are used as *abstract in the analysis.*

7. How many rods in 252 inches? In 1008 inches? In 39 feet? In 76 feet?

8. How many rods in 80 chains? In 50 chains? In 70 chains? In 90 chains?

9. How many rods in 75 links? In 87 links?

### ORAL AND WRITTEN EXERCISES.

**Problem.** — How many miles in 14267 feet?

PROCESS.

**Step I.** 3 ft.)14267 ft.
4755 (yds.) + 2 ft.

**Step II.** 4755 (yds.)
2 h. yds.
9510 h. yds.

**Step III.** 11 h. yds.)9510 h. yds.
864 (rds.) + (6 h. yds.) 3 yds.

**Step IV.** 320 rds.)864 (rds.)
2 mi. + 224 rds.

14267 ft. = 2 mi. 224 rds, 3 yds. 2 ft.

**Anal. Steps.** — (1.) *Change feet to yards.*
* (2.) *Change yards to half-yards.*
* (3.) *Change half-yards to rods.*
(4.) *Change rods to miles.*

**Analysis.** — (1.) Since 3 feet = 1 yard, 14267 feet = as many yards as 3 feet are contained times in 14267 feet; or, 4755 (yds.) with 2 feet remainder.

(2.) Since 1 yard = 2 half-yards, 4755 yards = 4755 times 2 half-yards; or, 9510 half-yards.

(3.) Since 11 half-yards (5½ yds.) = 1 rod, 9510 half-yards = as many rods as 11 half-yards are contained times in 9510 half-yards; or, 864 (rds.) with (6 h. yds.) 3 yards remainder.

(4.) Since 320 rods = 1 mile, 864 rods = as many miles as 320 rods are contained times in 864 rods; or, 2 (mi.) with a remainder of 224 rods.

---

* Pupils who have studied fractions may, in this and all similar cases, divide by the mixed *number*, as in division of fractions.

**PROOF.**

| | |
|---|---|
| STEP I. | 2 mi. + 224 rds. + 3 yds. + 2 ft. |
| | 320 (rds.) |
| | 640 " |
| | 224 " |
| | 2)864 (rds.) |
| STEP II. | 5½ (yds.) |
| | 432 (yds.) |
| | 4320 " |
| | 4752 " |
| | 3 " |
| | 4755 (yds.) |
| STEP III. | 3 (ft.) |
| | 14265 (ft.) |
| | 2 " |
| | 14267 " |

ANAL. STEPS. — (1.) *Change miles to rods, and add rods.*

(2.) *Change rods to yards, and add rods.*

(3.) *Change yards to feet, and add feet.*

ANALYSIS. — NOTE TO TEACHER. — The teacher should rigidly hold the pupils to a correct analysis of each step, and not allow them, by carelessness of expression, to fall into the fallacy of multiplying rods by yards, &c. The denominations are affixed to the numbers to aid the memory: the analysis alone determines which are used as abstract.

15. How many feet in 24 inches? In 36 inches? In 72 inches? In 39 inches? *W.* In 367 inches? In 872 inches?

16. How many yards in 6 feet? In 36 inches? In 47 inches? In 25 feet? In 27 feet? *W.* In 2347 feet? In 9673 inches?

17. How many rods in 16½ feet? In 33 feet? In 11 yards? *W.* In 27 yards? In 2473 feet? In 138 yards?

18. *W.* At 30 cents a foot, what will 13 yards 2 feet of wire railing cost? At 40 cents? At $1.13?

19. *W.* At 88 cents a rod, what will 80.75 chains of fence cost? 63.25 chains? 47.50 chains?

20. *W.* What will it cost to carpet a room 51 feet long, if it takes 4 breadths of carpet to cover the floor, *at $1.75 per yard?* At $2.25 per yard?

21. What will 5 feet 9 inches of wire railing cost, at $1.50 per foot?

PROCESS.

12)$\underline{\$1.500}$ × 5 = $7.500 cost of 5 feet.
.125 × 9 = 1.125 cost of 9 in.
**$8.625 total cost.**

**ANAL. STEPS.** — (1.) *Find cost of* 5 *feet.*
(2.) *Find cost of* 1 *inch.*
(3.) *Find cost of* 9 *inches.*
(4.) *Find total cost.*

**ANALYSIS.** — (1.) Since 1 ft. costs $1.50, 5 ft. will cost 5 times as much; or, $7.50.

(2.) Since 12 inches (1 ft.) costs $1.50, one inch will cost one-twelfth as much; or, $.125.

(3.) Since 1 inch costs $.125, nine inches will cost 9 times as much; or, $1.125.

(4.) The total cost will be the sum of $7.50 and $1.125; or, $8.625.

22. What will 4 yards and 1 foot of brass wire cost, at 12 cents a yard? *W.* What will 17 yards 2 feet cost, at 18 cents a yard?

23. At 36 cents a yard, what will 2 feet of cloth cost? *W.* What will 7 yards, 6 inches cost? 14 yards, 2 feet, 6 inches? 13 yards, 2 feet?

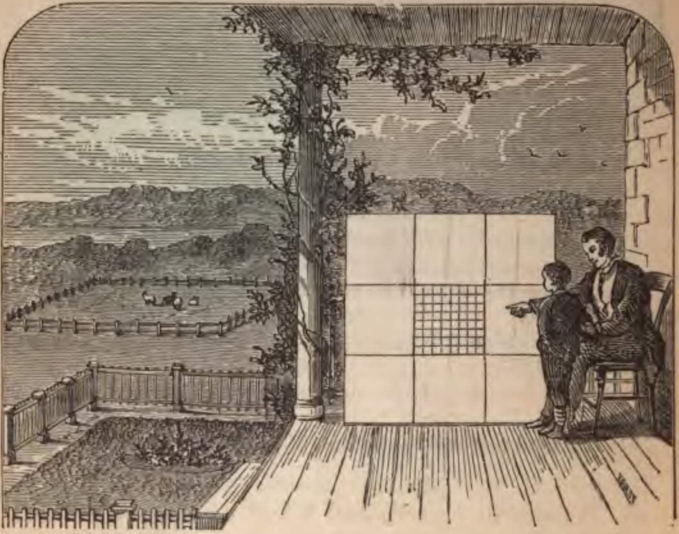
24. *W.* At $1.80 a rod, what will 7 rods and 2 yards of stone fence cost? 4 rods and 4 yards?

25. *W.* What will it cost to fence a piece of land 18.24 chains long and 11.38 chains wide, at $1.10 per rod?

---

**REVIEW.** — What is meant by the simple value of a figure? (10.) By the local value of a figure? (11.) What is numeration? (12.) What is the use of ciphers in writing a number? *Repeat the numeration table by periods from right to left. From left to right.*

## SQUARE MEASURE.



If you will take a piece of paper, like the one on the board in the picture, one *foot* square, and divide it into 12 rows of squares, with 12 squares in each row, making 144 pieces of equal size, each piece will contain just one *square inch*. It will take 9 square feet of paper to cover the surface of the board, which contains a *square yard*. The grass-plat in the door-yard is 30¼ times as large as the surface of the board, and contains one *square rod*. The field in which you see the sheep is 160 times as large as the grass-plot, and contains one *acre of land*.

**119.** ***Square Measure*** is used for calculating areas of land, roofing, flooring, paving, painting, &c.

---

*Review.* — How many units of any order make one of the next higher? (13.) *How does a change* of one place to the right affect the value of a digit? How does a change of one place to the left affect its value?

### SURFACE MEASURE.

144 square inches (*sq. in.*) = 1 square foot (*sq. ft.*).
9 square feet = 1 square yard (*sq. yd.*).
$30\frac{1}{4}$ square yards (121 quarter square yards) = 1 square rod or perch (*sq. rd.* or *pch.*).
160 square rods = 1 acre (*A.*).

### SURVEYORS' SQUARE MEASURE.

625 square links (*sq. lk.*) = 1 square rod or perch (*sq. rd.* or *pch.*).
16 square rods = 1 square chain (*sq. ch.*).
10 square chains = 1 acre (*A.*).
640 acres = 1 section (*sec.*).
36 sections = 1 township (*T.*)

**NOTE.** — A section is a piece of land a mile square, and is equal to a square mile.

### TABLE OF EQUIVALENTS.

1 A. = 160 sq. rds. = 4840 sq. yds. = 43560 sq. ft. = 6272640 sq. in.
1 sq rd. = $30\frac{1}{4}$ " = $272\frac{1}{4}$ " = 39204 "
1 sq. yd. = 9 " = 1296 "

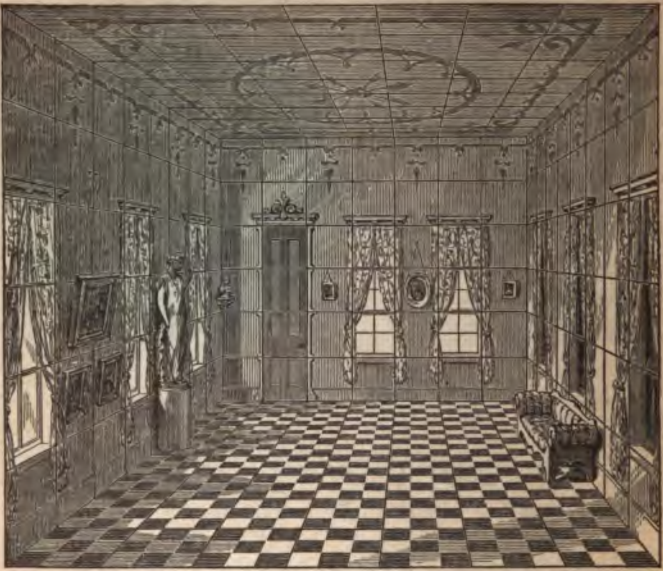
### SLATE AND BLACKBOARD EXERCISES.

(See Drill Card, Table VI.)

Exercise. I. — Reduce **V** acres to square rods.
II. — Reduce **V** square rods to square yards.
III. — Reduce **V** square yards to square feet.
IV. — Reduce **V** square feet to square inches.
V. — Reduce **W** square rods to square chains.
VI. — Reduce **W** square chains to acres.
VII. — Reduce **X** acres to sections.
VIII. — Reduce **X** sections to townships.

Prepare the above exercises in the same manner as on page 205, the results to be read as a class-exercise.

### ANALYSIS OF SQUARE MEASURE.



1. In the above illustration is seen a room 24 feet long, 21 feet wide, and 18 feet high: what is the size of the room in yards? How many square yards in the first row on the right side of the room? How many square yards in the entire floor?

ANALYSIS.—Since in 1 row (of square yards) there are 8 square yards, in 7 rows there are 7 times 8 square yards; or, 56 square yards.

2. How many square yards in the right hand wall? How many square yards in the left hand wall? How many in the end wall? How many in the ceiling? How many walls in a room? How many floors? How *many ceilings*?

3. How many square yards in the two side walls? How many in the two end walls? How many in all the walls taken together?

4. Since there are 9 tiles one foot square in each square yard, how many tiles in one row of square yards? How many in the entire floor?

**ANALYSIS.** — (1.) Since in 1 square yard there are 9 tiles, in 8 square yards (1 row) there are 8 times 9 tiles; or, 72 tiles.

(2.) Since in 1 row of 8 yards there are 72 tiles, in 7 rows (the entire floor) there are 7 times 72 tiles; or, 504 tiles.

***120.*** TO FIND THE AREA OF A SQUARE OR PARALLELOGRAM: —

**RULE.** — ***Multiply the length by the breadth, and the product will be the area.***

**NOTE.** — Usage, and the want of any other equally terse form of language, sanction this rule. Logically it is not true, any more than multiplying deer by grain will produce buckwheat. If the argument of the above paragraphs is thoroughly understood, the form of the rule can do no harm.

***Artificers*** usually estimate their work —

1. By the square foot; as in stone-cutting.
2. By the square yard; as in papering, lathing, plastering, paper-hanging, &c.
3. By the 100 square feet; as in flooring, roofing, siding, slating, &c.
4. By the thousand bricks, or by the square yard; as in bricklaying.

### WRITTEN EXERCISES.

26. How many square inches in a cardboard 5 inches long and 3 inches wide?

**FORMULA.** 5 square inches × 3 (rows) = 15 square inches.

**ANALYSIS.** — Since in one row (5 inches long) there are 5 square inches, in 3 rows (3 inches wide) there are 3 times 5 square inches; or, 15 square inches.

---

**REVIEW.** — Define a rule. (14.) Recite the rule for notation. (15.) Recite the rule for *numeration. (16.)* Define addition. (17.)

27. How many square inches in a board 26 inches long by 9 inches wide? 40 inches by 13 inches?

28. How many square feet in a plat of ground 75 feet by 100? 85 by 76 feet? 67 by 86 feet?

29. How many square inches in a board 108 inches long by 12 inches wide? How many square feet?

30. How many square feet in a pavement 156 feet long and 8 feet wide? How many square yards?

31. How many square feet in the side wall of a room 15 feet long and 8 feet high? How many square yards?

32. How many square feet in the floor of a room 36 feet long and 18 feet wide? How many square yards?

33. How many square feet in the ceiling of a room 81 feet long and 18 feet wide? How many square yards?

34. How many square feet in the two end walls of a room 25 feet wide and 8 feet high? How many square yards?

35. How many square yards in the four walls of a room 44 feet long, 18 feet wide, and 11 feet high?

36. How many square yards in the ceiling and walls of a room 26 feet long, 18 feet wide, and 12 feet high?

37. How much will it cost to lay the tiles on the floor of a room 24 feet long and 21 feet wide, at 50 cents per square yard? (See illustration.)

38. How many square yards of paper will it take to paper the walls of the same room, the ceiling being 18 feet in height? What will it cost, at 18 cents per square yard, no allowance being made for windows or doors?

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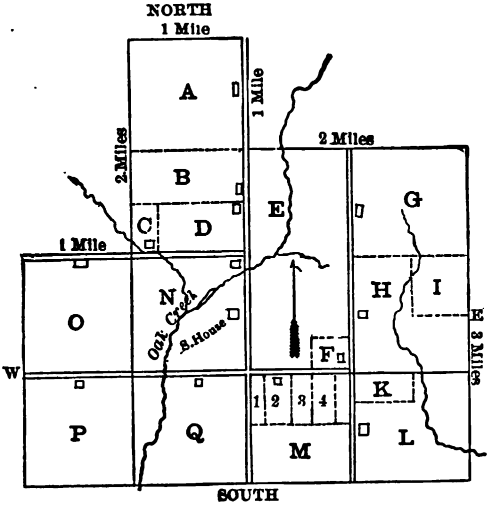
REVIEW.—Define "the sum" in addition. (18.) Describe the sign of addition, and illustrate the use. (19.) Describe the sign of equality, and give its use. (20.) Describe the dollar-sign, and give its use. (21.) What is an axiom? (22.) Write *the analysis of* a short example in addition. (23.)

39. What will it cost to paint the ceiling of the same room, at $1.14 per square yard?



40. The above is the map of Oakdale school district. A.'s farm lies in the square in the northern part. The north line is one mile long, and the east line is 80 chains: how many acres in his farm? What is it worth, at $$33\frac{1}{3}$ per acre?

41. B.'s farm is 80 chains long, and contains 320 acres: how wide is it? What is it worth, at $35.20 per acre?

42. D. had a farm of the same shape and size as A.; but he sold to B. 320 acres, the north half: how wide was the remainder? He then sold to C. a piece 40 chains long and 80 rods wide, from the west part of what remained: how many acres did C. buy? How many acres did D. sell in all? How much did he receive for it, at $15.75 per acre?

43. E. has the largest farm in the district: it was 2 miles long and 1 mile wide before he sold to F. a piece of land 35 rods long and 8 chains wide from the south-east corner. How many acres did he sell? How many acres had he before he sold any? How many acres had he left? What is it worth, at $10 per acre.

44. G. owns a farm containing a section of land situated in a square, for which he paid $18.42 per acre. He paid 1928 dollars for building his house and barn, and $0.87½ per rod for fencing it: what was the entire cost?

45. H. had a farm in shape and size like G.; but he sold the north-east quarter-section to I., at $22⅓ per acre. How much did he sell? How much should he receive?

46. I. paid 87 cents per rod for enclosing his land with a hedge: how much did it cost him?

47. K. bought a strip of land south of H., which was 160 rods long and 45 rods wide, located in the north-west part of the south-east section of the district: how much will it cost him to fence it, at 96 cents per rod? How much is L.'s land worth, at $31.35 per acre, including buildings?

48. M. owns a section of land, and he wishes to divide the north half into 5 equal parts: how wide a front will each piece have on the road at the north? How many acres will each contain? What will the first be worth, at $17 per acre? The second, at $42.30 per acre? The third, at $14 per acre? The fourth and fifth, at $9.75 per acre? What is the whole north half worth? What is the south half worth, at $11 per acre? He

---

**Review.—Define subtraction. (31.) The subtrahend. (32.) The minuend. (33.) The remainder. (34.) Describe the minus-sign. (35.) Give the axioms. (36.) *Give the analysis of a short problem.* (37.)**

paid $3000 for the whole farm: is the present value more or less than the cost, and how much? What is the gain or loss?

49. N. owns a section, and O. owns the section directly west: how much will it cost each to surround his land with fence at 90 cents per rod, supposing each to make his half only of the line[*] fences?

50. How many acres in the entire district?

51. The school district is assessed as follows:—

| | |
|---|---|
| Mr. A. at $7 per acre. | Mr. I. at $8 per acre. |
| " B. " 8½[†] " " | " K. " 8½ " " |
| " C. " 9 " " | " L. " 5 " " |
| " D. " 6¾ " " | " M. " 9½ " " |
| " E. " 4½ " " | " N. " 7 " " |
| " F. " 11 " " | " O. " 7 " " |
| " G. " 9⅓ " " | " P. " 7½ " " |
| " H. " 9⅞ " " | " Q. " 4 " " |

What is the assessed value of the land of the entire district?

52. What is the average assessed value of the district per acre?

**Suggestion.**—Divide the entire valuation by the number of acres in the district.

53. What will a board 6 feet 3 inches (75 in.) long, and 7 inches wide, cost, at $.288 a square foot?

**Formula.** (1.) 6 ft. 3 in. = 75 in.; 75 sq. in. × 7 = 525 sq. in.
(2.) 288 mills ÷ 144 = 2 mills.
(3.) 2 mills × 525 = $1.05.

54. *W.* At $1.30 per square yard, what will it cost to pave a walk 105 feet long and 7 feet wide? 47 feet long by 9 feet wide?

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[*] Road fences are not considered line fences.

[†] *Change the fractions to cents and mills.*

55. *W.* What will it cost to paint the floor of a room 25 feet long and 18 feet wide, at 40 cents per square yard? To paint a floor 55 feet long by 20 feet wide, at the same price?

56. *W.* What will it cost to plaster the walls and ceiling of a room 18 feet long, 16 feet wide, and 9 feet high, at $1.20 per square yard?

57. *W.* What will a piece of land 28 chains long and 5 chains wide, cost, at $14.72 per acre?

58. How many square feet in the floor of a room 16 feet long and 8 feet wide? *W.* 13 feet long and 17 feet wide? 25 feet long and 17 feet wide?

59. How many square feet in the ceiling of a room 9 feet long and 8 feet wide? *W.* 14 feet long and 12 feet wide? 14 feet long and 11 feet wide?

60. *W.* How many square feet in the end walls of a room 25 feet long, 15 feet wide, and 8 feet high? 22 feet long, 11 feet wide, and 9 feet high?

61. *W.* How many square feet in the side walls of a room 17 feet long, 13 feet wide, and 9 feet high? In a room 14 feet long, 12⅔ feet wide, and 7 feet high?

62. *W.* How much will it cost to plaster the walls and ceiling of a room 24 feet long, 15 feet wide, and 9 feet high, at 80 cents per square yard?

63. *W.* What will it cost to floor a room 16 feet long, and 12 feet wide, and 9⅔ feet high, at $12.50 per square of 100 feet?

64. What will it cost to paint the outside of a house 46 feet long, 27 feet wide, and 20 feet high, at $1.20 per square yard?

## CUBIC MEASURE.

**121. *Cubic Measure*** is used in estimating the contents of solids; as, wood, masonry, the capacity of cellars, cisterns, &c.

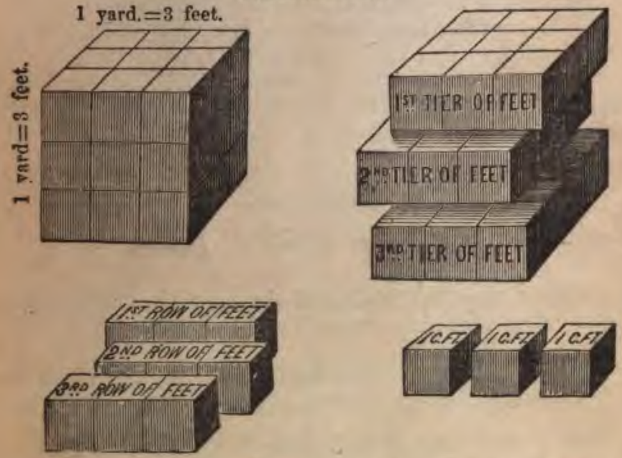
### TABLE OF CUBIC MEASURE.

1728 cubic inches (*cu. in.*) = 1 cubic foot (*cu. ft.*).
27 cubic feet = 1 cubic yard (*cu. yd.*).
128 cubic feet = 1 cord (*cd.*).
24¾ cubic feet = 1 perch of stone (*pch.*).

**Note.** — 99 quarter cubic feet make 1 perch of stone.

## ANALYSIS OF CUBIC MEASURE.

### ILLUSTRATION.



Anal. Steps. — (1.) *Find the number of rows of cu. ft. in* 1 *cu. yd.*

Analysis. — Since there are 3 *rows* of cubic feet in 1 *tier* of feet, in 3 *tiers*, there are 3 times 3 *rows*, which is 9 *rows*.

(2.) *Find the number of cubic feet in* 1 *cubic yard.*

Analysis. — Since in 1 row there are 3 cubic feet, in 9 rows, there are 9 *times 3 cubic feet*, which is 27 cubic feet.

***122.*** To find the solid contents of a body, we have the following

RULE. — *Multiply the length by the breadth, and that product by the thickness.*

**NOTES.** — (1.) A load of earth contains a cubic yard, and weighs about 3250 pounds. A cubic foot of water weighs 62½ pounds.

(2.) Railway and transportation companies estimate light freight by the number of cubic feet it occupies; but heavy freight is estimated by weight.

(3). A pile of wood 4 feet wide, 4 feet high, and 8 feet long, contains 1 cord; and a cord-foot is 1 foot in length of such a pile.

(4.) A brick is usually 8 inches long, 4 inches wide, and 2¼ inches thick.

(5.) Sawed lumber, as joists, plank, and scantlings, are now generally bought and sold by board measure. The dimensions of a foot of board measure are 1 foot long, 1 foot wide, and 1 inch thick. Thus a plank 12 feet long, 1 foot wide, and 2 inches thick, contains 24 feet board measure, or as much as two boards of the same length and breadth, and 1 inch thick. But boards that are less than 1 inch thick are measured as if they were an inch thick.

### ORAL EXERCISES.

1. How many cubic feet in 2 cubic yards?

**ANALYSIS.** — Since in 1 cubic yard there are 27 cubic feet, in 2 cubic yards, there are 2 times as many; or, 54 cubic feet.

2. How many cubic inches in 1 cubic foot and 19 cubic inches?
3. How many cubic feet in 2 cords of wood?
4. How many cubic feet in 1 cord and 17 cubic feet?
5. How many perch of stone in 99 cubic feet?

---

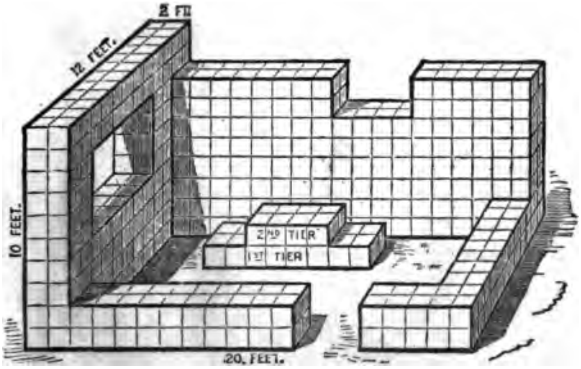
REVIEW. — Give the use of the parenthesis. (39.) Define multiplication. (40.) The multiplicand. (41.) The multiplier. (42.) The product. (43.) Which are factors? (44.) Describe the sign of multiplication, and illustrate its use. (45.) Give the axiom. (46.) Give the principles. (47.)

6. How many cubic inch blocks will it take to fill a mortise one foot long, 3 inches wide, and 6 inches deep?

7. How many quarter cubic inch blocks will it take to fill a mortise one inch each way? One foot each way?

8. How many cubic feet in a block of stone 5 feet long, 3 feet wide, and 3 feet thick?

8. What is the difference in volume between a cube 5 inches on each edge, and 5 cubic inches?


2 FT.
12 FEET.
10 FEET.
2ND TIER
1ST TIER
20. FEET.

In the above illustration will be seen a part of the walls and foundation of a building 20 feet long, 12 feet wide, and 10 feet high, the walls being 2 feet thick.

### WRITTEN EXERCISES.

65. How many cubic feet in the left-hand wall?

**Formula** (1.) 12 cubic feet × 2 = 24 cubic feet.
(2.) 24 cubic feet × 10 = 240 cubic feet.

**Analysis.** — (1.) Since in one row (12 feet long) there are 12 cubic feet, in two rows (2 feet thick), there are 2 times as many, or 24 cubic feet.

(2.) Since in 1 tier (12 feet long by 2 feet thick) there are 24 cubic feet, in 10 tiers (10 feet high), there are 10 times as many, or 240 cubic feet.

**Note.**—In estimating the contents of walls or masonry, no allowance is made for windows, doorways, or corners. All measurements are understood as being outside measurements, unless otherwise specified.

66. How many cubic feet in the highest side wall?

67. How many cubic feet in the foundation of the lowest end wall? In the foundation of the lowest side wall, making a deduction for the doorway?

68. How many cubic feet in the walls of the building as they now stand?

69. How many cubic feet of masonry will there be in the walls when ten feet high?

70. How many cubic feet of bricks in the pile on the floor of the building?

71. How many cubic feet of bricks will it take to cover the floor inside the walls, one foot thick? How many cubic inches?

72. How many bricks will it take to make the floor one foot thick? What will they cost, at $5 per thousand?

**Note.**—Allowing for mortar, 21 bricks of the common size ($8 \times 4 \times 2\frac{1}{4}$) make a cubic foot.

73. How many bricks will it take to build the walls as they now stand? What will they cost, at $7.30 per thousand?

74. How many bricks will it take to build the walls 10 feet high? How much will it cost, including labor, at $11.20 per thousand?

---

**Review.**—Write the analysis of a short example in multiplication. (48.) How may a number be multiplied by 10, 100, 1000, &c? (49.) Give table of U.S. currency. (50.) Give the definition of division. (51 ) Of dividend (52.) Of divisor. (53). Of quotient. (54.) Of remainder. (55.) Give use of the sign. (56.) Of the fractional sign. (57.) How does short division differ from long division? (58, 59.)

75. How many perch of stone would be required to make the walls 10 feet high?

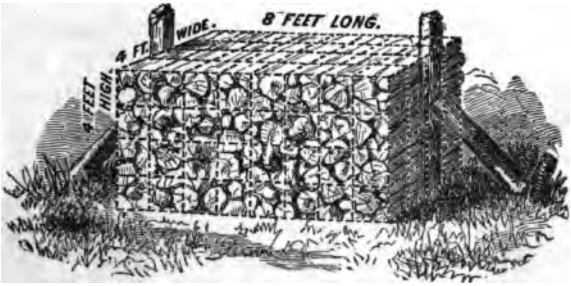
SUGGESTION. — Change the cubic feet to quarter cubic feet; then quarter cubic feet to perch.

76. Suppose the walls completed 12 feet high, how many perch of stone would they contain?

77. Suppose the walls to be built 10 feet high, how many cubic feet of wood would it take to fill the inside of the building level with the top?

78. How many cubic inch blocks would it take to fill it?

79. 231 cubic inch blocks are equal in volume to a gallon of water: how many gallons of water will fill a cistern of the same size?


4 FT. WIDE.
8 FEET LONG.
4 FEET HIGH.

CORD OF WOOD.

Arrange 128 cubic foot blocks into a pile 8 feet long, 4 feet wide, and 4 feet high, and it will represent the shape and size of a *cord*. One foot of the length of such a pile is called a *cord foot*, and contains 16 cubic feet. 8 cord feet make a cord.

---

REVIEW. — Give the axiom used in division. (60.) Name the principles. (61.) Write the analysis of an example in short division. (62.) How may a number be divided by 10, 100, 1000, &c.? (63.) Write the analysis of a short example in long division. (64.) Give the table of U. S. currency. (65.) Give the table of aliquot parts. (66.) Of what are coins made? (67.) What can you say about the coins? Of what does paper money *consist?* (68.) What is a bill, or invoice? (69.) Which is *the debtor? which the creditor?* What is an account?

90. How much will it cost to shingle the above barn, each shingle covering 6 square inches, at $1.75 per thousand shingles.

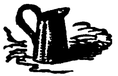
91. What is the estimated cost of the lumber of the above building?

### LIQUID MEASURE.

***123.*** *Liquid Measure* is used for measuring liquids.

TABLE.

4 gills (*gi.*) = 1 pint (*pt.*).
2 pints = 1 quart (*qt.*).
4 quarts = 1 gallon (*gal.*).


PINT.

GILL.

TABLE OF EQUIVALENTS.

| *gal.* | *qt.* | *pt.* | *gi.* |
|---|---|---|---|
| 1 = | 4 = | 8 = | 32 |
| | 1 = | 2 = | 8 |

QUART.

GALLON.

NOTE. — In calculating the capacity of cisterns, a hogshead is estimated to hold 63 gallons.

ORAL AND WRITTEN EXERCISES.

92. How many gills in 3 pints? In 2 quarts? In 2 gallons? In 1 gallon and 3 quarts? In 3 quarts and 1 pint?

93. How many gallons in 12 quarts? In 24 quarts? In 72 gills? *W.* In 463 quarts? In 463 gills?

94. How many pints in 13 quarts of berries? How much are they worth at 5 cents a pint?

95. *W.* How many quarts of milk in 43 gallons? How much is it worth at 9 cents a quart?

---

REVIEW. — For what is square measure used? (119.) Give the table of surface measure. Of surveyors' square measure. What is meant by a section of land? Give the rule for finding the area of a square or parallelogram. (120.) How do artificers estimate their work?

96. *W.* I have three cows, each gives 4 gallons of milk a day: how much milk will they yield in a week? How much would it be worth at 8 cents a quart?

### DRY MEASURE.

***124.*** *Dry Measure* is used for measuring grain, fruit, and for estimating the capacity of boxes, bins, corn-cribs, &c.

PECK.

QUART.

#### TABLE.

2 pints (*pt.*) = 1 quart (*qt.*).
8 quarts = 1 peck (*pk.*).
4 pecks = 1 bushel (*bu.*).

PINT.

BUSHEL.

#### TABLE OF EQUIVALENTS.

*bu.* *pks.* *qts.* *pts.*
1 = 4 = 32 = 64
1 = 8 = 16

#### TABLE OF COMPARISON.

1 gal. liquid mea. = 231 cu. in.
1 qt. " = 58 "
1 qt. dry measure = 67+ "
1 litre = 61 "

1 bushel = 2150 cu. in.
1 peck = 537½ "
8 qts. liq. mea. = 462
1 hectolitre = 6102+ "

NOTES — (1.) A bushel measure is 18½ inches in diameter, and 8 inches deep: it contains 2150.42 cubic inches.

(2.) In measuring grain, seeds, peas, beans, and small fruit, the measure must be *even* full. In measuring large fruits, — as apples, pears; coarse vegetables, as potatoes, beets, and other bulky articles, — the measure must be *heaping* full. Four heaped measures should equal five even measures. Five gills liquid measure make about one pint dry measure.

---

REVIEW. — Give the use of cubic measure. (121.) Write out the table. How many steps in changing cords to cubic inches? Cubic inches to cords? Give the rule for finding the contents of a cubic body. (122.) About how many pounds does a cubic yard of earth weigh? How much does a cubic foot of water weigh? What are the dimensions of a cord of wood?

### ORAL AND WRITTEN EXERCISES.

97. How many pints in a peck? In 3 pecks? *W.* In 75 pecks? In 47 bushels?

98. In 80 quarts how many pints? How many pecks? How many bushels?

99. At 6 cents a pint, what will be the cost of a peck of chestnuts? *W.* of 1 bushel? Of 138 bushels?

100. *W.* I bought a bushel of strawberries for $6.40, and sold them at 40 cents a quart: what did I gain?

101. *W.* I bought a barrel of apples, containing 2 bushels and 3 pecks, and sold them at the rate of 15 cents for 2 quarts: how much did I receive for them?

102. *W.* A grocer paid $5.75 for a barrel of potatoes containing 2 bushels and 2 pecks, and sold them at the rate of 20 cents for 2 quarts: did he make, or lose, and how much? Would he have made, or lost, if he had sold them at the rate of 20 cents for 3 quarts?

103. *W.* How many 3 pint bottles can be filled from 10 gallons of vinegar? How many pint bottles? How many gill cups and pint cups, each an equal number?

104. What will 2 gallons and 1 quart of wine cost, at $2 per gallon? *W.* At $2.12 per gallon? At $2.38?

105. At 10 cents a quart, what will 1 gallon, 1 pint, of vinegar cost? *W.* 21 gallons and 3 quarts? 47 gallons, 1 quart?

### WRITTEN EXERCISES.

106. Change 47 quarts to bushels.

107. What will a bushel and a peck of wheat cost at $2 a bushel? At $2.37½?

---

**Review.**—How are joists, planks, and scantlings generally sold? (122.) What is *meant by board measure?*

**108.** What will 3 bushels and 3 quarts of clover seed cost, at $10 per bushel?

PROCESS.

cost 1 bu.
4)$10.000 × 3 = $30.000
8)2.500
.312 × 3 (qts.) = .936
$30.94

**Anal. Steps.*** — (1.) *Find cost of 3 bush.*
(2.) *Find cost of 1 pk.*
(3.) *Find cost of 1 qt., then 3 qts.*
(4.) *Find cost of 3 bush. and 3 qts.*

**109.** What will 9 bushels, 3 pecks, of pears cost, at $2.50 per bushel? At $3.43 per bushel?

**110.** What will 39 gallons, 2 pints, of molasses cost, at $1.05 per gallon? At $1.37½?

**111.** What will 11 gallons, 1 quart, 3 gills, of wine cost, at $2.25 per gallon? At $3.13½?

**112.** How many bushels in a box 12 feet long, 8 feet wide, and 4 feet deep?

PROCESS.

(1.) 12 × 8 × 4 = 384 cu. ft.
(2) 1728 cu. in. × 384 = 663552 cu. in.
(3.) 663552 cu. in. ÷ 2150 = 308 bu. 2 pks.

**Anal. Steps.** — (1.) *Find the cubic contents.*
(2.) *Change to cubic inches.*
(3.) *Reduce to bushels, pecks, &c.*

**Analysis (Step 3).** — Since 2150 cubic inches = 1 bushel, 663552 cubic inches (contents of box) = as many bushels as 2150 cubic inches are contained times in 663552 cubic inches; or, 308 bushels, with a remainder of 1352 cubic inches. Since 537 cubic inches = 1 peck, 1352 cubic inches = as many pecks as 537 are contained times in 1352 cubic inches; or, 2 pecks +. (The reduction may be carried to quarts and pints in the same way.)

---

* We much prefer this method of solving problems of this class, because of its brevity, the simplicity of its analysis, and the avoidance of troublesome fractions.

113. How many bushels will a box 5 feet long, 3 feet wide, 3 feet deep, contain? 8 feet long, 4 feet wide, and 3 feet deep? 18 inches long, 12 inches wide, and 10 inches deep?

114. I have a corn-crib 26 feet long, 8 feet wide, and 10 feet high: how many bushels will it hold?

115. My wagon-box is 11 feet long, 6 feet wide, and 22 inches deep: how many bushels of shelled corn will it contain?

116. How many gallons of water will a circular tank or cistern contain, 5 feet in diameter, and 8 feet deep?

NOTE. — The distance across a cistern or tank, through the centre, is called the *diameter*. To find the number of cubic inches in a circular cistern or tank, use the following

RULE. — *Multiply the diameter in feet by itself, this product by* 1357, *and the result by the depth.*

FORMULA. (1.) $5 \times 5 \times 1357 \times 8 = 271400$ cu. in.
(2.) 271400 cu. in. $\div$ 231 cu. in. $= 1174 +$ gal.

ANAL. STEPS. — *(1.) Find the contents* in cubic inches.
*(2.) Reduce the* cubic inches to gallons, quarts, &c.

117. How many cubic inches in a circular cistern 4 feet in diameter and 7 feet deep? How many gallons will it hold?

118. How many gallons will a circular tank hold 4 feet across the top and 4 feet deep? If 8 feet across the top and 7 feet deep?

119. A railroad tank, the average diameter of which is 13 feet, and depth 12, contains how many gallons?

In addition to the above, the teacher should dictate a number of problems in estimating the contents of boxes, tanks, and cisterns.

## APOTHECARIES' WEIGHT AND MEASURE.

**125. *Apothecaries' Weight and Fluid Measure*** are used in preparing prescriptions; but drugs and medicines are bought and sold by avoirdupois weight.

### TABLE OF WEIGHTS.

20 grains (*gr.*) = 1 scruple (*sc.*, or ℈).
3 scruples = 1 drachm (*dr.*, or ʒ).
8 drachms = 1 ounce (*oz.*, or ℥).
12 ounces = 1 pound (*lb.*, or ℔.).

### TABLE OF FLUID MEASURE.

60 minims (*m.*) = 1 fluid drachm (f. ʒ).
8 fluid drachms = 1 fluid ounce (f. ℥).
16 fluid ounces = 1 pint (O.).
8 pints = 1 gallon (*Cong.*).

NOTE. — (1). The pint (O.) and gallon (Cong.) are the same as in liquid measure.

(2.) Apothecaries' fluid measure is treated out of its logical order, for the sake of brevity and convenience.

REVIEW. — For what is liquid measure used? (123.) Write the table. How many steps in changing gallons to gills? In changing gills to gallons? Which is reduction ascending? Which reduction descending? In calculating the capacity of cisterns, how many gallons is a hogshead estimated to hold?

120. How many pills of 3 grains each can be made from an ounce of quinine? How many of 4 grains each?

121. Three pints of the essence of peppermint, costing a dollar a pint, were put in ounce phials, and sold at 15 cents each. How much was made by the sale?

## TROY WEIGHT.

***126. Troy Weight*** is used in weighing gold, silver, coins, jewels, &c.

TABLE.

24 grains (*gr.*) = 1 pennyweight (*pwt.*).
20 pennyweights = 1 ounce (*oz.*).
12 ounces = 1 pound (*lb.*).

**NOTE. — A carat is a weight of about 3.2 grains, and is used by jewellers to weigh diamonds. The term *carat* is also used to denote the fineness of gold. When gold contains 18 parts pure gold and 6 parts alloy, which is usually silver and copper, it is said to be 18 carats fine. Gold 14 carats fine contains 14 parts pure gold and 10 parts alloy, &c.**

122. What will a gold chain weighing 12 pennyweights cost, at 80 cents per pennyweight? At 67 cents?

123. What will a gold chain weighing one ounce and 7 pennyweights cost, at 75 cents per pennyweight?

**NOTE. — For examples in oral and written reduction, use Drill Card.**

## AVOIRDUPOIS WEIGHT.

***127. Avoirdupois Weight*** is used for weighing all common goods.

TABLE.

16 ounces (*oz.*) = 1 pound (*lb.*).
100 pounds = 1 hundred wt. (*cwt.*). 1 cental (*c.*).
*20 hundred wt. = 1 ton* (*T.*).

TABLE OF COMPARISON.

1 lb. Troy = 5760 grs. Troy
1 lb. apoth. = 5760 " "
1 lb. avoir. = 7000 " "
1 kilogram = 15432 " "

***128.*** Nearly all the States have adopted the weights in the following

TABLE.

196 lbs. = 1 barrel of flour.
200 " = 1 " beef, pork, fish.
280 " = 1 " salt.
32 " = 1 bushel of oats.
56 " = 1 " corn or rye.
60 " = 1 " wheat, beans, peas.
14 " = 1 " bluegrass-seed.
60 " = 1 " clover-seed.
48 " = 1 " barley.
56 " = 1 " flaxseed.

44 lbs. = 1 bushel hemp-seed.
60 " = 1 " potatoes.
45 " = 1 " timothy-seed.
57 " = 1 " onions.
28 " = 1 " apples, pears (dry).
70 " = 1 " corn in ear.
52 " = 1 " buckwheat.
50 " = 1 " salt.
80 " = 1 " bituminous coal.

ORAL AND WRITTEN EXERCISES.

124. How many ounces in 2 pounds? In 5 pounds? *W.* In 75 pounds? In 647 pounds?

125. How many pounds in 5 hundred weight? In 7 cwt.? In 9 cwt.? *W.* In 13 cwt. and 46 lbs.? In 12 cwt. 37 lbs.?

126. How many hundred weight in 3 tons? *W.* In 5 T. 7 cwt.? In 14 T. 13 cwt.?

127. How many ounces in 4 lbs. 6 oz.? *W.* In 27 lbs. 12 oz.?

128. *W.* How many hundred weight in 347 lbs.? In 476 lbs.? In 2467 ounces? In 41 T.?

129. *W.* What will 913 lbs. of flour cost, at $4.25 per hundred weight?

PROCESS.

100) $4.250 × 9 cwt. = $38.25
.042 × 13 lbs. = .546
$38.80

ANAL. STEPS.—(1.) *Find cost of* 9 *cwt.*
(2.) *Find cost of* 13 *lbs.*
(3.) *Find cost of* 9 *cwt.* 13 *lbs.*

NOTE.—When the mills equal or exceed 5, add one to cents.

---

**REVIEW.**—For what is dry measure used? (124.) Write the table. How many steps in changing pints to bushels? In changing bushels to pints? How many cubic inches in a gallon of liquid measure? In a quart? In a quart dry measure? In a liter? In a bushel? In a peck? In 8 quarts liquid measure? In a hectoliter?

**130.** *W.* What will be the cost of a cask of sugar weighing 657 pounds, at $7\frac{1}{2}$ cents per pound? At \$8.385 per cwt.?

**131.** *W.* What will 5 lbs. 3 oz. of tea cost, at \$$1.87\frac{1}{2}$ per pound? At \$1.53 per pound? At \$2 per pound?

**132.** *W.* What will 274 pounds of butter cost, at \$43.42 per cwt.? At \$28.97 per cwt.? At \$35.46 per cwt.?

**133.** *W.* How many bushels of flaxseed in 470 pounds? What will it be worth, at \$1.83 per bushel?

**134.** *W.* Change 3478 lbs. of oats to bushels. Of corn. Of wheat. Of barley. Of potatoes. Of onions. Of salt.

**135.** *W.* In 439 pounds of rye, how many bushels? Of beans? Of clover-seed? Of corn in ear? Of hemp-seed?

**136.** *W.* What cost 7 lbs. 6 oz. rice, at \$.08 per pound?

ANAL. STEPS. — (1.) *Find cost of* 7 *lbs.*
(2.) *Find cost of* 1 *oz.*
(3.) *Find cost of* 6 *oz.*

**137.** *W.* What cost 2 lbs. 8 oz. of spice, at $37\frac{1}{2}$ cents per pound?

**138.** *W.* What cost 4 lbs. 4 oz. honey, at 38 cents per pound?

**139.** *W.* What cost 9 lbs. 5 oz. butter, at 55 cents per pound?

**140.** *W.* Add cost of 12 lbs. 13 oz. butter, at 20 cents per lb.; 11 lbs. 5 oz. rice, at 11 cents per lb.; 7 lbs. 3 oz. sugar, at 15 cents per lb.

---

**REVIEW.** — Give the dimensions of a bushel measure. (124.) The measure should be even full in measuring what grain and fruits? When should it be heaping full? *How many gills* make a pint liquid measure? A pint dry *measure?*

141. *W.* Add cost of 3 lbs. 6 oz. pepper, at 38 cents per pound; 4 lbs. 3 oz. spice, 40 cents per lb.; 18 lbs. 2 oz. cinnamon, at 80 cents per lb.

142. *W.* Add cost of 5 lbs. 7 oz. cloves, at $1.80 per lb.; 9 lbs. 11 oz. dates, at $.37½ per lb.; 4 lbs. 8 oz. honey, at 35 cents per lb.

143. *W.* Add cost of 1438 lbs. flour, at $6 per cwt.; 418 lbs. beef, at $18 per bbl.; 186 lbs. pork at $12 per bbl.

144. *W.* Add cost of 327 lbs. fish, at $14 per bbl.; 27 lbs. bluegrass-seed, at $4.30 per bu.; 58 lbs. beans, at $2.25 per bu.

145. *W.* Add cost of 75 lbs. clover-seed, at $9.50 per bu.; 119 lbs. onions, at $1.75 per bu.; 812 lbs. potatoes at $1.05 per bu.

### MEASURE OF TIME.

**129.** ***Time*** is the measure of duration.

#### DIVISION OF THE YEAR.

| Season. | Name of months. | No. of days. | Abbreviations. |
|---|---|---|---|
| Winter, | 1. January, | 31 | Jan. |
| | 2. February, | 28 or 29 | Feb. |
| Spring, | 3. March, | 31 | — |
| | 4. April, | 30 | — |
| | 5. May, | 31 | — |
| Summer. | 6. June, | 30 | — |
| | 7. July, | 31 | — |
| | 8. August, | 31 | Aug. |
| Autumn, | 9. September, | 30 | Sept. |
| | 10. October, | 31 | Oct. |
| | 11. November, | 30 | Nov. |
| Winter, | 12. December, | 31 | Dec. |
| | | 365 or 366 | |

---

REVIEW. — How can gallons liquid measure be reduced to bushels dry measure? (124.) ANS. — *By changing gallons to cubic inches, then the cubic inches to bushels.* How can gallons be reduced to liters? How can liters be reduced to quarts, liquid measure? To quarts, dry measure?

TABLE.

| | | |
|---|---|---|
| 60 seconds (*sec.*) | = 1 minute | (*min.*). |
| 60 minutes | = 1 hour | (*hr.*). |
| 24 hours | = 1 day | (*da.*). |
| 7 days | = 1 week | (*wk.*). |
| 4 weeks 2 days<br>30 days | = 1 month | (*mo.*). |
| 365 days<br>52 weeks 1 day<br>12 months | = 1 year | (*yr.*). |
| 100 years | = 1 century | (*C.*). |

NOTES. — (1.) The exact length of the solar year is 365 days 5 h. 48 m. 46.05 sec.; but, for convenience, the civil year is reckoned 11 min. 13.95 sec. more than this, or 365 da. 6 h. = 365¼ days. In four years, this ¼ day makes 1 day, which, in every fourth year, called bissextile or leap year, is added to the shortest month, giving it 29 days. The numbers denoting leap years are exactly divisible by 4; as, 1856, 1860, 1864. Years whose number can be divided without a remainder by 100, but not by 400, are not leap years, although divisible by 4.

(2.) In most business transactions, 30 days are called a month, and 52 weeks a year.

ORAL PROBLEMS.

1. How many seconds in 2 minutes? In 3 minutes? In 4 minutes? In 12 minutes?

2. How many minutes in 120 seconds? In 3 hours? In 240 seconds? In 5 hours?

3. How many days in 3 months? In 3 months, 12 days? In 9 months, 7 days?

4. How many hours from 6, A.M., to 11, A.M.? From 4, A.M., to 12, M.? From 9, A.M., to 3, P.M.?

5. How many minutes from 20 minutes past 9 to 10 minutes past 10? Between 13 minutes past 11 o'clock, A.M., and 12 minutes past 2 o'clock, P.M.?

---

REVIEW. — *How can bushels be reduced to hectoliters?* (124.) *How can hectoliters be reduced to pecks?*

### WRITTEN EXERCISES.

146. How many years in 783064 seconds?

147. How many years in 84160 minutes?

148. How many weeks in 71834 seconds?

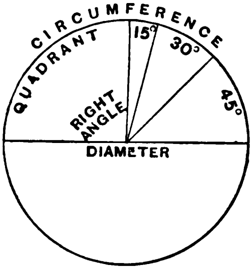
149. How many seconds in 3 years of 365 days?

150. How many seconds in the winter months of a common year?

151. How many in the winter of a leap year?

152. At a speed of 3 rods per second, how long will it take a railroad car to run 138 miles?

## CIRCULAR OR ANGULAR MEASURE.



**130. *A Circle*** is a plane figure bounded by a line, every point of which is equally distant from a point within, called the *centre.*

(*a.*) The *circumference* of a circle is its bounding line. An *arc* is any part of the circumference. A *quadrant* is one-fourth of a circumference.

(*b.*) The *diameter* is the distance across a circle through its centre. A *radius* is one-half of a diameter.

TABLE.

| | | | |
|---|---|---|---|
| 60 seconds (″) | = | 1 minute | (′). |
| 60 minutes | = | 1 degree | (°). |
| 30 degrees | = | 1 sign | (*sig.*). |
| 90 degrees | = | 1 quadrant | (*quad.*). |
| | | 1 right angle | (*r. a.*). |
| 4 quadrants or 12 signs | = | 1 circumference or circle | (*cir.*). |

**Review.** — What is the use of apothecaries' weight? (125.) What of apothecaries' fluid measure? How are drugs and medicines bought and sold? Write the *table. Name and give* the number of steps in reduction ascending. Give and *name the number of* steps in reduction descending.

**Notes.** — (1.) Circular or angular measure is used to measure angles, or the arcs of circles. It is used in astronomy, geography, navigation, and surveying, and for calculating differences of time.

(2.) The greatest circumference of the earth's surface is about 24,930 miles; 1° of that circumference is one 360th of 24,930 miles, which is 69¼ miles.

(3.) Since the sun apparently goes around the earth once in 24 hours, in 1 hour it must pass over *one* 24th of 360°, which is 15°: hence, if, in the sun's apparent motion, 15° of long. require 1 hour, or 60 minutes of time, one deg. will require *one* 15th of 60 minutes, which is 4 minutes.

(4.) Since 1° or 60′ long. requires 4 min., or 240 sec. of time, 1′ long. will require *one* 60th of 240 seconds, which is 4 seconds: therefore, 15° long. require 1 hour of time, 1° long. requires 4 minutes of time, 1′ long. requires 4 seconds of time. Hence the following: —

**Rule.** — I. *Multiply the difference of longitude by* 4 *to find difference of time.*

2. *To find the time at places east, add the difference to the given time; to find the time at places west, subtract it from the given time.*

### WRITTEN EXERCISES.

153. The difference of longitude between two places is 24° 35′: what is the difference of time?

TABLE.

| *long.* | *time.* |
|---|---|
| 1° (*deg.*) = | 4 min. |
| 1′ (*min.*) = | 4 sec. |

PROCESS.

24° 35′
4
———
1 hr. 38 min. 20 sec.

**Analysis.** — Since 1′ of long. requires 4 sec. of time, 35′ will require 35 times 4 sec., or 140 sec; 140 sec. = 2 min. 20 sec.

Since 1° of long. requires 4 min. of time, 24° will require 24 times 4 min., which, with 2 min., = 98 min. = 1 hr. 38 min.

154. What is the difference in time between Greenwich and Washington, the difference of longitude being 77 degrees?

Give the differences of time corresponding to the following differences of longitude: —

155. 4° 37′　　157. 97° 38′
156. 36° 42′　　158. 72° 22′

Give the equivalent in longitude of 17 min. 40 sec.

ANALYSIS. — (1.) Since 4 min. = 1°, 17 min. = 4°, with 1 min. remaining.

(2.) 1 min. = 60 sec., 60 sec. + 40 sec. = 100 sec. Since 4 sec. = 1′, 100 sec. = 25′. Ans. 4° 25′.

Give the differences of longitude corresponding to the following differences of time: —

159. 4 hr. 3 min. 16 sec.　　161. 1 hr. 2 min. 18 sec.
160. 2 hr. 46 min. 21 sec.　　162. 4 hr. 1 min. 2 sec.

## *131.* STATIONERS' TABLE.

12 units = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

163. What will 6 dozen penholders cost, at 5 cents apiece? At 4½ cents?

164. Bought 6 gross of pens, at $2.50 per gross, and retailed them at 2 cents apiece: did I make, or lose? and how much?

## ENGLISH MONEY.

***132. English Money*** is the currency of Great Britain.

4 farthings (*far.*) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound or sovereign (£.)

SILVER, 24 cts.

SOV., GOLD, $4.866.

SILVER, 19.3 cents.

NOTES. — (1.) The gold coins are the sovereign (£) and the half-sovereign (10*s.*). A sovereign is worth $4.866 in gold.

(2.) The silver coins are the crown (5*s.*), the half-crown (2*s.* 6*d.*), the florin (2*s.*), the shilling (12*d.*), sixpenny piece (6*d.*), and the three-penny piece (3*d.*).

(3.) The bronze coins are the penny, the half-penny, and farthing.

(4.) The franc is the unit of the French currency, and is worth $0.193. The denominations are francs and centimes.

(5.) The denominations of the Canadian currency are the same as United States currency.

166. How many shillings in 3 pounds? In 2 pounds? In 327 pounds?

167. How many shillings in £34? How many pence? How many farthings?

## ADDITION OF DENOMINATE NUMBERS.

### WRITTEN PROBLEMS.

***133.*** What is the sum of 14 bu. 2pks. 4 qts., 29 bu. 11 pks. 8 qts., 23 bu. 5 qts.?

PROCESS.

| bu. | pk. | qt. |
|---|---|---|
| 14 | 2 | 4 |
| 29 | 11 | 8 |
| 23 | 0 | 5 |
| 69 | 3 | 1 |

ANALYSIS. — (1.) For convenience write the numbers so that units of the same denomination will stand in the same column.

(2.) 5 qts. + 8 qts. + 4 qts. = 17 qts. = 2 pks. 1 qt. Write 1 in the place of quarts, and add the 2 pecks to the sum of the pecks.

(3.) 11 pks. + 2pks. + 2 pks. = 15 pks. = 3 bu. 3 pks. Write 3 in the place of pecks, and add the bushels to the sum of the bushels.

(4.) 23 bu. + 29 bu. + 14 bu. + 3 bu. = 69 bu.; which write in the proper place.

168. What is the sum of 14 bu. 3 pks. 6 qts., 2 bu. 2 pks. 5 qts., 37 bu. 2 pks. 8 qts., 5 bu. 8 qts., 9 pks. 4 qts.

169. What is the sum of 8 lbs. 5 oz. 11 pwt., 7 lbs. 2 oz. 13 pwt., 27 lbs. 16 pwt., 9 lb. 8 oz.

170. What is the sum of 44 A. 23 p., 10 A. 68 p., 25 A. 105 p., 17 A. 19 p.?

171. What is the sum of 25 rds. 12 ft. 5 in., 28 rds. 9 ft. 10 in., 18 rds. 10 ft., 12 rds. 14 ft. 9 in.?

172. I received at one time £25 6s. 4d.; at another, £36 18s.; at another, £12 10d.: how much was paid me in all?

173. A man owned three farms. The first contained 144 A. 27 P.; the second, 82 A. 16 P.; the third, 152 A. 37 P.: how many acres in all?

### SUBTRACTION.

***134.*** From 47 gals. 3 qts. 1 pt. 1 gi. take 29 gals. 2 qts. 1 pt. 3 gi.

PROCESS.

| gal. | qt. | pt. | gi. |
|---|---|---|---|
| 47 | 3 | 1 | 1 |
| 29 | 2 | 1 | 3 |
| 18 | 0 | 1 | 2 |

ANALYSIS. — (1.) For convenience write the subtrahend under the minuend so that units of the same denomination will stand in the same column.

(2.) Since 3 gills cannot be taken from 1 gill, take the 1 pint = 4 gills; 4 gills and 1 gill are 5 gills; 3 gills from 5 gills leave 2 gills, which we write under the gills.

(3.) Since the 1 pint was added to the gills, leaving no pints in the minuend, take 1 quart from 3 quarts, 1 quart = 2 pints, and from it subtract 1 pint, leaving 1 pint, which write under pints.

(4.) 2 quarts from 2 quarts leave no quarts: write a cipher under the quarts.

(5.) 29 gallons from 47 gallons leave 18 gallons, which write under gallons.

---

REVIEW. — What is the use of troy weight? (126) Write the table. Name and give the number of steps in reduction ascending. In reduction descending. What is the weight of the carat used in weighing diamonds? What is meant by term *carat*, in denoting the fineness of gold? What is meant by gold 18 car- fine? 14 carats fine? What is alloy? What metals are used to alloy gold?

174. From £78 11s. 5d. 2 far. take £41 13s. 3d. 3 far.

175. From 15 lbs. 3 oz. 12 pwt. 14 gr. take 9 lbs. 11 oz. 17 pwts. 21 gr.

176. From 15 lbs. 7℥ 1ʒ 2℈ 15 gr. take 11 lbs. 9℥ 7ʒ 1℈ 19 gr.

177. From 3 mi. 31 rds. 1 yd, take 2 mi. 9 rds. 8 yds.

178. What is the difference in time between July 3, 1875, and Sept. 1, 1877?

PROCESS.

| year | mo. | da. |
|---|---|---|
| 1877 | 9 | 1 |
| 1875 | 7 | 3 |
| 2 | 1 | 28 |

**Analysis.** — (1.) Sept. 1, 1877, denotes the 1st day of the 9th month of the 1877th year of the Christian era. As it is the later date, it is written as the minuend. July 3, 1875, the earlier date is written as the subtrahend, placing the years, months, days, each in its own column.

(2.) Subtract as in subtraction of denominate numbers.

What is the difference in time between each of the following dates? —

179. Aug. 1, 1872, and Sept. 21, 1875?

180. July 5, 1855, and Aug. 13, 1871?

181. Dec. 3, 1867, and March 5, 1870?

182. Nov. 1, 1859, and April 11, 1869?

183. Sept. 5, 1872, and Nov. 1, 1872?

184. July 5, 1874, and July 1, 1876?

185. Nov. 4, 1875, and July 1, 1877?

186. July 1, 1870, and July 9, 1870?

187. What is the difference in longitude between 132° 3′ 2″ east, and 67° 14′ 38″ east.

188. A ship starting from an island 37° 18′ 10″ west longitude was wrecked in 63° 10′ west. How far had she sailed?

---

**Review.** — What is the use of avoirdupois weight? (127.) Write the table. Give the steps. How many grains in 1 lb. troy? In 1 lb. apothecary? In 1 lb. avoirdupois? What is the difference between the weight of a pound avoirdupois and a pound troy? *Between a pound troy and a pound apothecary? Which is the heavier, — an ounce troy, or an ounce avoirdupois?*

**189.** What is the difference between 19° 30′ east, and 37° 6′ west?

SUGGESTION.—Add the longitudes of the two places to find the difference.

**190.** What is the difference between 120° 14′ E., and 36° 46′ W.?

**191.** What is the difference in time between 44° 8′ E., and 7° 10′ 4″ W.?

**192.** A grocer bought a hogshead of molasses containing 80 gallons. He sold 54 gals. 3 qts. 1 pt.: how much remained?

**193.** A firkin of butter contained 133 pounds: how much remained after selling 35 lbs. 7 oz.?

**194.** From three quarter-sections of land, there were sold 258 A. 34 p.: how much remained?

## MULTIPLICATION OF COMPOUND NUMBERS.

### WRITTEN PROBLEMS.

***135.*** What will 6 boxes of grain contain, if each hold 11 bu. 3 pks. 5 qts.?

PROCESS.

| bu. | pks. | qts. |
|---|---|---|
| 11 | 3 | 5 |
| | | 6 |
| 71 | 1 | 6 |

ANALYSIS.—(1.) For convenience, write the multiplier under the lowest denomination of the multiplicand.

(2.) 6 times 5 quarts = 30 quarts = 3 pecks, 6 quarts; write the quarts in the proper place, and add the pecks to the product of the pecks.

(3.) 6 times 3 pecks = 18 pecks, which with 3 pecks added = 21 pecks, or, 5 bushels, 1 peck; write the peck in the proper place, and add the bushels to the product of bushels.

(4.) 6 times 11 bushels = 66 bushels, which with 5 bushels added = 71 bushels, which write in the proper place.

---

REVIEW.—How can pounds avoirdupois be reduced to pounds troy? (127.) ANS.—*By changing the pounds avoirdupois to grains, then the grains to pounds troy.* How many steps? Which is reduction ascending? Which reduction *descending?*

195. Multiply £5 7 s. 8 d. 2 far. by 3. By 7.

196. Multiply 2 rds. 4 yds. 2 ft. 6 in. by 6. By 18.

197. Multiply 5 hrs. 42 min. 37 sec. by 13. By 42.

198. How many cranberries in 16 boxes, each containing 2 bu. 2 pks. 6 qts.?

199. If 2 lbs. 5 oz. grapes are put in a box, what will be the weight of a crate containing 24 boxes? [illegible] boxes?

200. What will be the weight of 2 dozen spoons, each weighing 1 oz., 5 pwt., 12 grs.? What will they cost, at 7 cents per pwt.?

## DIVISION OF COMPOUND NUMBERS.

### WRITTEN PROBLEMS.

***136.*** Divide 12 lbs. 3 pwt. 11 gr. by 8.

PROCESS.

| | lbs. | oz. | pwt. | grs. |
|---|---|---|---|---|
| 8 ) | 12 | 0 | 3 | 11 |
| | 1 | 6 | 0 | 10$\frac{3}{8}$ |

ANALYSIS. — (1.) For convenience, arrange the example as in simple numbers.

(2.) 1 eighth of 12 lbs. is 1 lb. with a remainder of 4 lbs. Write the 1 lb. in the quotient, and reduce the 4 lbs to ounces. 4 lbs = 48 oz.

(3.) 1 eighth of 48 oz. is 6 oz. Write the 6 oz. in the quotient.

(4.) Eight is not contained in 3 pwt. Write a cipher in the quotient, and reduce the pennyweights to grains. 3 pwts. = 72 grs., which, added to 11 grs. = 83 grs.

(5.) 1 eighth of 83 grains is 10$\frac{3}{8}$ grains, which write in the quotient.

201. Divide 35 lbs. 8 oz. 4 pwts. by 8. By 5.

202. A teamster drew 450 bu. 16 lbs. wheat at 15 loads, how much did he average per load?

203. A half-dozen spoons weighed 10 oz. 3 pwt., what was the weight of each?

204. 472 gal. 3 qts. ÷ 7 = what?

---

**REVIEW.** — Give the number of pounds in a barrel of flour. (128.) Beef. Pork. Fish. Salt. Give the number of pounds in a bushel of oats. Corn. Rye. Wheat. Beans. Peas. Barley.

205. 341 A. 24 p. ÷ 4 = what?

206. 243 mi. 43 rds. ÷ 19 = what?

207. 57 bu. 9 qts. ÷ 4 = what?

208. £35 3 s. 4 d. ÷ 14 = what?

209. How many spoons weighing 8 pwts. can be made from 11 oz. of silver?

[illegible]GGESTION. — Reduce both divisor and dividend to the lowest [illegible]ination mentioned in either, then divide as in simple numbers.

210. A man feeds his horse five quarts of oats a day, how long will 13 bu. 3 pks. last him?

211. How many bottles, each containing 1 qt. 1 pt., can be filled from 36 gallons of vinegar?

212. How many packages, each containing 3 oz., can be filled from 136 pounds of pepper?

## DENOMINATE FRACTIONS.

### 137. *Reduction Descending.*

WRITTEN PROBLEMS.

(*a.*) Reduce $\frac{4}{7}$ of a mile to inches.

(*b.*) Reduce $\frac{4}{7}$ of a mile to integral numbers of lower denominations.

(*a.*) PROCESS.

$$\overset{\text{m.}}{\tfrac{4}{7}} \times 320 \times \tfrac{11}{2} \times 3 \times 12 = \overset{\text{in.}}{\tfrac{253440}{7}} = \overset{\text{in.}}{36205\tfrac{5}{7}}$$

(*b.*) PROCESS (1).

| | m. | rds. | yds. | ft. | in. |
|---|---|---|---|---|---|
| 7) | 4 | 0 | 0 | 0 | 0 |
| | | 182 | 4 | 2 | $1\frac{5}{7}$ |

SUGGESTION. — Since $\frac{4}{7}$ of one mile = $\frac{1}{7}$ of 4 miles, the result may be found by the process used in division of denominate numbers.

REVIEW. — What is time? (129.) How is the year divided? Which are the winter months? Which the spring? Which the summer? Which the autumn? Give the number of days in each. Write the table.

(*b.*) PROCESS. (2.)

1. 320 rds. × $\frac{4}{7}$ (mi.) = 182 ($\frac{6}{7}$) rds.
2. $\frac{11}{2}$ yds. × $\frac{6}{7}$ (rds.) = 4 ($\frac{5}{7}$) yds.
3. 3 feet × $\frac{5}{7}$ (yd.) = 2 ($\frac{1}{7}$) feet.
4. 12 in. × $\frac{1}{7}$ (ft.) = 1 ($\frac{5}{7}$) in.

ANS. — 182 rds. 4 yds. 2 ft. 1$\frac{5}{7}$ in.

(*b.*) **ANAL. STEPS.** — (1.) *Change the fraction of a mile to rods.*
(2.) *Change the fraction of a rod to yards.*
(3.) *Change the fraction of a yard to feet.*
(4.) *Change the fraction of a foot to inches.*

(*b.*) **ANALYSIS** — (1.) Since in 1 mile there are 320 rods, in $\frac{4}{7}$ of a mile there are $\frac{4}{7}$ of 320 rods; or, 182$\frac{6}{7}$ rods.

(2.) Since in 1 rod there are $\frac{11}{2}$ yards, in $\frac{6}{7}$ of a rod there are $\frac{6}{7}$ of $\frac{11}{2}$ yards; or, 4$\frac{5}{7}$ yards.

(3.) Since in 1 yard there are 3 feet, in $\frac{5}{7}$ of a yard there are $\frac{5}{7}$ of 3 feet; or, 2$\frac{1}{7}$ feet.

(4.) Since in 1 foot there are 12 in., in $\frac{1}{7}$ of a foot there are $\frac{1}{7}$ of 12 in.; or, 1$\frac{5}{7}$ inches.

Reduce .5714 of a mile to integers of lower denominations.

PROCESS.

```
   .5714 mi.
     320 (rds.)
  ------
  114280
 17142
 -------
 182|.8480 rds.
       5½ (yds.)
 -------
    424
   4240
  ------
    4|.664 yds.
        3 (ft.)
  ------
    1|.992 ft.
       12 (in.)
  ------
   11 904 in.
```

ANS. — 182 rds. 4 yds. 1 ft. 11.904 in.

---

**REVIEW.**— *What is meant by leap year?* (128.) *What is meant by July 4, 1876? What is meant by the Centennial?*

4. In 1 fourth of a unit, how many ten
many hundredths? How many thou
many millionths?

FORMULA. 10 tenths ÷ 4 = .2¼.
100 hundreds ÷ 4 = .25.

ANALYSIS. — Since in 1 unit there are 10 tenths,
unit there are 1 fourth of 10 tenths, or 2½ tenths.

5. Divide .8 by 4. .14 by 7. 1.4 by 7.

6. What is 1 fifth of 2.5? Of 35? Of .35?

7. What is 1 eighth of 24? Of 4.8?
.72?

8. What is 1 twelfth of 72? Of 7.2?
Of $9\frac{6}{10}$?

**108.** PRINCIPLE. — *Any order divided by
order gives a quotient of units.*

Decimals, like common fractions, *must* be changed
nations, before one fraction can be divided by another
tient must be units, or parts of a unit: hence the place
point can be determined by inspection, by reducing
dividend to like denominations, and placing an accent
over the place of the decimal point in the quotient,
if necessary; thus, —

ILLUSTRATIONS.

| (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|
| 4.7)4.7′ | .007)2.000′ | 4)6.08 | .2).0′8 |
| 1 | 285.71+ | 1.52 | .4 |

---

REVIEW. — What is meant by a decimal fraction?
decimal? (102.) How are decimals written? (103.) In how
mals be written? How does the notation of United States
decimals? (104.) What is the first place at the right of
The third? The fourth? The first at the left of the un
The third at the right? The third at the left? The fou
*left? How are* decimals added? (105.) How is subtractio
*(106.)*

Since in 3 feet there is 1 yard, in $\frac{1}{4}$ of a foot there are as many feet are contained times in $\frac{1}{4}$ of a foot; or, $\frac{1}{12}$ of a foot.

ince in $\frac{11}{2}$ yards ($5\frac{1}{2}$ yards) there is 1 rod, in $\frac{1}{12}$ of a yard e as many rods as $\frac{11}{2}$ yards are contained times in $\frac{1}{12}$ of a , $\frac{1}{66}$ of a rod.

What part of a rod is 4 yds. 2 ft. 5 in.?

Reduce 4 yds. 2 ft. 5 in. to decimal of a rod.

CONTRACTED PROCESSES.

| (*a.*) | (*b.*) | |
|---|---|---|
| (in.) | 12 in.)5.000 | (in.) |
| (ft.) | 3 ft.)2.416+ | (ft.) |
| (yd.) | $\frac{11}{2}$yd.)4.805+ | (yd.) |
| rd.) | .873+ | (rd.) |

NOTE.—Since the analytical steps and the analysis are essentially the same as in the previous processes, they are not repeated.

duce 2 ft. 10 in. to the fraction of a yard. decimal of a yard.

Change 7 rds. 2 yds. 1 ft. to the fraction of a To the decimal of a mile.

What part of a gallon is 3 qts. 2 pts. 3 gi.? 3 qts. 2 pts. 3 gi. to the decimal of a gallon.

hat is the cost of 2 pks. 2 qts. of apples, at per bushel?

TION.—2 pks. 2 qts. = $\frac{9}{16}$ bush. \$1.60 × $\frac{9}{16}$ (bush.) = ?
2 qts. = .5625 bush. \$1.60 × .5625 bush. = ?

What will 3 qts. 1 pt. 2 gills of vinegar cost, at per gallon?

Change .3 in. to the decimal of a rod. $\frac{7}{8}$ yd. the fraction of a rod.

hange 7 yds. 2 ft. 8 in. to the decimal of a he fraction of a rod.

---

is meant by currency? What is English money? (132.) ve the coins and the value of each. What is the value of rency? How does the Canadian currency differ from U. S. ny units make a dozen? (131.) Recite the entire table.

**Note.** — It will be observed that the analysis of the reduction of denominate fractions is the same as that of denominate numbers. When the multiplier is a common fraction or a decimal, the word "of" is used instead of "times."

213. Change $\frac{3}{5}$ of a mile to integers of lower denominations. In the same way, change .3 of a mile.

214. Reduce $\frac{4}{11}$ of a rod to integers of lower denominations. In the same way, reduce .17 of a rod.

215. How many inches in .5 of a yard? In 2.7 of a foot? In $2\frac{3}{5}$ of a rod?

216. What is the sum of $\frac{5}{9}$ mile and $\frac{3}{7}$ rod?

217. What is the sum of .785 ton and 3.65 cwt.

218. Find the amount of $\frac{3}{7}$ lb. + $2\frac{3}{5}$ oz. + $\frac{3}{7}$ pwt.; of .375 mi. + 3.625 rd. — .275 yds.

219. What is the cost of $3\frac{2}{3}$ lbs. of gold, at $16 per oz.?

220. Bought 37.85 lbs. of gold-dust, at $0.80 per pwt.: what did it cost? What will $\frac{3}{11}$ lb. cost?

### *138. Reduction Ascending.*

(*a.*) Reduce 3 inches to the fraction of a rod. (*b.*) In the same way, reduce 3 inches to the decimal of a rod.

| (*a.*) PROCESS. | (*b.*) PROCESS. |
|---|---|
| (1.) 3 in. $\div$ 12 in. $= \frac{1}{4}$ (ft.) | (1.) 3 in. $\div$ 12 in. $= .25$ (ft.) |
| (2.) $\frac{1}{4}$ ft. $\div$ 3 ft. $= \frac{1}{12}$ (yd.) | (2.) .25 ft. $\div$ 3 ft. $= .083+$ (yd.) |
| (3.) $\frac{1}{12}$ yd. $\div \frac{11}{2}$ yd. $= \frac{1}{66}$ (rd.) | (3.) .083 yd. $\div \frac{11}{2}$ yd. $= .0150+$ (rd.) |

**Anal. Steps.** — (1.) *Reduce inches to the fraction of a foot.*
(2.) *Reduce fraction of a foot to fraction of a yard.*
(3.) *Reduce fraction of a yard to fraction of a rod.*

**Analysis** (Process *a*). — (1.) Since in 12 inches there is 1 foot, in 3 inches there are as many feet as 12 inches are contained times in 3 inches; or, $\frac{1}{4}$ of a foot.

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**Review.** — What is a circle? (130.) Where is the center of a circle? What is meant by the circumference of a circle? By an arc? By a quadrant? By a diameter? Write the table. What is the use of angular measure?

(2.) Since in 3 feet there is 1 yard, in $\frac{1}{4}$ of a foot there are as many yards as 3 feet are contained times in $\frac{1}{4}$ of a foot; or, $\frac{1}{12}$ of a foot.

(3.) Since in $\frac{11}{2}$ yards ($5\frac{1}{2}$ yards) there is 1 rod, in $\frac{1}{12}$ of a yard there are as many rods as $\frac{11}{2}$ yards are contained times in $\frac{1}{12}$ of a yard; or, $\frac{1}{66}$ of a rod.

(*a.*) What part of a rod is 4 yds. 2 ft. 5 in.?

(*b.*) Reduce 4 yds. 2 ft. 5 in. to decimal of a rod.

CONTRACTED PROCESSES.

| (*a.*) | | (*b.*) | |
|---|---|---|---|
| 12 in.)5 | (in.) | 12 in.)5.000 | (in.) |
| 3 ft.)$2\frac{5}{12}$ | (ft.) | 3 ft.)2.416+ | (ft.) |
| $\frac{11}{2}$yd.)$4\frac{29}{36}$ | (yd.) | $\frac{11}{2}$yd.)4.805+ | (yd.) |
| $\frac{173}{198}$ | (rd.) | .873+ | (rd.) |

NOTE.—Since the analytical steps and the analysis are essentially the same as in the previous processes, they are not repeated.

221. Reduce 2 ft. 10 in. to the fraction of a yard. To the decimal of a yard.

222. Change 7 rds. 2 yds. 1 ft. to the fraction of a mile. To the decimal of a mile.

223. What part of a gallon is 3 qts. 2 pts. 3 gi.? Reduce 3 qts. 2 pts. 3 gi. to the decimal of a gallon.

224. What is the cost of 2 pks. 2 qts. of apples, at $1.60 per bushel?

SUGGESTION.—2 pks. 2 qts. = $\frac{9}{16}$ bush. $1.60 × $\frac{9}{16}$ (bush.) = ? or, 2 pks. 2 qts. = .5625 bush. $1.60 × .5625 bush. = ?

225. What will 3 qts. 1 pt. 2 gills of vinegar cost, at 33 cts. per gallon?

226. Change .3 in. to the decimal of a rod. $\frac{7}{8}$ yd. 4 ft. to the fraction of a rod.

227. Change 7 yds. 2 ft. 8 in. to the decimal of a rod, to the fraction of a rod.

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REVIEW.—What is meant by currency? What is English money? (132.) Recite the table. Give the coins and the value of each. What is the value of a franc in U. S. currency? How does the Canadian currency differ from U. S. currency? How many units make a dozen? (131.) Recite the entire table.

228. What is the cost of 7 yds. 2 ft. 8 in. of picket fence, at $24.20 per rod?

229. What is the value of 3 gals. 3 qts. ½ pt. of domestic wine, at $3.75 per gallon?

## DRILL CARD EXERCISE.—TABLE VI.

### DENOMINATE FRACTIONS.

Exercise I.—Reduce **Z** mi. to its value in lower denominations.

II.—Reduce **Z** sq. mi. to integers of lower denominations.

III.—What is the value of **Z** cords?

IV.—Place a decimal point before each number in **W**, and, calling it the decimal of a mile, change it to integers of lower denominations.

V.—In the same manner, make each number in **W** the decimal of a square mile, and give its value in lower denominations.

VI.—In the same way, change **W** to decimals of a ton, and reduce to ounces.

VII.—What is the value of **Z** cwt. avoirdupois?

VIII.—What is the value of **Z** months?

IX.—Reduce **V** feet to the fraction of a rod.

X.—Reduce **W** square yards to a fraction of an acre.

XI.—Reduce **W** cubic feet to the decimal of a cord.

XII.—Change **W** pints to gallons, and decimals of a gal.

XIII.—Change **W** quarts to bushels.

XIV.—Change **W** pennyweights to pounds.

XV.—Change **W** oz. avoirdupois to hundred-weights.

XVI.—Change **W** days to the fraction of a year.

---

REVIEW.—Write the analysis of a short example in addition of denominate numbers. (133.) Write the analysis of a short example in the subtraction of denominate numbers. (134.) Write the analysis of a short example in the multiplication of denominate numbers. (135.) Write the analysis of a short example in the division of denominate numbers. (136.)

## REVIEW. — BUSINESS PROBLEMS.

### WRITTEN EXERCISES.

**230.** What must I give for 26 acres of land, at £5 2 s. 6 d. per acre? How many dollars U. S. currency will that amount to?

PROCESS.

12)6.00
20)2.50
£5.125
26 (acres)
£133.250
$4.866 × 133.25 = $648.39

or,

PROCESS.

(1.) £5 2*s.* 6*d.* = £5$\frac{1}{8}$
(2.) £5$\frac{1}{8}$ × 26 = £133$\frac{1}{4}$
(3.) $4.866 × 133$\frac{1}{4}$ = $648.39

ANAL. STEPS. — (1.) *Change 2s. 6d. to the decimal of a pound.*
(2.) *Find cost of 26 acres.*
(3.) *Find the value in U. S. currency.*

**231.** What is the cost of 48 bushels of wheat, at 8s. 5 d. per bushel? What is the cost in U. S. currency?

**232.** Bought 148 bushels of wheat at 48 d. per bushel: what is the cost in U. S. currency?

**233.** Bought 978 pounds of pork at 4 d. per pound: what is the cost in U. S. currency?

**234.** What is the value of a silver vessel weighing 1 lb. 7 oz. 3 pwt., at $9 per lb.?

**235.** What is the value of a gold chain weighing 1 oz. 3 pwt., at $18.37 per oz.?

**236.** What will 6 silver spoons cost, at 75 cents per oz., if each spoon weighs 12 pwt. 11 gr.?

**237.** How many silver spoons can be made from 4 oz. of silver, and how much silver will remain, if each spoon weighs 8 pwt. 7 gr.?

**238.** What will 5$\frac{3}{4}$ tons of coal cost, at $5.37 per ton?

**239.** What will 3 cwt. 39 lbs. of coal cost, at $7.35 per ton? *At $9.57 per ton?*

*3 cwt. 39 lbs. = .1695 ton.*

240. What must I give for 87 lbs. 6 oz. flour, at $3 per cwt.?

241. What must be given for 16 lbs. 8 oz. butter, at 37½ cents per lb.? At 45 cents per lb.?

242. If 7 pounds of sugar are worth $1.20, what is 1 lb. worth? What are 5⅜ pounds worth? What are 9 lbs. 9 oz. worth?

243. What is the value of 15½ bbls. flour, at $2.75 per cwt.?

244. What is the value of 15 lbs. pork, at $2.55 per cwt.?

245. How many doses in 7℥ 6ʒ 2℈ of tartar-emetic, allowing 20 gr. for each dose?

246. How many pills can be made from 9℥ 4ʒ 2℈ 8 gr. of calomel, allowing 12 gr. for each pill?

247. At $1.75 per yard, what cost 4 and 3 quarter yards of cloth? 3⅝ yards? $2\frac{7}{16}$ yards?

248. What will it cost to lay 3 miles 120 rds. of railroad track, at $1500 per mile?

249. How many yards of carpeting 1 yard wide will it take to cover the floor of a room 30 feet long and 18 feet wide? 50 feet long and 24 feet wide?

250. How many square feet in the ceiling and walls of a room 38 feet by 67, and 11 feet high?

251. What will it cost to paper the walls of a room 36 feet by 24, and 11 feet high, at $1.37½ per square yard?

252. What will it cost to paint the walls of a room 36 feet by 54, and 10 feet high, at 87½ cents per square yard?

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REVIEW — What is a composite number? (70.) What is a prime number? (71.) What is meant by a factor or divisor? (72.) Find the prime factors of 24. (73) What is the greatest common divisor of two or more numbers? (74.) What is a common multiple of two or more numbers? (75.) What is the least common multiple of two or more numbers? (76.) Illustrate the method of cancellation. (77.) Give the principles upon which it is based. (78.)

253. How many square feet in a board 9 feet long and 1 foot wide? How much will it cost, at 2 cents per square foot?

254. What will it cost to slate a roof 44 feet by 63, at $3.80 per square of 100 square feet each.

PROCESS.

44 ft. × 63 ft. = 2772 sq. ft.
2772 sq. ft. ÷ 100 = 27.72 squares
$3.80 × 27.72 = $105.34

ANAL. STEPS. — (1.) *Find number of square feet.*
(2.) *Find number of squares of 100 square feet.*
(3.) *Find cost of 27.72 squares.*

255. What will it cost to have a tin roof put on a house, each slope being 60 feet by 37, at $2.27 per square of 100 feet?

256. What will it cost to pave a walk 300 feet long, 8 feet wide, at $3 per 100 square feet?

257. What will it cost to pave a street 80 rods long, and 79 feet between curb-stones, at $7.25 per 100 square feet?

258. How much will it cost to plaster the walls and ceiling of a room 17 feet long, 13 feet wide, and 9 feet high, at $7.11 per 100 square feet?

259. 1 cubic yard is considered a load: how many loads of earth must be removed to dig a cellar 40 feet long, 18 feet wide, and 9 feet deep?

260. What will it cost to dig a cellar 27 by 60 feet, 10 feet deep, at $.75 per load?

261. How many cubic feet in a bin $8\frac{1}{4}$ feet long, 3 feet wide, and 4 feet deep?

---

**REVIEW.** — What is meant by a fraction? (79.) What does the denominator show? (80.) What does the numerator show? (81.) What is a proper fraction? (82.) An improper fraction? (83.) A mixed number? (84.) Name the principles affecting the terms of a fraction. (85.) How does multiplying the terms of a fraction by the same number affect the value? (86.) How does dividing the terms by the same number affect the value? (88.)

262. Making no allowance for thickness of boards or corners, how many square feet of boards will it take to make a box and cover 15 feet long, 8 feet wide, and 4 feet high? What will they cost, at $2.50 per hundred?

263. Making no allowance for corners, how many perches of stone are required in the walls of a cellar 37 feet long, 18 feet wide, 6 feet high, and 3 feet thick?

264. What will 24 gallons 1 quart of domestic wine cost, at $2.75 per gallon?

265. What will 25 gallons 3 quarts of molasses cost, at 60 cents per gallon?

266. What should I give for a pint of domestic wine, at $2 per gallon?

267. Bought 14 barrels of cider, holding 40 gallons each, at 18 cents a gallon, to be paid for in potatoes, at $1.80 per bushel: how many bushels must be given in payment?

268. How many bushels are contained in a bin 4 feet long, 3 feet wide, and 2 feet deep?

269. How much sugar at 12 cents per pound must be given for 359 pounds of pork at 7 cents per pound?

270. What is .32 of a ton of iron worth, at 2 cents a pound?

271. At 11 cents a pound, what is the value of ⅗ of a hundred-weight of rice? Of .024 of a cwt. of sugar?

272. How many loads of corn, 22 bushels each, can

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**Review.** — When is a fraction said to be at its lowest terms? (87.) Reduce 2¾ to an improper fraction, and write an analysis. (89.) Reduce 305 eighths to a mixed number, and write an analysis. (90.) When are fractions said to have a common denominator? (92.) Upon what principle does the addition of fractions depend? (93.) Add ⅔ and ¾, and write the analysis. Find the difference of ⅔ and ¾, and write the analysis. (94.) Give the rule for multiplying a fraction by an *integer. (95.)* For multiplying an integer by a fraction. (96.) For multiplying a *fraction by a fraction.* (97.)

be taken from a bin 8 feet wide, 6 feet deep, and 9 feet long?

273. A farmer has 1000 bushels of potatoes: how many barrels are required to hold them, if each barrel holds 2 bushels, 3 pks?

274. Bought 200 gallons of molasses, paid $0.45 per gallon, and sold at the rate of 3 gallons for $2: what was the total gain?

275. How many rods of fence will enclose a farm $\frac{1}{3}$ of a mile square?

## ANALYSIS OF SECTION X.

### I. Compound Denominate Numbers.

1. ILLUSTRATION.
2. DEFINITIONS.
3. REDUCTION.
   - (*a.*) *Descending.*
     - Analysis.
   - (*b.*) *Ascending.*
     - Analysis.
4. SPACE.
   - (*a.*) *Length.*
     - Long Measure.
     - Surveyors' Measure.
     - Analytical Steps.
   - (*b.*) *Area.*
     - Square Measure.
     - Surveyors' Square Measure.
     - Analytical Steps.
   - (*c.*) *Volume.*
     - Cubic Measure.
     - Liquid Measure.
     - Dry Measure.
     - Apothecaries' Fluid Meas.
     - Analytical Steps.
5. WEIGHT.
   - (*a.*) *Apothecaries' Weight.*
   - (*b.*) *Troy Weight.*
   - (*c.*) *Avoirdupois Weight.*
     - *Analytical Steps.*
6. DURATION.
   - *Time.*
     - Analytical Steps.
7. GEOMETRICAL MEASURE.
   - *Circular Measure*, or
   - *Angular Measure.*
     - Definitions.
     - Analytical Steps.
8. CURRENCY.
   - (*a.*) *U. S. Currency.*
   - (*b.*) *English Currency.*
   - (*c.*) *French Currency.*
     - Analytical Steps.
9. ADDITION DENOMINATE NUMBERS.
   - *Analytical Steps.*
10. SUBTRACTION DENOMINATE NUMBERS
    - *Analytical Steps.*
11. MULTIPLICATION DENOM. NUMBERS.
    - *Analytical Steps.*
12. DIVISION DENOMINATE NUMBERS.
    - *Analytical Steps.*
13. PROCESSES.
    - *Analysis.*
14. APPLICATION.
    - *Analysis.*

## II. Denominate Fractions.

1. COMMON.
2. DECIMAL.
3. REDUCTION.
   (*a.*) *Descending.*
      Analytical Steps.
   (*b.*) *Ascending.*
      Analytical Steps.
4. PROCESSES.
      *Analysis.*
5. APPLICATIONS.
      *Analysis.*

## TEST QUESTIONS FOR EXAMINATION.

**113.** What is a denominate number?

**114.** What is the reduction of a denominate number? How is the value affected by reduction? Give an illustration.

**115.** What is meant by reduction descending? How is the value of the number affected? How is the number of units affected? Give an illustration.

**116.** What is meant by reduction ascending? How is the value of the number affected? How is the number of units affected? Which of the fundamental operations are used in reduction descending? Which in reduction ascending?

**117.** What is meant by space? Give an illustration. What is meant by a solid? Give an example of a solid. Illustrate the meaning of surface. Of a line. Is the space of a room a solid? With what is it filled? Give examples of surfaces. Can a solid exist without a surface? What is a line? What dimension has it? Give examples of a line. Is a spider's web a line? ANS. — *It is not: it is a solid.* Why? Can a line exist without a surface? Can a surface be found that is not bounded by a line?

**118.** What are the measures of length? What is an inch? Make a mark an inch in length on the blackboard, or on paper. How many inches in length make a foot? Recite or write the table of long measure. Give the denominations. Give the analytical steps for reducing miles to inches. Inches to miles. Recite or write the table of surveyors' measure. For what is it used? What is the length of a surveyor's chain? How are surveyors' measurements recorded? How long is a link of a surveyor's chain? How long is a link of an engineer's chain? How do engineers record their measurements? How is cloth measured? Which is the unit? How many halves make a yard? Quarters? Eighths? Sixteenths? What is the difference between a statute mile and a geographical or nautical mile? Which is the longer? What is meant by saying that a ship sails ten knots an hour?

**119.** For what is square measure used? What is a surface? What is a square? What is a parallelogram? What is a square foot? Make a square inch on paper. For what is surveyors' square measure used? What are the dimensions of a square foot? Of a square rod? Of a square chain? Of an acre? Of a section? What are the dimensions of a square yard? How many square feet does it contain? *How many times* is the square foot repeated along one side? How many times *are the rows* repeated? (See illustration.)

**120.** What is the common rule for finding the area? What work is estimated by the square foot? What by the square yard? By the 100 square feet?

**121.** What is a cube? What are the dimensions of a cubic foot? Of a cubic yard? How many cubic feet in a cubic yard? Show how cubic feet are arranged to make a cubic yard. How many cubic feet in each row? How many rows in each tier? How many tiers make up the cube? Give the table of cubic measure. For what is it used?

**122.** What is the rule for finding the solid contents of a body? What is the weight of a cubic yard of earth? What is the weight of a cubic foot of water? What are the dimensions of a cord of wood? What is meant by a cord foot? What is a perch of masonry? What are the usual dimensions of a brick? What is meant by board measure? How is sawed lumber sold?

**123.** For what is liquid measure used? Recite the table. In calculating the capacity of cisterns, how much is a hogshead estimated to hold?

**124.** For what is dry measure used? Recite the table. How many cubic inches in a gallon? In a bushel? In a liter? How can gallons be reduced to bushels? Bushels to gallons? Bushels to liters? Liters to gallons? What are the dimensions of a bushel? When is even measure used? When is heaped measure used? How many gills liquid measure make a quart dry measure? How can cubic feet be changed to gallons? To bushels? Bushels to cubic feet? Gallons to cubic feet?

**125.** For what are apothecaries' weight and fluid measure used? How are drugs and medicines bought and sold? Recite the tables.

**126.** For what is Troy weight used? Recite the table. What is the weight of a carat? What is meant by saying gold is twenty carats fine?

**127.** For what is avoirdupois weight used? Recite the table. What is the number of grains in a pound Troy? In a pound apothecary? In a pound avoirdupois? How can Troy pounds be reduced to avoirdupois pounds? How can avoirdupois pounds be reduced to pounds apothecary? How can bushels of corn be reduced to pounds? Bushels of oats to pounds? Pounds of clover-seed to bushels?

**129.** What is time? Write the table. Write the table of the divisions of the year. Give the abbreviations for the months. Give the number of days in each. When has February 29 days? What is the exact length of a year? What is the cause of leap year? How often does it occur? Why? The usual business month contains how many days? What is meant by July 4, 1876, 9 o'clock, A.M.?

**130.** What is a circle? What is the center of a circle? What the circumference? An arc? A quadrant? A diameter? A radius? What is the difference between a diameter and a radius? Which radius of a circle is the longest? Which diameter is the least? How is the angle measured? What is the measure of a right angle? What is a degree? A minute? Are the degrees of one circle equal to those of another circle? How can degrees of different circles be used to measure the same angle? Write the table. What is the greatest circumference of the earth? One degree of longitude is equal to how many minutes of the apparent motion of the sun? Give the rule for finding the difference of time between places, the difference of longitude being known.

**130.** What apparent daily motion has the sun? What causes this apparent motion? How often does the earth rotate? In what direction does the earth turn? *In what direction does the sun seem to move?* When is it noon at any place? Does *noon come to all places on the* earth at the same instant? How does the time noon differ in two places? How far does the sun seem to move in 24 hours? In 1 hour? One degree of motion requires how much time? One minute of motion? Of two places, which has the faster time? Which has the faster time, — Chicago, or New York? Chicago, or San Francisco? New Orleans, or Cincinnati? London, or New York?

**132.** What is money? What is meant by currency? What is used by savages as money? By civilized nations? What is a coin? What is paper money? What kinds of paper money are now used in the United States? What are United States legal tender notes? National bank notes? What is the currency of Great Britain? Recite the table. Name the gold coins. Silver coins. The bronze coins. What is the unit of the French currency? What is its value? How does the Canadian currency differ from Federal money?

**133, 134.** How are denominate numbers written for addition and subtraction?

**135.** What is the difference between the multiplication of denominate numbers and simple numbers?

**136.** Write the analysis of an example in the division of denominate numbers.

**137.** Write the analysis of an example reducing a common fraction of a higher denomination to integers of lower denominations. In the same manner, reduce a decimal fraction.

**138.** Write the analysis of an example reducing a denominate number of several terms to a common fraction of a higher denomination. To a decimal of a higher denomination.

Dry-Goods' Commission House. Salesroom.

# SECOND PART.

## PERCENTAGE.

1. One hundredth of a unit, or number of units, is said to be one *per cent* of it: what part of a unit is one *per cent* of it? *Ans.* .01. What is 3 per cent of a unit? 5 per cent? 17 per cent? 43 per cent?

*Per cent* is shortened from the Latin words *per centum*, "by the hundred."

2. What is 8 per cent of a unit? 75 per cent of a unit? 83 per cent?

3 *How many* gallons in 3 per cent of 12 gallons?

**Analysis.** — Since 3 per cent of 1 gallon is 3 hundredths of a gallon, 3 per cent of 12 gallons is 12 times 3 hundredths, or 36 hundredths of a gallon.

***1. Rate per Cent*** shows how many ***hundredths*** of a unit or number are taken.

The rate per cent forms the numerator of a fraction, of which the denominator is 100: thus, 5 per cent equals $\frac{5}{100}$; 57 per cent equals $\frac{57}{100}$; 247 per cent $= \frac{247}{100}$, &c. The words *per cent* are represented by the sign %: thus, 4 per cent is written 4 %.

***2. The Base*** is the number on which the *percentage* is estimated.

***3. The Percentage*** is that portion of the base which is taken.

The term **Percentage** is also applied to all computations in which rate per cent is employed as a factor, including profit and loss, commission, brokerage, stocks, insurance, &c.

***4. The Amount*** is the sum of the *base* and *percentage.*

***5. The Difference*** is the difference between the *base* and *percentage.*

Note. — In the statement, $8 is 4 % of $200, $200 is the *base,* $8 is the *percentage,* and 4 is the *rate per cent.* The sum of $200 and $8 is the *amount,* and $200 minus $8 is the *difference.*

Any given per cent of a unit equals that per cent written as an equivalent decimal expression.

The *percentage* is always of the same kind or denomination as the base. *The rate is always abstract.*

***6. Signs.*** The initial letters of the terms often stand for the terms themselves; thus,

B = Base. R = Rate per cent.
P = Percentage. A = Amount.
D = Difference.

### EQUIVALENT EXPRESSIONS.

| Rate per Cent. | Decimal Form. | Fractional Form. | Express in the same manner the following rates per cent: — | | |
|---|---|---|---|---|---|
| 1 % = $\frac{1}{100}$ or | .01 | = $\frac{1}{100}$ | 6 % | $\frac{1}{2}$ % | $3\frac{1}{2}$ % |
| 2 % = $\frac{2}{100}$ or | .02 | = $\frac{1}{50}$ | 12 % | $\frac{3}{4}$ % | $5\frac{3}{4}$ % |
| 5 % = $\frac{5}{100}$ or | .05 | = $\frac{1}{20}$ | 24 % | $\frac{2}{3}$ % | $12\frac{2}{3}$ % |
| 25 % = $\frac{25}{100}$ or | .25 | = $\frac{1}{4}$ | 25 % | $\frac{1}{8}$ % | $10\frac{1}{8}$ % |
| 100 % = $\frac{100}{100}$ or | 1.00 | = 1 | 36 % | $\frac{5}{8}$ % | $41\frac{5}{8}$ % |
| 125 % = $\frac{125}{100}$ or | 1.25 | = $1\frac{1}{4}$ | 50 % | $\frac{7}{8}$ % | $75\frac{7}{8}$ % |
| $\frac{1}{2}$ % = $\frac{\frac{1}{2}}{100}$ or | .005 | = $\frac{1}{200}$ | 112 % | $\frac{3}{8}$ % | $121\frac{8}{9}$ % |
| $\frac{3}{4}$ % = $\frac{\frac{3}{4}}{100}$ or | .0075 | = $\frac{3}{400}$ | 125 % | $\frac{3}{5}$ % | $100\frac{3}{4}$ % |
| $12\frac{1}{2}$ % = $\frac{12\frac{1}{2}}{100}$ or | .125 | = $\frac{1}{8}$ | 150 % | $\frac{4}{5}$ % | $37\frac{1}{100}$ % |

### ORAL EXAMPLES.

1. How many hundredths of a number is 3 % of it? 7 % of it? 23 %? 75 %?

2. How many hundredths of a number is 5 % of it? $2\frac{1}{2}$ % of it? $\frac{1}{2}$ of 1 % of it? $6\frac{3}{4}$ %?

3. What fractional part of a number is 50 % of it? 25 %? $33\frac{1}{3}$ %? $12\frac{1}{2}$ %? $16\frac{2}{3}$ %?

4. What fractional part of a number is 10 % of it? 20 %? 30 %? 40 %? 50 %? 60 %?

5. What fractional part of a number is 70 % of it? 80 %? 90 %?

6. What fractional part of a number is $6\frac{1}{4}$ % of it? $12\frac{1}{2}$ %? $37\frac{1}{2}$ %? $62\frac{1}{2}$ %? $87\frac{1}{2}$ %?

## FELTER'S

# ARITHMETICAL DRILL CARDS.

### TABLE VII.

### PERCENTAGE.

| No. Exam. | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | 4 | 21 | $\frac{1}{2}$ | .5 | 41.03 | 243 |
| 2 | 3 | 95 | $\frac{2}{3}$ | .8 | 8.651 | 867 |
| 3 | 7 | 75 | $\frac{3}{4}$ | .21 | 95.17 | 341 |
| 4 | 6 | 90 | $\frac{3}{5}$ | .07 | 2.014 | 214 |
| 5 | 2 | 35 | $\frac{3}{7}$ | .11 | 91.34 | 807 |
| 6 | 1 | 70 | $\frac{3}{10}$ | .3 | 75.83 | 609 |
| 7 | 17 | 24 | $\frac{4}{5}$ | .05 | 2.167 | 413 |
| 8 | 10 | 40 | $\frac{4}{7}$ | .4 | 853.4 | 210 |
| 9 | 9 | 45 | $\frac{4}{11}$ | .9 | 71.24 | 702 |
| 10 | 12 | 33 | $\frac{5}{6}$ | .08 | 10.03 | 305 |
| 11 | 8 | 60 | $\frac{6}{7}$ | $.4\frac{3}{4}$ | 5.001 | 634 |
| 12 | 5 | 25 | $\frac{7}{11}$ | $.6\frac{4}{5}$ | 60.41 | 591 |
| 13 | 16 | 58 | $\frac{5}{8}$ | $.8\frac{3}{8}$ | 9.503 | 321 |
| 14 | 11 | 36 | $\frac{8}{9}$ | $.7\frac{2}{5}$ | 86.47 | 768 |
| 15 | 13 | 27 | $\frac{9}{11}$ | $.01\frac{1}{4}$ | 39.24 | 341 |
| 16 | 20 | 78 | $\frac{7}{8}$ | $.\frac{3}{8}$ | 1.057 | 243 |
| 17 | 18 | 50 | $\frac{8}{11}$ | $.\frac{7}{5}$ | 63.91 | 847 |
| 18 | 15 | 55 | $\frac{9}{10}$ | $.0\frac{2}{5}$ | 5.370 | 645 |
| 19 | 14 | 100 | $\frac{10}{11}$ | $.1\frac{1}{5}$ | 6.830 | 110 |
| 20 | 19 | 30 | $\frac{11}{12}$ | $.\frac{3}{8}$ | 40.10 | 701 |

N.B. — For answers to the problems on the above Drill Card, see *Key.*

### DRILL CARD EXERCISES. — TABLE VII.

Exercise I. — Change **A** % to equivalent decimal expressions.
II. — Change **B** % to equivalent decimal expressions.
III. — Change **D** % to equivalent decimal expressions.
IV. — Change **D** to equivalent expressions having the sign of %.
V. — Change **A** % to equivalent common fractions.
VI. — Change **B** % to equivalent common fractions.
VII. — Change **D** % to equivalent common fractions.
VIII. — Change **C** % to equivalent decimal expressions.
IX. — Change **C** % to equivalent decimal expressions having the sign of per cent.

ILLUSTRATION.

| Exer. I. | Exer. II. | Exer. III. | Exer. IV. |
|---|---|---|---|
| Exam. **A** %. | Exam. **B** %. | Exam. **D** %. | Exam. **D**. |
| **1.** 4 % = .04 | **1.** 21 % = .21 | **1.** .5 % = .005 | **1.** .5 = 50 % |
| **2.** 3 % = .03 | **2.** 95 % = .95 | **2.** .8 % = .008 | **2.** .8 = 80 % |
| **3.** 7 % = .07 | **3.** 75 % = .75 | **3.** .21 % = .0021 | **3.** .21 = 21 % |
| **4.** 6 % = .06 | **4.** 90 % = .9 | **4.** .07 % = .0007 | **4.** .07 = 7 % |
| &c. | &c. | &c. | &c. |

**7.** ***Profit and Loss*** is a term often applied in percentage to those questions which relate to business transactions in which money is either gained or lost.

1. The **Cost** of a thing is the money value given for it, or expended in producing it, and is the *base* in percentage.

2. The **Selling Price** of any thing is the money value asked or received for it; and is the *amount* in percentage, when more than the cost, and the *difference*, when less than the cost.

The price of the seller becomes the cost of the buyer.

3. **Profit** or **Gain** is the *percentage* of gain on the cost price, or *base*.

4. **Loss** is the percentage of discount or loss, when the article is sold for less than its value.

The gain or loss is the difference in value between the selling price and cost price.

*Net Profit* is the amount gained after deducting all expenses connected with the transaction.

EQUIVALENTS.

*Cost Price* $= B.$

*Selling Price* $\begin{cases} = A, \text{ when more than cost.} \\ = D, \text{ when less than cost.} \end{cases}$

*Gain or Loss* $= P.$

*Rate per Cent* $= R.$

**8. *Commission*** is an allowance made to an agent, factor, or broker, for transacting business for another, and is reckoned at a certain rate per cent on the business done.

**The base of commission is the sum paid for a purchase, or received for a sale on commission.**

1. A **Factor,** or **Commission Merchant,** is one who is intrusted with the property, to dispose of for the owner.

2. A **Broker** acts as a medium between buyer and seller, usually without having any property in his own possession.

3. The term **Agent** includes both factor and broker. When a distinction is made, it means one who acts for another under definite instructions.

A *consignment* is an invoice of goods sent to be sold on commission.
The *consignor* is the sender.
The *consignee* is the person to whom the goods are sent.

4. **Net Proceeds** is the sum left from a sale or collection, after deducting the commission and charges for expenses.

5. An **Account Sales** is the written statement of the commission merchant, or factor, showing the amount of sales, the price for which sold, the expenses, and the net proceeds.

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REVIEW. — Define rate per cent. (1.) Base. (2.) Percentage. (3.) Amount. (4.) Difference. (5.) Give the letters representing base, &c. (6.) What is the cost of a thing? (7.) The selling-price? When is the selling-price the amount? When the difference? What is profit or gain? Loss? Net profit? Give the equivalents of cost price. Selling-price. Gain or loss. Rate per cent. What is commission? (8.) What is a factor? A broker? An agent? A consignment? Who is the consignor? Consignee? Define net proceeds. What is an account sales?

## PROPOSITION I.

***9.*** **THE BASE AND RATE GIVEN, TO FIND THE PERCENTAGE.**

**PROBLEM.** — What is 4 % of 200?

**FORMULA.** (*a.*) $200 \times .04 = 8$

**FORMULA.** (*b.*) $200 \times \frac{4}{100} = 8$

**ANALYSIS.** — (*a.*) Since 4 % of 1 (unit of the base) = 4 hundredths, 4 % of 200 (the whole base) = 200 times 4 hundredths, or 800 hundredths = 8.

**ANALYSIS.** — (*b.*) Since 4 % $= \frac{4}{100}$, 4 % of 200 $= \frac{4}{100}$ (4 times $\frac{1}{100}$) of 200, or 8.

**RULE.** — *Multiply the base by the rate per cent (written as a decimal).*

**FORMULA.** $B \times R = P$.

### ORAL EXERCISES.

1. What is 10 % of 100? 6 %? 4½ %? ½ %?
2. What is 6% of 150? 4 %? 8%? 12 %?
3. What is 5 % of 180? Of 60 sheep? Of 120 pounds? Of 600 acres?
4. If I earn $600, and spend 30 %, how much do I spend? If I spend 20 %? 4½ %? 50 %?
5. If a man make a profit of 8 % on a yard of cloth which cost him $6, what is his gain? 20 %? 9 %?
6. If a dealer make 25 % profit on the sale of a horse which cost him $240, what is his gain?

### DRILL CARD EXERCISES. — TABLE VII.

Exer. X. — Find **A** % of **B**.
XI. — Find **A** % of **F**.
XII. — Find **B** % of **F**.
XIII. — Find **A** % of **C**.
Exer. XIV. — Find **A** % of **D**.
XV. — Find **B** % of **A**.
XVI. — Find **B** % of **E**.
XVII. — Find (**B** + **C**) % of **F**.
Exer. XVIII. — *How much is* **B** % of **F** gallons, plus **F** gallons?
*XIX.* — *How much is* **E** acres minus **A** % of **E** acres?

### ORAL AND WRITTEN EXERCISES.

1. What is 8 % of 3? *W.* 13 % of 21? 27 % of 69?

2. What is 3 % of 12 pounds? *W.* 24 % of 35 bushels?

**PROCESS.**

35 (bu.)*
.24 bu.
1.40
7.0
8.40 bu.

**ANALYSIS.** — (*a, concrete.*) Since 24 % of 1 bushel = .24 of a bushel, 24 % of 35 bushels = 35 times .24 of a bushel, or 8.40 bushels; or,

(*b.*) Since 24 % = 24 hundredths, 24 % of 35 bushels = .24 of 35 bushels, or 8.40 bushels.

3 What is 7 % of 8 bushels, and 3 % of 7 bushels? *W.* Of 23 % of 49 gallons, and 75 % of 83 gallons?

#### ANALYTICAL STEPS.

1. .07 bu. × 8 (bu.)* = .56 bu. = 7 % of 8 bushels.
2. .03 bu. × 7 (bu.) = .21 bu. = 3 % of 7 bushels.
3. .56 bu. + .21 (bu.) = .77 bu. = the required sum.

Arrange the following problems by steps as above.

4. What is 9 % of 12 cords, and 11 % of 9 cords? *W.* Of 36 % of 247 gallons, and 27 % of 345 gallons?

5. I bought 9 % of 8 bushels of apples, and sold 3 % of 7 bushels: how much had I left? *W.* I bought 25 % of 123 gallons of cider, and sold 17 % of 150 gallons: how many gallons had I left?

6. If I earn $25 in a week, and spend 1 % of it, how much do I spend? *W.* If I spend 26 % of it?

7. *W.* Mr. Smith sold $930 worth of corn, and received 8 % of the amount for his trouble: how much did he receive? How much was left after paying him?

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* The numbers the denominations of which are enclosed in a parenthesis should in every case be used in the analysis as abstract.

8. I own a watch assessed at \$100, on which I pay an annual tax of 2 %: how much tax do I pay on the watch? *W.* At the same rate, how much must be paid on a house and lot, the assessed value of which is \$3430?

### WRITTEN EXERCISES.

9. I bought a cow for \$29, and sold her the next day at a profit of 17 %: how much did I make?

**Analysis.**—(*a.*) Since on a cost of \$1 I gained \$.17, on a cost of \$29 I gained 29 times \$.17, or \$4.93.

(*b.*) Since 17 % = $\frac{17}{100}$, 17 % of \$29 = $\frac{17}{100}$ of \$29, or, \$4.93.

10. I paid \$133 for a horse, but was obliged to sell him at a loss of 19 %: how much did I lose?

11. Bought a yoke of oxen for \$100, and after keeping them 3 weeks, at an expense of \$2.50 per week, I sold them at a profit of 6 % of their total cost: how much did I make by the transaction?

12. A merchant deducted 6 % from a bill of \$363.50: what was the amount deducted?

13. Lost 8 % on the sale of goods costing \$2180: how much did I lose?

14. An agent expended \$1300 in purchasing cattle. He charged 3 %: what was his bill?

15. What will it cost to collect \$4250, at 2 % commission?

16. 5 % was deducted from the rent of a house. The rent was \$526.50: what was the deduction?

When the mills in the answer equal five or more, add one cent; when less than five, reject them.

17. Bought 96 tons of coal, at \$5.62½ per ton, and sold it at a gain of 24 %: what was the profit on the sale?

18. What is $2\frac{3}{4}$ % of 25? $5\frac{6}{7}$ % of 134?

PROCESS.

$$\begin{array}{r} 134 \\ .05\frac{6}{7} \\ \hline 7)8.04 \\ \hline 1.14\frac{6}{7} \\ 6.70 \\ \hline 7.84\frac{6}{7} \text{ Ans.} \end{array}$$

$(134 \times \frac{6}{7} = 114\frac{6}{7}.)$

**FORMULA.** $B \times R = P$.

**ANALYSIS.** — Since $5\frac{6}{7}$ % of $1 = .05\frac{6}{7}$, $5\frac{6}{7}$ % of $134 = 134$ times $.05\frac{6}{7}$, or $7.84\frac{6}{7}$.

19. What is $5\frac{2}{3}$ % of 63 yards of cloth? $4\frac{1}{8}$ % of 86 gallons of wine?

20. What is $\frac{4}{5}$ % of 140 barrels of pork? $15\frac{1}{2}$ % of 400 apple-trees?

21. A real estate-agent sells a house and lot for $4000, and charges a commission of $1\frac{1}{2}$ %: how much does he receive?

22. I pay $1\frac{2}{5}$ % for insuring goods valued at $550: what is the cost of insurance?

23. I own a building valued at $6000: what will it cost me annually to have it insured at $\frac{3}{4}$ %?

24. The amount paid for insurance is called the "premium:" at $4\frac{1}{2}$ %, what is the amount of premium on a store valued at $35000?

25. At $1\frac{2}{3}$ % premium, what must be paid for insuring a case of silk valued at $1000?

26. Bought 250 pounds of beef at $8\frac{1}{2}$ cents per pound, and sold it at $76\frac{4}{5}$ % of its cost: what did I receive for it?

27. What is $66\frac{2}{3}$ % of 36? $12\frac{1}{2}$ % of 75?

*PROCESS.*

*(1.)* $66\frac{2}{3}\ \% = \frac{2}{3}$

*(2.)* $36 \times \frac{2}{3} = 24$

**ANALYSIS.** — (*a.*) Since $66\frac{2}{3}$ % of $1 = \frac{2}{3}$, $66\frac{2}{3}$ % of $36 = 36$ times $\frac{2}{3}$, or 24; or,

(*b.*) Since $66\frac{2}{3}\ \% = \frac{2}{3}$, $66\frac{2}{3}$ % of $36 = \frac{2}{3}$ of 36, or 24.

When the rate per cent is an aliquot or even part of 100, it is more convenient to reduce the rate to a common fraction.

TABLE OF ALIQUOT PARTS.

5 % = $\frac{1}{20}$.
6¼ % = $\frac{1}{16}$.
10 % = $\frac{1}{10}$.
12½ % = $\frac{1}{8}$.
16⅔ % = $\frac{1}{6}$.
20 % = $\frac{1}{5}$.
25 % = $\frac{1}{4}$.
33⅓ % = $\frac{1}{3}$.
37½ % = $\frac{3}{8}$.
50 % = $\frac{1}{2}$.
62½ % = $\frac{5}{8}$.
66⅔ % = $\frac{2}{3}$.
75 % = $\frac{3}{4}$.
80 % = $\frac{4}{5}$.
87½ % = $\frac{7}{8}$.

28. A farmer invested $375 in raising poultry, and lost 6¼ % of his investment: what was the amount of his loss?

29. Bought 375 yards of velvet at $3.54 per yard, and sold it at an advance of 16⅔ %: what was the gain? What was the amount received? What was the selling price per yard?

30. What will be the loss in buying a house for $3784, and selling it at 62½ % less than cost? What will be the selling price?

31. An auctioneer sold goods at a commission of 12½ %: what was his charge on a sale of $785.20?

32. What is 6¼ % of 2400 men? Of 2480 sheep?

33. What is 25 % of 14 gallons, 3 quarts, and 1 pint?

**Analysis.** — 3 qts. 1 pt. = .875 galls.; 14 galls. 3 qts. 1 pt. = 14.875 galls.; 25 % of 14.875 = 3.71875 galls.; .71875 galls. = 2 qts. 1¾ pts.: therefore 25 % of 14 galls. 3 qts. and 1 pt. = 3 galls. 2 qts. and 1¾ pts.

34. In a consignment of 7 cwt. 90 lbs. of sugar, 30 % was found worthless: what was the amount lost?

**Suggestion.** — As in the 33d example, reduce to the decimal of the highest denomination named. Find the percentage, and reduce *the decimals to equivalent* integers of lower denominations.

35. From a cask containing 64 gallons, 2 quarts, 8 % leaked out: how much was lost?

36. Sold 27 % of 3 tons, 7 cwt. of coal: how much remained? How much did I receive at \$7 per ton?

## PROPOSITION II.

***10.*** **THE BASE AND RATE GIVEN, TO FIND THE AMOUNT OR THE DIFFERENCE.**

PROBLEM.—Sold a cow which cost \$60 for 12 % more than I gave for her: what was the selling-price?

PROCESS (*a*).

(1.) $\$60 \times .12 = \$7.20.$

(2.) $\$60 + \$7.20 = \$67.20.$

FORMULA. $B + P = A.$

PROCESS (*b*).

$\$60 \times 1.12 = \$67.20.$

FORMULA. $B \times (1 + R) = A.$

ANAL. STEPS.—(*a*.) 1. *Find the percentage of gain as in Prop. I.*

2. *Find the amount.*

ANALYSIS.—(*b*.) The cost of the cow = the base of percentage = 100 %. Since the cow was sold for 12 % more than the cost, she was sold for 112 % of the cost. 112 % of \$60 = \$67.20.

PROBLEM.—A man having 250 sheep lost 20 % of them by disease: how many had he left?

PROCESS (*a*).

(1.) $250 \times .20 = 50.$

(2.) $250 - 50 = 200.$

PROCESS (*b*).

$250 \times (1 - .20) = 200.$

FORMULA. $B \times (1 - R) = D.$

ANAL. STEPS.—(*a*.) 1. *Find the percentage of loss as in Prop. I.*

2. *Find the difference.*

ANALYSIS.—(*b*.) Since he lost 20 % of the sheep, he must have had left 100 % — 20 %, or 80 %. $250 \times .80 = 200.$

Hence the following

RULE.—*Multiply the base by* 1 PLUS *the rate to find the amount, or by* 1 MINUS *the rate to find the difference.*

### ORAL EXERCISES.

1. What is 20 % more than 60? than 40? than 75?

2. What is 600 increased by 3 % of itself? by 5 %?

3. If I add 20 % of itself to 15, what will be the sum? to 40? to 65?

4. If 75 is decreased by 20 % of itself, what is the remainder?

5. If I buy cloth for $2, and sell it at a loss of 10 %, what is the selling-price?

6. At a loss of 10 % what is the selling-price of wheat costing $0.80 a bushel?

### DRILL CARD EXERCISES. — TABLE VII.

**Exercise XX.** — What will be the sum of **F** dollars, plus a gain of **A** %?

**XXI.** — At a loss of **A** %, what will be the selling-price of an article costing **F** dollars?

**XXII.** — What will be the amount of **E** dollars, at an advance of **A** + **C** %?

### ORAL AND WRITTEN EXERCISES.

37. If I spend 25 % of my money, what % of it will be left? If I add to my money 25 % of itself, what % of it shall I then have? *W.* If my money amounts to $2500, and I spend 25 % of it, how much will be left? If I add 25 % of my money to what I have, how much will there be?

38. A quantity of flour was sold at a loss of 26 %: what % of the cost is the selling-price? What if sold at a gain of 29 %? *W.* If the cost of the flour was $343, what is the selling-price at 26 % loss? At 29 % gain?

39. Henry gave 18 % of his oranges to his sister, and 33 % to his brother: what % did he give to both? What % had he left? *W.* If he had 300 oranges at first, how many had he left?

40. Mr. J. Willets gave 26 % of his estate to his son, 16 % to each of his two daughters, and the remainder to his wife: what % did his wife receive? *W.* His estate was valued at $26500: what was the amount received by each?

41. Bought 90 % of a flock of sheep; I sold at one time a number equal to 20 % of the entire flock, at another 14 % of the same: what % of the whole flock remained? *W.* The flock contained 1250 sheep: how many did I buy? How many did I sell? How many had I left?

42. In a school, 24 % of the pupils were in the first grade, 12 % in the second grade, 35 % in the third grade, 16 % in the fourth grade; the remainder were in the fifth grade: what % were in the fifth grade? *W.* The school contained 600 pupils: how many were in the fifth grade?

### WRITTEN EXERCISES.

43. A farmer bought a horse for $160, and sold him for 20 % more than he gave: what did he receive?

44. How shall I sell wheat costing $1.50 per bushel, that the gain may be 10 %?

45. A book costing $5, was sold 25 % below cost: what is the selling-price?

46. In a cargo of 2500 bbls. of flour, 6 % was spoiled by water: how many barrels were uninjured?

47. A collector, whose commission was $2\frac{1}{2}$ %, made collections amounting to \$3750: how much should he pay over to his employer?

48. What should be asked for butter costing 40 cents per lb., to gain 15 %?

$$\$.40 \times (1 + .15) = ?$$

49. If a coat costs \$6.40, and sells at 30 % profit, what is the selling-price? At $12\frac{1}{2}$ %? At 25 %?

50. A merchant investing \$3750 in a business, lost 37 %: how much had he left?

$$3750 \times (1 - .37) = ?$$

51. Sold 40 yds. silk, costing \$$4.37\frac{1}{2}$ per yd., at 20 % advance: what was the amount received? What was the gain?

52. A quantity of flour costing \$8.75 per bbl. was damaged, and sold at a loss of 12 %: what was the selling-price?

53. I sent to market 3500 baskets of berries, which were sold at 8 cts. per basket: what did I receive after paying a commission of $2\frac{1}{2}$ %?

54. A man owing \$6000 paid 35 % of his debt: how much did he still owe?

55. A house and lot cost \$6320: for how much must it be sold to gain $5\frac{3}{4}$ %?

56. A farm contained 840 acres. 40 % was planted in wheat, 20 % was meadow, and the rest was woodland: how many acres of woodland?

57. An agent sold goods amounting to \$37580, at a commission of 15 %: what amount should he return to his employer?

58. If it cost \$24 to make a sewing-machine, and it sells for 166⅔ % of its cost, what is the selling-price of the machine?

59. Bought a horse and carriage for \$465, and sold them at a loss of 8⅕ %: what did I receive for them?

60. The number of pupils in a certain school last year was 352. This year the school has increased 12½ %: how many pupils now in the school?

61. Bought 175 barrels of apples for \$350, and sold them so as to gain 35 %: what was the selling-price per barrel?

62. At 20 % profit, what must be the selling-price of each item in the following list of goods: —

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (1.) | Calico | cost | 10 cts. | per yd. | (7.) | Gents' hose | cost | 20 cts. | per pr |
| (2.) | Muslin | " | 12 " | " | (8.) | Gloves | " | 88 " | " |
| (3.) | Merino | " | \$1.20 | " | (9.) | Handk'ch'fs | " | 32 " | " |
| (4.) | Sheeting | " | 8 " | " | (10.) | Collars (paper) | | 18 " | box. |
| (5.) | Toweling | " | 35 " | " | (11.) | Collars (lin.) | | \$2.20 | doz. |
| (6.) | Silk | " | 3.75 | " | (12.) | Cravats | | 37½" | each. |

## PROPOSITION III.

**11.** — THE BASE AND PERCENTAGE GIVEN, TO FIND THE RATE.

NOTE. — As has been shown (¶ 1), the rate is the value in hundredths of any given part of a number. Hence, to find the rate, the percentage, or part of the number taken, placed as the numerator of a fraction of which the given number or base is the denominator, will be the rate % when the fraction is reduced to a decimal of two places.

Any number is $\frac{100}{100}$, or the whole of itself.

PROBLEM. — 6 is what % of 8?

| PROCESS. | ANALYSIS. |
|---|---|
| (*a.*) 8)6.00.<br>.75=75 %. | (*a.*) 6 is ¾ of 8; ¾ = .75, or 75 % of 8. |
| (*b.*) *1 % of 8=.08.*<br>*6÷.08=75.* | (*b.*) Since 1 % of 8 is .08, 6 is as many % of 8 as .08 is contained times in 6, or 75 %. |

Since the *percentage* is the product of the *base* and *rate*, it follows that the *percentage* divided by the *base* must equal the *rate:* hence the following

RULE. — *Divide the percentage by the base.*

FORMULA. $\frac{P}{B} = R.$

### ORAL EXERCISES.

1. 5 is what fractional part of 20? What decimal part? What %? 6 is what fractional part of 12? What decimal part? What %? 6 is what fractional part of 18? What decimal part? What %? 9 is what fractional part of 45? What decimal part? What %? 25 is what fractional part of 125? What decimal part? What %? 80 is what fractional part of 60? What decimal part? What %?

2. 5 is what % of 5? $3\frac{1}{2}$ is what % of $3\frac{1}{2}$? 1 is what % of 1?

3. 3 is what % of itself? $2\frac{1}{2}$ is what % of itself? $23\frac{4}{7}$ is what % of itself?

4. What % of 8 is 4? Is 2? Is 12?

5. 5 is what % of 15? 15 is what % of 5? 9 is what % of 12? 12 is what % of 9? Of 2? Of 24?

6. What % of 48 barrels are 12 barrels?

7. 25 gallons are what % of 125 gallons? Of 150 gallons? Of 5 gallons? Of 75 gallons?

### DRILL CARD EXERCISES. — TABLE VII.

Exer. XXIII. — A is what % of B?
XXIV. — B is what % of A?

Exer. XXV. — A is what % of C?
XXVI. — C is what % of D?

### ORAL AND WRITTEN EXERCISES.

63. If I receive \$3 for the use of \$60, what % do I receive? *W.* If I receive \$8.40 for the use of \$140, what % do I receive?

PROCESS.

(*a.*) $\frac{3}{60} = \frac{1}{20}, = .05$, or 5 %.

(*b.*) \$.60)\$3.00.

5 %.

**ANALYSIS.** — (*a.*) If, for the use of \$60, I receive \$3, for the use of \$1 I receive $\frac{1}{60}$ of \$3; or, \$$\frac{3}{60}$ = \$.05 = 5 %.

(*b.*) Since 1 % of \$60 is 60 cents, \$3 will be as many per cent of \$60 as \$.60 is contained in \$3, or 5 %.

64. Bought 12 barrels of flour, of which 9 were damaged: what % of the flour was damaged? *W.* If, of 340 barrels, 85 were damaged, what % was damaged?

65. *W.* Bought 630 barrels of flour, of which 105 were worthless: what was the % of loss?

66. A man bought a horse for \$200, and sold it at an advance of \$50: what % did he gain? *W.* If he bought it for \$130, and gained \$6.50, what % did he gain?

67. A man bought a house for \$3200, and sold it at an advance of \$800: what % did he gain? *W.* What % did he gain, if he sold it at an advance of \$240?

68. A farmer raised 250 bushels of potatoes, and sold all but 75 bushels: how many did he sell? *W.* What % did he sell?

69. A tax of \$4 was paid on property valued at \$400: what was the rate of taxation? *W.* If the tax is \$25.65 on an assessed valuation of \$5130, what is the rate %?

70. 3⅓ is what part of 10? What %? *W.* 8¼ is what % of 198? 198 is what % of 8¼?

71. A man bought an Elgin watch for $100, and sold it for $150: what % did he make?

**Anal. Steps.** — (1.) *Find the gain:* $150 — $100 = ?
(2.) *Find the gain % :* 50 ÷ 100 = *what %* ?

72. A boy bought a pearl for 1 cent, and afterward sold it for 75 cents: what % did he make? *W.* He bought a diamond for $1, and sold it for $38.50: what was his gain %?

73. *W.* A boy bought 5 sleds at $2 each. He sold the first for $3, the second for $4, the third for $4.50, the fourth for $5.20, and the fifth for $5.80: what was the % of profit on each? What was his gain % on the whole?

74. A book that cost $2.00 sold for $1.50: what was the % of loss? *W.* A book that cost $2.64 was sold for $2.40: what was the rate % of loss?

75. 12½ cents is what % of 50 cents? *W.* Of $2? Of $23?

76. 5 lbs. is what % of 10 lbs? *W.* Of 36 lbs? Of 100 lbs. or 1 cwt.?

77. 2 ft. is what % of 3 ft., or 1 yard? Of 2 yards? *W.* Of 5½ yards?

## WRITTEN EXERCISES.

78. Bought a cow for $90, and sold her for $115: what was the gain %?

79. If tea is bought for 70 cents, and is sold for 75 cents, what is the gain %?

80. Bought sugar for 11½ cents, and sold it for 14 *cents: what was the* gain %?

81. An agent charged \$37.50 for selling goods amounting to \$1875: what was his % of commission?

82. A man received \$15 for collecting \$20: what % did he receive as commission? Had he collected \$82.50, what % would he have received?

83. An auctioneer charged \$$9\frac{1}{2}$ for selling goods worth \$475: what % did he receive as commission? What would have been his rate %, if he had received \$$30.87\frac{1}{2}$?

84. A lot of land costing \$900 was sold for \$855: what was the loss %?

85. A grocer bought a lot of butter at $28\frac{1}{2}$ cts.; but, finding it injured, he sold it for $21\frac{3}{8}$ cts.: what was the loss %?

86. A broker bought cotton at $16\frac{1}{2}$ cts. a lb. and sold it for $17\frac{1}{2}$ cents: what was the gain %?

87. What was the % of gain on each of the following articles, if bought and sold at the following prices?—

Bought trunk locks at 32 cents, and sold for 42 cents.
" chest locks at 44 cents, and sold for 48 cents.
" buck-saws at \$1.50, and sold for \$2.00.
" nails at $6\frac{1}{2}$ cents a lb. and sold for $9\frac{3}{4}$ cents.

88. Received \$17 for selling goods worth \$510: what was the rate of commission?

89. \$16 was spent in repairing a wagon which cost \$87. It was then sold for \$126.60: what was the rate % of profit?

90. 320 bushels of potatoes were sold at 85 cents a bushel: what was the rate % of gain, if they cost 50 cents a bushel?

91. Of 680 barrels of apples, 170 bbls. were found *worthless*: what was the rate % of loss?

92. Of a crop of 310 bushels of corn, 175 bushels were sold: what % was sold? What % remained unsold?

93. A farmer received 940 bushels of wheat; he sold 141 bushels at 90 cents per bushel, 329 bushels at $1, and the remainder at $1.10: what % was sold at each price? How much did he receive for the wheat?

94. A man having 18 gallons of wine sold 4 gallons, 2 qts.: what % of the wine did he sell?

**Suggestion.**—Since like numbers only can be compared, both base and percentage must be reduced to the same denominations.

**Anal. Steps.**—(1.) 4 *galls.* 2 *qts.* = 18 *qts.* = *P.*
(2.) 18 *galls.* = 72 *qts.* = *B.*
(3.) 18 ÷ 72 = .25 = 25 % = *R.*

95. 4 lbs. 8 oz. is what % of 36 lbs.?

96. In a cask containing 96$\frac{1}{8}$ lbs. of indigo, 7$\frac{1}{2}$ lbs. were damaged: what % of the indigo was injured?

97. A contractor agreed to construct a mile of railroad-track, but left 8 rods unfinished: what % of the track was not completed? He was to receive $2000 a mile: how much should be deducted from his bill?

98. If I buy eggs for 25 cts. a doz., and sell them for 50 cts. a doz., what is my rate % of gain? If I sell at 30 cts.? At 75 cts.?

99. If eggs are bought at 25 cents a dozen, and sold at 20 cts., what % is lost? If sold at 12$\frac{1}{2}$ cts.? At 6$\frac{1}{4}$ cents? If given away?

100. An army of 12000 men lost 160 men by desertion: what was the rate % of loss? What was the rate % remaining?

101. An army went into battle with a muster-roll of 7250 men. After the battle, there were only 4930: what was the loss %?

102. A real-estate agent sold a farm for $15750, and charged $393.75 for commission: what was the rate of brokerage?

## PROPOSITION IV.

***12.*** — THE RATE AND PERCENTAGE GIVEN TO FIND THE BASE.

### ORAL EXAMPLES.

1. 12 is 5 % of what number?

PROCESS (*a*).

$12 \div 5 \times 100 = 240.$

PROCESS (*b*).

$$\begin{array}{r} .05)12.00 \\ \hline 240 \end{array}$$

ANALYSIS. — (*a.*) Since 12 is 5 % of some number, 1 % of that number $= \frac{1}{5}$ of 12, and 100 %, or the number $= 100$ times $\frac{1}{5}$ of 12, or 240.

(*b.*) Since 12 is the product of some number multiplied by .05, that product (12) divided by .05 will give the number multiplied, or base required. $12 \div .05 = 240.$

(*c.*) Since every .05 of the percentage requires a base of one unit, the whole percentage, will require a base containing as many units as .05 are contained times in 12, or 240.

2. 16 is 10 % of what number? Is 20 % of what number? Is 25 % of what number?

3. 8 is 4 % of what number? 45 is 9 % of what number? 24 is $66\frac{2}{3}$ % of what number?

4. If I sell goods at 4 % commission, and receive $60, what amount have I sold?

PROCESS (*a*).

$60 ÷ $.04 = $1500.

PROCESS (*b*).

$60 ÷ 4 × 100 = 1500.

ANALYSIS. — (*a.*) 4 % commission = $.04 on each dollar's worth sold. Since I received $.04 on each dollar's worth sold, to receive $60 I must have sold as many dollars' worth as $.04 are contained times in $60, or $1500.

(*b.*) Since $60 is 4 % of the amount sold, 1 % is ¼ of $60, and 100 % is 100 times ¼ of $60, or $1500.

5. A man selling his horse at 2 % profit gained $12: what was the cost of the horse?

6. What did a horse cost, if, by selling at 20 % loss, he loses $80?

7. 36 sheep is 12 % of how many sheep?

Since the percentage is the *product* of the base and rate, it follows that the percentage divided by the rate must equal the base: hence the following

**RULE.** — *Divide the percentage by the rate per cent.*

FORMULA. $\frac{P}{R} = B$.

DRILL CARD EXERCISES. — TABLE VII.

Exercise XXVII. — **A** is **B** % of what number?
XXVIII. — **B** is **C** % of what number?
XXIX. — **C** is **D** % of what number?

ORAL AND WRITTEN EXAMPLES.

**103.** The annual rental of a house is $210, which is 7 % of its present value: what is its value? *W.* $210 is 12½ % of its value in 1870: what was its value at that time?

**104.** $60 is the amount received annually as interest on a sum of money loaned at 6 %: what is the amount loaned? *W.* $60 is the amount annually received on money loaned at 18 %: what is the amount loaned?

105. I paid $20 for selling five car-loads of potatoes, which sum was 2 % of the amount for which they were sold: for how much were they sold? *W.* What were the net receipts after paying the expense of selling? The potatoes cost $925: what was the net gain or loss? What was the % of gain or loss?

NOTE.—The expense of selling, as well as the amount paid for the potatoes, must be included in the cost.

ANAL. STEPS.—(1.) $20 ÷ .02 = $1000 = *amount received.*
(2.) $1000 — $20 = $980 = *net receipts.*
(3.) $980 — $925 = $55 = *net gain.*
(4.) $55 ÷ $945 = 5.8 + % = *gain %.*

106. *W.* Five bushels of apples is 5 % of the quantity sold to Mr. McAfee, 10 % of the quantity sold to Mr. Searle, 20 % of the quantity sold to Mr. Nellis, and 25 % of the quantity sold to Mr. Forbes: how many bushels were sold to each?

107. 24 is 6 % of what number? Is 8 % of what number? Is 1 % of what number? *W.* Is $\frac{1}{4}$ % of what number? Is $\frac{3}{8}$ % of what number?

108. 2.5 is 25 % of what number? Is 5 % of what number? *W.* Is $6\frac{1}{4}$ % of what number? Is $1\frac{1}{4}$ % of what number?

109. *W.* $42.30 is 42.3 % of what number? Is 21.15 % of what number? Is $33\frac{1}{3}$ % of what number?

110. $4.60 is 1 % of what number? *W.* Is $\frac{1}{2}$ % of what number? Is 23 % of what number? Is $2\frac{3}{10}$ % of what number?

### WRITTEN EXAMPLES.

111. A man sold a horse for 20 % above cost, and thereby gained \$33: what did the horse cost?

112. A factor, whose commission was 3½ %, received \$105 for the sale of a consignment: what was its value?

113. The rent of a house is \$812, which is 7 % of its value: what is the house worth?

114. By selling at 20 % advance, a merchant gained \$368.76: what was the cost of the goods sold?

115. 16⅔ % advance on a bill of goods gives the merchant a gain of \$686: what was the cost of the goods?

116. A grocer charging 8⅓ % advance on his butter gained \$.03¼ per lb.: what was the cost of the butter?

117. A bar of soap lost 3.2 oz. in drying, which was 5 % of its weight: what did the bar weigh at first?

118. A cask of vinegar lost by leakage 3 galls. 2 qts., which was 5⅚ % of its entire contents: how much did the cask contain at first?

119. The commission on the sale of a farm at 2½ % was \$86: what was the selling-price of the farm?

120. If a farmer sells his produce at a loss of 12 %, and loses \$68.40, what is its cost?

121. A man settling with his creditors at 62½ %, paid \$3784: what was his entire indebtedness?

## PROPOSITION V.

**13.** THE RATE AND THE AMOUNT OR DIFFERENCE GIVEN, TO FIND THE BASE.

PROBLEM.—What number increased by 25 % of itself equals 80?

PROCESS.

(a.) $80 \div 125 \times 100 = 64$.

(b.) $80 \div 1.25 = 64$.

FORMULA.

$$\frac{A}{1+R} = B.$$

ANALYSIS.—(a.) Since the required number is 100 % of itself, 80 is 100 % + 25 %, or 125 % of the number. If 80 is 125 % of the number, 1 % of the number is $\frac{1}{125}$ of 80, and 100 % is 100 times $\frac{1}{125}$ of 80, or 64.

(b.) 80 is the number increased by 25 % of itself, or $1 + .25 = 1.25$ times the number. If 80 is 1.25 times the number, then $80 \div 1.25 = 64$, the number required.

PROBLEM.—What number decreased by 25 % of itself equals 60?

PROCESS.

(a.) $60 \div 75 \times 100 = 80$.

(b.) $60 \div .75 = 80$.

FORMULA.

$$\frac{D}{1-R} = B.$$

ANALYSIS.—(a) Since the number is 100 % of itself, 60 is 100 % − 25 %, or 75 % of the number. If 60 is 75 % of the number, then 1 % of the number is $\frac{1}{75}$ of 60, and 100 % is 100 times $\frac{1}{75}$; or $\frac{100}{75}$ of $60 = 80$.

(b.) 60 is the number decreased by 25 % of itself, or $1 - .25 = .75$ times the number. If 60 is .75 times the number, then $60 \div 75 =$ the number, or 80.

Hence the

RULE.—*Divide the amount by* 1 PLUS *the rate, or the difference by* 1 MINUS *the rate.*

ORAL EXERCISES.

1. Nine is 50 % more than what number?

2. 160 is 20 % less than what number? 250 is 25 % more than what number?

3. A number diminished by 30 % of itself is 140: t is the number?

4. What number decreased by 10 % of itself equals 180?

5. A man received $90 for a carriage, which was 50 % more than its cost: what was the cost of the carriage?

PROCESS.
$90 ÷ $1.50 = 60.

FORMULA.

$$\frac{A}{1+R} = B.$$

ANALYSIS. — At a profit of 50 %, every $1 of the cost was sold for $1.50 ($1.00 + $.50). Since every $1.50 of the selling-price cost $1, the carriage must have cost as many dollars as $1.50 is contained times in $90, or $60.

6. If I sell a watch for $80, and lose 20 % of the cost, what did the watch cost me?

PROCESS.
$80 ÷ .80 = $100.

FORMULA.

$$\frac{D}{1-R} = B.$$

ANALYSIS. — At a loss of 20 %, every $1 of the cost was sold for 80 cents ($1.00 — $.20). Since every 80 cents of the selling-price cost $1, the watch must have cost as many dollars as 80 cents is contained times in $80, or $100.

### DRILL CARD EXERCISES. — TABLE VII.

Exercise **XXX.** — What number increased by **A** % of itself equals **B**?
**XXXI.** — What number increased by **B** % of itself equals **E**?
**XXXII.** — What number increased by (**B** + **C**) % of itself equals **F**?
**XXXIII.** — What number decreased by **A** % of itself equals **B**?
**XXXIV.** — What number decreased by **B** % of itself equals **E**?

### WRITTEN EXERCISES.

**122.** A boy sold a knife for 87½ cents, thereby gaining 25 %: what did the knife cost him?

**123.** If, by selling a horse at $375 I gain 20 %, what did the horse cost me?

**124.** My agent returned me $1287, retaining his commission of 2½ %: what was the amount of the sale?

**125.** What is the cost price of cloth on which I make 8⅓ % by *selling* it at $3.25½?

126. Sold a farm for $8420, which was 25 % more than it cost me: what was the cost?

127. The population of a certain city has increased 29 % in 10 years. It is now 29025: what was it 10 years ago?

128. Sold goods for my correspondent, deducted 2½ % for my services, and sent him $3851.25: what did I receive for the goods?

129. Bought coal at $7.50 per ton, which was 20 % less than the market price: what was the market-price?

130. If the price marked is 40 % advance upon the cost, what is the cost of the following articles?

| | | | | | |
|---|---|---|---|---|---|
| (1.) Vest patterns, marked | | $5.60. | (4.) Overcoats, marked | | $24.50. |
| (2.) Vests, | " | 7.00. | (5.) Umbrellas, | " | 4.20. |
| (3.) Broadcloth, per yd. | | 3.50. | (6.) Cravats, | " | 1.26. |

131. My flock of sheep has increased 250 %, and now numbers 525: what was the original number?

132. If I send $2499 to my agent, to be invested in cotton, after deducting his commission of 2 %, how much can he invest for me? To how much will his commission amount?

ANALYSIS. — Since, for every $1 expended, the agent is to receive $.02, he can purchase as many dollars' worth of cotton as $1.02 ($1 + $.02) is contained times in $2499, or $2450; and the agent receives as many dollars as the difference of $2499 and $2450, or $49.

NOTE. — The agent should receive commission only on the amount actually expended, and not on the amount sent.

133. I send cotton to the amount of $2500 to my agent for sale. How much should he send me after deducting his commission of 2 %? To how much did *his commission* amount?

**Analysis.**—Since, for every dollar's worth of cotton sold, the agent receives $.02, for selling $2500 he receives 2500 times $.02, or $50; and he should send me the difference of $2500 and $50, or $2450.

**Note.**—In this transaction the agent should receive commission on the entire amount of cotton sold.

134. How much flour, at $8 per barrel, can my agent purchase with my draft for $3250, after deducting his commission of 4 %?

135. A broker received $3936 to invest in stocks, after deducting his commission of 2½ %: what amount should he invest?

136. A Boston merchant sent to his factor in Chicago $1624.98 to invest in wheat, after deducting his commission of 4½%: how much did he invest? How many bushels did he buy, at $1.25 per bushel?

137. A merchant sold the following articles at the prices named, which was an advance of 33⅓ %: what was the cost of each article?

(1.) 1 dress coat, $27.50.
(2.) 1 sack coat, 17.38.
(3.) 2 pairs pants, at 8.30.
(4.) 2 vests, at 5.00.
(5.) ½ doz. cravats, at 50 cts.
(6.) ⅓ " boxes collars, at 40 cts.
(7.) 1 " boxes cuffs, at 40 cts.
(8.) 2 " shirts, at $3.50.

138. What was the cost of the whole? What did he receive for the whole? What was his profit on the whole?

139. 150 is 100 % more than what number? Is 50 % more than what number? Is 25 % more? Is 1 % more? Is ¼ % more? Is ⅜ % more?

140. Sold a farm for $3450, which was 3 % more than it cost: what did it cost?

141. Sold a horse for $350, which was 16⅔ % less than its value: what was its value?

142. I sold my horse for $380, which was 95 % of its cost: what was the cost?

143. A boy sold a knife for 46 cents, which was 92 % of its cost: what was its cost?

144. A man sold 45 bushels of potatoes, which was 75 % of what he raised: how many did he raise?

145. A clerk spent $38, which was 3.2 % of his salary: what was his salary?

146. 2 % of $5 is 8 % of how many dollars? 2 % of $50 is 75 % of how many dollars?

147. A clerk spent 20 % of his monthly salary for board, 40 % for clothing, 7 % for books, and had $15.18 left: what was his salary?

148. 64 of a crew deserted, 8 died, 64 % of the entire crew remained: what was the full number?

149. A man who owned 70 % of a vessel sold 25 % of his share for $3500: what was the value of the vessel?

150. A merchant sold goods to the amount of $42600, at a profit of 20 %: what did they cost him?

151. A boy sold chestnuts at 5 cents a gill, and made a profit of 150 %: what did they cost per peck?

152. A market-woman sold peas at 13 cents a quart, and by so doing made a profit of 225 %: what did they cost a bushel?

153. A man sold sugar for 14 cents per pound, and lost 12½ %: what did it cost him?

154. Sold a cow for $42, which was 12½ % less than her value: what was the cow worth?

155. My income this year is $950, and is 5 % less than it was last year: what was my income last year?

156. A gentleman sold his library of 350 volumes for $700, at a loss of 20 %: what was the average cost of each book?

### FORMULAS.

1. To find the percentage, base and rate being given: $B \times R = P.$

2. To find the amount or difference, base and rate being given: $\begin{cases} B \times (1 + R) = A. \\ B \times (1 - R) = D. \end{cases}$

3. To find the rate per cent, base and percentage being given: $\frac{P}{B} = R.$

4. To find the base, rate and percentage being given: $\frac{P}{R} = B.$

5. To find the base, the rate and amount, or the rate and difference, being given: $\begin{cases} \frac{A}{1 + R} = B. \\ \frac{D}{1 - R} = B. \end{cases}$

### MISCELLANEOUS PROBLEMS.

157. A broker sold a farm for $15000, and charged $300: what was the rate of commission?

158. An army went into battle numbering 7200 men. After the battle, there were only 4500: what was the loss %?

159. A man bought all the resin in market at $2.25 per bbl.; he then advanced the price to $5 per barrel: what was the gain %?

160. A tailor bought a piece of cloth containing 55 yards; it shrunk 1 yard: give the rate % of shrinkage.

161. A broker negotiated a loan of $1500000 for a railroad company, and received $5625 as his brokerage: what % did he receive?

162. An imported book cost $12, and 20 % for duties: what was the entire cost?

163. The retail price of a book is $5: I bought it for 20 % less: what did I pay?

164. In a ship's cargo of 6500 bushels of wheat, $8\frac{1}{2}$ % was spoiled by leakage: how many bushels were saved?

165. My agent sells a house for $2575, and charges 4 % commission: what do I receive for the house after the agent deducts his commission?

166. What shall I ask for cloth costing $3.40, so as to gain 25 %?

167. Bought railway shares at $120: for how much must they be sold so as to gain 8¾ %?

168. If hats cost $5.50, for how much must I sell them to gain 22 %?

169. If it costs $15 to make a sewing-machine, and it sells for 280 % of its cost, what is the selling-price of the machine?

170. An army of 12000 men lost 160 men by desertion: what % deserted? What % remained?

171. A man having an income of $6580 spent 73 %: what % did he save?

172. Sold 60 barrels of flour, costing $5.87½ per bbl., at 20 % advance: what was the amount received? What was the amount of gain?

173. Cloth marked at $5.75 per yard, being damaged, was sold at a loss of 20 %: what was the selling-price? What was the loss per yard?

174. My broker sells for me 280 bushels of wheat, at $1.12½ per bushel: what should I receive, if he charges 3½ % commission?

175. A merchant failed, owing $5200. He paid 35 cents on the dollar: how much does he still owe?

176. A travelling agent sold goods amounting to $25675 in one year, at a commission of 15 %: what amount should he return to the principal?

177. Bought a horse for $275, and a carriage for $230. The horse I sold at a gain of 5 %, the carriage at a loss

of 5 %: what did I get for each? Did I make, or lose, by the transaction, and how much?

178. Bought 375 barrels of apples for $750, and sold them so as to gain 28 %: what was the selling-price per bbl.?

179. A merchant sold goods to the amount of $231.20, and then deducted 5 % for cash payment: what was the amount of cash received?

180. A house which was insured to the amount of $\frac{3}{4}$ of its value was burned. Its value was $2400: what % did its owner lose? How many dollars?

181. I bought 15 pieces of silk, each containing 15 yds., for $270, and sold them at a gain of $33\frac{1}{3}$ %: what was the selling-price per yard? What was the total profit?

182. Bought 25 bbls. apples. On opening them, 8 % proved worthless: how many were good?

183. A man owed $8341.25. He paid his creditors 20 %: what % was unpaid? What amount did he pay?

184. A man having $4320, spent 30 % of it for a farm, 20 % of it in building a house, and $12\frac{1}{2}$ % of it in fences and improvements: how much money had he remaining? What % of his money did he spend? What % had he left?

185. Two men engaged in the lumber trade, each with a capital of $3000. At the end of one year, one had made $28\frac{3}{4}$ % of his capital, and the other had made $33\frac{3}{8}$ % of his capital: how much did one gain more than the other? What % did one gain more than the other? What was the average % of gain of both?

186. A father gave his two boys, Albert and Arthur, $5 each. At the end of a month, Albert had lost 7 % of his money, and Arthur had gained 7 % of his money:

**what amount did each then have? How much more did Arthur have than Albert?**

187. I bought goods valued at $3458, at a discount of 18 % from valuation, after which I obtained an additional discount of 5 % on my bill for cash payment: how much cash must I pay?

188. At 25 % profit, what must be the selling-price of each item in the following list of goods: —

| | | | | |
|---|---|---|---|---|
| (1.) | Calico | cost | 10 cts. | per yd. |
| (2.) | Muslin | " | 12 " | " |
| (3.) | Merino | " | $1.20 | " |
| (4.) | Sheeting | " | 8 " | " |
| (5.) | Toweling | " | 35 " | " |
| (6.) | Silk | " | 1.75 | " |
| (7.) | Gents' hose | cost | 20 cts. | p. pr. |
| (8.) | Gloves | " | 88 " | " |
| (9.) | Handk'ch'fs | " | 32 " | " |
| (10.) | Collars (paper) | | 18 " | box. |
| (11.) | Collars (lin.) | | $2.20 | doz. |
| (12.) | Cravats | | 37½ " | each. |

189. A boy received a recitation-mark of 4 on Monday, of 3 on Tuesday, of 3 on Wednesday, of 5 on Thursday, and of zero on Friday: what was his average for the week of five days?

PROCESS.

(1.) $4+3+3+5+0=15$ total marks.
(2.) $15\div5$ (No. days) $=3$ average.

190. On Monday there were 33 pupils in attendance at school; on Tuesday, 23; on Wednesday, 39; on Thursday, 35; and on Friday, 30: what was the average daily attendance for the week?

191. The average standing of a pupil for the week is 3: what is the per cent, if marked on a scale of 5?

192. The total number of marks received by Laura in geography for the month (20 days) is 72. She was excused two days for absence: what is her average? What is her per cent for the month, if marked on the scale of 5? If marked on the scale of 10?

193. Charles has attended 55 days during a term of 3 months (60 days): what is the per cent of his attendance?

194. In a school of 40 pupils, the average daily attendance was 35: what was the per cent of attendance?

195. A man bought 3 barrels of sugar, at $25 per barrel. He sold it so as to gain 25 % on cost: how much did he gain?

196. A grocer gained 13 % on a box of tea that cost him $39, and 33 % on a bag of coffee that cost him $12: how much did he gain on both?

197. I had $830 in my purse. I gave Henry 7 %, John 9 %, and Mary 11 % of it: what % did I give away? How many dollars did I give to each? How many dollars did I have left?

198. I lost 3 % on a barrel of sugar costing $43, and gained 8 % on a barrel costing $33: did I make, or lose, on the transaction, and how much? What %?

199. 12½ % of 360 ft. is 2½ % of how many feet?

200. A man sold 25 % of 800 bushels of potatoes. The remainder was 20 % of his entire crop: what was his crop?

201. ¾ of 64 is 5 % of what number?

202. .75 of 64 is ½ % of what number?

203 75 % of 64 is .05 of what number?

204. 3 times 25 is 3 % of what number?

205. .03 of 25 is 3 times what number?

206. 8 % of 25 is .03 of what number?

207. The amount is 1500, the rate is 25 %: what is the base? The base is 625, the rate 4½ %: what is the amount? The percentage? The difference?

208. *The difference* is 742, the rate is 30 %: what is

the base? The base is 840, the difference is 714: what is the rate? What is the percentage?

### REVIEW EXAMPLES.

[The following problems are intended for advanced classes who wish to review by a short and comprehensive course in analysis. They may be omitted (at the teacher's option) by regular classes, until a final review of the subject.]

209. A farm contained 363 A. 126 rods. 20 % is woodland, 10 % is pasture, and the rest cultivated: how much land is there of each kind?

210. A merchant bought 475 bales of cotton for $71250. He sold 40 % of it at $210 per bale, 20 % of it at $180 per bale, and the remainder at cost: how much did he gain?

211. If a cask containing 36 gallons loses 5 gallons $1\frac{3}{8}$ qts. by leakage, what is the % loss?

212. A broker sells a farm for $9750, and returns to the owner $9311.25: what is the rate of brokerage?

213. If a coal mine is sold for $\frac{7}{8}$ of what it is worth, what is the gain % of the buyer?

214. A grocer sold tea for $112\frac{1}{2}$ % of its cost, and gained 25 cts. per lb: what was the cost?

215. My factor receives $7389.90 to invest in wheat, at $3\frac{1}{2}$ % commission: how many bushels can he buy, at $1.50 per bushel?

216. If I send a broker $2583, how many barrels of flour, at $12 a barrel, can he purchase, his commission being 5 %?

217. A merchant sent his Southern agent a draft for $17630 to invest in cotton, at $16\frac{1}{2}$ cts. per lb., after deducting his commission of $2\frac{1}{2}$ %: how much cotton *did the broker* purchase?

218. A commission merchant was allowed 10 % on sales, he paying all expenses. On an invoice which sold for $8500, his expenses were $37.50 for storage, $52.50 for cartage, and $80 for advertising: what was his net commission?

219. My factor, whose commission is 5 %, sends me $5250.27: what was the amount of his sales?

220. A farmer harvested 2600 bu. of wheat. He sold 25 % at $1.12 per bu., 30 % at $1.15, 15 % at $1.25, and the remainder at $1: what did he receive for his wheat?

221. A dry-goods merchant bought 150 yds. of flannel for $60: by accident 45 yds. were destroyed. He sold the remainder at $0.52 per yd: what % did he lose?

222. A man who owned 70 % of a vessel sold 40 % of his share for $3500: what was the value of his share? What was the value of the vessel?

223. How much must I remit my factor, commission included, that he may purchase for me 3500 yds. muslin, at 8½ cts. per yd., his commission being 2½ %?

224. My correspondent makes sales on my account amounting to $12350: how much does he place to my credit if his commission is 5⅝ %?

225. A man bought a house and lot for $4500. Paid $375.50 for repairs, $360 for painting, and 2 % on first cost for taxes: for what sum must he sell it to gain 15 % on his whole investment?

226. A man bought 95 yds. of silk for $190. 15 yds. were unsalable; and he disposed of the rest at 15 % advance on first cost per yard: how much did he lose?

227. How much shall I remit my broker, that he may purchase 5000 barrels of flour at $8.50 per barrel, allowing him 3 % commission, and $315 for freight?

228. How many barrels can the broker buy on my order, with the same commission and allowance for freight, if I send him a draft for $7143?

229. If in selling cloth at $8.40 per yard, the gain is 40 %; what will be the gain if I sell it at $7.20? What will be the gain %?

230. I sold my potatoes at $0.87$\frac{1}{10}$ per bushel, and lost 4$\frac{2}{7}$ %: what ought I to have received to realize a profit of 14$\frac{2}{7}$ %?

231. If I sell a mowing-machine for $88, and gain 10 %, would I gain, or lose, by selling at $75, and what %?

232. A man bought a horse for $320, which was 20 % less than his real value, and sold him for 30 % more than his real value: how much did he receive for the horse?

233. If I mark cloth costing $0.44 per yard at an advance of 25 %, what will it cost a customer whom I allow a discount of 20 % from the marked price?

234. If I ask 10 % advance on cost for my coal, and sell it at 10 % reduction from asking-price, do I gain, or lose, and what %?

235. If I fall 20 % from my asking-price, which is 30 % advance on cost, what % will I gain?

236. If I fall 10 % from my asking-price, which is 30 % advance on cost, what % will I gain?

237. What % of the value of a case of goods shall be my asking-price, that I may fall 10 %, and still gain 17 %?

238. A merchant sold two coats for $25 each: on one he gained 20 %, on the other he lost 20 %: did he gain, or lose, and how much?

239. A commission merchant sold 2800 yds. of cloth at $1.20, paid [illegible] for insurance, $39.80 for freight,

and charged 2½ % commission for the sale: what amount did he deduct from the receipts?

240. An agent tries to collect a bill of $375; he finally consents to settle for 90 %: what is his commission at 5½ %?

241. A merchant shipping 42390 bush. grain lost ⅜ of the cargo by the wreck of a vessel: what % shall he direct his agent to advance on the cost-price of the remainder, that he may lose nothing?

242. Sold goods damaged by fire at 40 % below marked price, and lost 10 % of the cost-price. My loss was $1200: what was the marked value of my goods? What was the cost-price?

243. Bought 35 bales of wool at $36 per bale; found 5 bales so damaged that I sold them at 15 % discount. How must I sell the rest per bale to make 20 % on my purchase?

244. The retail price of an encyclopædia was $5.50 per volume. The bookseller bought at 40 % discount, and sold at 40 % profit: what was his selling-price?

245. A merchant bought 25 bags of coffee at 20 % discount from net price, and sold it for $30.75 per bag, which was 25 % more than he paid: what was the net price?

246. A, owning 45 % of a coal mine, has an income of $3600: what is B's income, who owns 36 %? What is the value of the mine, if the earnings are 16 % of the cost of the mine?

247. A factor bought goods at ¾ % commission; paid $750 for expenses, and sent a bill for $23343.75: what was the amount of the purchase?

248. How shall I direct my agent to sell sewing-

machines for me which cost $20, so that I may net 128 % profit when his commission is 5 %?

249. A dishonest dealer pretended to sell his sugar at cost, but so arranged his scales, that 14 oz. of sugar balanced a pound-weight: what % profit did he make?

250. Bought a bale of silk costing $3.50 per yard for $224. Finding 4 yds. damaged, I want to sell the rest so as to make 25 %: what shall I ask per yard?

251. A merchant bought the following bill of goods:—

| | |
|---|---|
| 1 case prints, 350 yds. @ $0.08. | 25 doz. handk'ch'fs, @ $3 00 p. doz. |
| 8 rolls carpet, ea. 80 " @ .84. | 3 bales ticking, ea. 53 yds. @ $0.16. |
| 3 pieces silk, " 35 " @ 1.50. | 5 pieces sheeting, ea. 33 yds. @ 9c. |

for what did he sell the whole bill, if he charged 12½ % advance?

## STOCKS.

***14. A Company*** is an association of persons for the transaction of business. It is either incorporated, or unincorporated.

***15. A Corporation*** is a company authorized by law to act as an individual.

It can sue and be sued, hold real estate, &c., the same as a man.

**A Charter** is an act of the legislature creating a corporation, and defining its powers and obligations.

***16. Stock,*** or ***Joint Stock,*** is the capital or money invested to carry on the business of a corporation.

The stock of bank, insurance, and railroad companies, is usually divided into SHARES of $100 each. The stock of incorporated manufacturing companies varies in the value of its shares. The par value of a share is $100, unless otherwise mentioned.

1. **A Certificate of Stock or Scrip** is the certificate issued by the company to each stockholder, to show that he owns one or more shares of stock.

Certificates are issued for single shares, or for any number of shares.

2. **The Par Value** is the value named in the certificate.

3. **The Market Value** of a stock is the price it sells for.

Stock is *at par* when the selling-price is the same as the par value.

Stock is *at premium*, or *above par*, when it sells for more than its par value.

Stock is *at discount*, or *below par*, when it sells less than its par value.

4. **Stock Jobbing** is the business of buying and selling stocks.

5. **A Stockbroker** is one whose business is the buying and selling of stocks, either for himself or for another.

6. **An Assessment** is a certain tax made by the company on each share, to cover losses.

***17. A partnership,*** or ***firm,*** is a union of two or more persons in business, acting without incorporation.

The PAR VALUE in stocks is the *base of percentage;* and all premiums, discounts, brokerages, dividends, and assessments are computed at a certain per cent of the PAR VALUE.

### DRILL CARD EXERCISES.—TABLE VII.

Exercise I.—Find the value of **B** shares of R.R. stock at 100 + **A**.
II.—Find the value of **B** shares of R.R. stock at 100 − **A**.

### WRITTEN EXERCISES.

252. The stock of the Farmer's National Bank is worth 127: what is the value of 10 shares? 37 shares? 137 shares?

253. Howard Savings stock is selling at 97½: what are 75 shares worth?

254. I order my broker to purchase 35 shares N.Y.

Central Railroad stock, quoted at 108: what is his bill for brokerage at ¼ %? What do the shares cost me?

NOTE. — Brokerage on *par value*.

255. Southern Michigan is quoted at 103: what will 42 shares cost? 130 shares?

NOTE. — These stocks are given as quoted in the newspapers: they may be understood to be railroad stocks, unless otherwise stated.

256. New Jersey Central is quoted at 105: what will 12 shares cost? 50 shares?

257. New York Central is quoted at 103: what will 10 shares cost? 50 shares?

258. Rock Island is quoted at 108⅝: what will 8 shares cost, with brokerage at ⅛ %?

259. What is obtained for 28 shares Atlantic and Great Western, at 128⅜, after deducting brokerage at ¼ %.

260. Bought 85 shares M. & E. R.R. at 91½, and sold it at 102½: how much do I make, paying ¼ brokerage each way; i.e., on each transaction?

261. Find the cost of 30 shares Northern Pacific at 104, brokerage ⅛ %.

262. How many shares Illinois Central at 97½ can I buy for $2346, brokerage ¼ %?

263. The Iron Hill Mining Company issued stock at a par value of $25. It was sold at $22½ per share: what % loss was that to the company?

264. A man sold 40 shares of mining stock at 8 % discount, and realized $1840: what is the par value of a share?

265. Bought 21 shares First National Bank stock at 95, and sold it at 104: what do I gain, brokerage ⅛ both *ways?*

266. A company which has a capital of $480000 has earned $29540 net in six months: it has declared a semi-annual dividend of 5 %, and has reserved the remainder for surplus. What is the surplus? What is the dividend on 25 shares?

267. A company which has a capital of $500000 has paid $103.50 as a dividend on 23 shares, and has reserved as a surplus $14265: what were the net earnings?

268. A company has a capital of $250000: its expenses are 85 % of its gross earnings: it has reserved $4355, and has paid a dividend of 7 %. What were its gross earnings?

269. A company which has a capital of $280000 has lost during the year $25386.50 more than it has earned: what per cent of assessment, avoiding fractions, must be laid to cover the deficiency? and what surplus will the assessment yield?

Note. — An assessment which would exactly pay the deficit would be laid at a very inconvenient fractional rate. The lowest rate, expressed without fractions, which will cover the deficit, will pay something more.

270. The capital stock of a railway company is $1250000: what per cent of installment must be called to buy the iron for a track 50 miles long, if each rail weighs 60 lbs. to the yard, and costs $55 per ton?

271. Bought 95 shares at 105½, brokerage ⅛; received a dividend of 5½ %; sold at 103⅛, brokerage ¼. What per cent did I make on my investment?

272. Bought 100 shares at 37½; paid 3 assessments of 15 % each; sold at 80. What did I lose?

273. Bought 216 shares Eureka Mining Company at 115; 81 shares Western Union Telegraph at 60; ex-

changed at the same prices for Rock Island at 108, and sold the Rock Island at 115: what was made by the transaction?

274. A ferry company makes an assessment of $3\frac{1}{4}$ % for repairs. How much must a stockholder pay who owns 13 shares?

275. When railroad stock is quoted at $85\frac{1}{3}$, how many shares can be bought for $6020, after paying $\frac{2}{3}$ % brokerage?

276. Sold 20 shares of bank stock at 110, and invested the money in cotton at 20 cts. per pound: how many pounds can I buy, after paying $\frac{1}{8}$ % brokerage for selling stock, and 2 % for buying the cotton?

277. If gold is worth 15 % premium, how much currency will buy $125 of gold?

SUGGESTION. — $1 gold = $1.15 currency: $125 gold = (?) currency.

278. Gold is selling at $1.10 currency. How much gold will $1 currency buy? $50 currency?

279. Sold 30 bales of cotton, averaging 565 lbs. each, at 20 cents per pound, and purchased railroad stock at 90, which I afterward sold at par. I then invested the money in school bonds ($100 each), at 95. How many bonds could I buy, after deducting the commission of 5 % for selling the cotton, and $\frac{3}{8}$ % for each transaction in buying and selling the bonds and stock?

280. Bought 20 school-district bonds ($100 each), at 90, and immediately sold them for 95; with the money I bought corn at 50 cts. per bushel. Paid 3 % insurance, $150 freight and charges, and lost 2 % by shrinkage. I then sold it at an advance of 15 cents per bushel. Did *I make*, or lose, and how much? What %?

## INSURANCE.

**18. *Insurance*** is a contract made by a company to pay a certain sum of money in case of loss or damage of a specified character.*

1. Insurance is of several kinds; viz.,

(1.) Insurance against loss of property.
(2.) Insurance against accident.
(3.) Insurance against loss of health.
(4.) Insurance against loss of life.

2. **Fire Insurance** is insurance against loss of houses, merchandise, &c., by fire.

3. **Marine Insurance** secures payment for damage to vessels and cargoes, for loss or injury by storm, fire, or accident. Insurance on a vessel is usually called *hull insurance*, and that upon the merchandise carried is called *cargo insurance*.

**19. *A Policy*** is the written contract of insurance.

**20. *An Insurer*,** or ***Underwriter*,** is the party agreeing to pay the sum insured in case of loss or damage.

1. The **Insurer** is generally a joint-stock company, called an *insurance company*.

2. A **Mutual Insurance Company** is one in which all the policy-holders share equally in the profits and losses.

3. The **Policy-Holder,** or **Insured,** is the person who is to receive the money in case of loss.

4. The **Premium** is the sum of money paid to the insurer, in consideration of which he agrees to pay the sum insured, or make good the loss or damage sustained by the insured. The premium is estimated at a certain per cent of the sum insured.

**Fire** and **Marine Insurance** are generally not taken on more than three-fourths of the value of the property insured.

### EQUIVALENTS.

Amount insured = B.
Premium = P.
Rate per cent = R.

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* *The amount to be returned in case of loss is also called insurance.*

### ORAL AND WRITTEN PROBLEMS.

1. What is the premium on \$3000 at 1 %? At 2 %? *W.* At $1\frac{1}{4}$ %?

PROCESS.

$\$3000 \times .01\frac{1}{4} = \$37.50.$

ANALYSIS. — (*a.*) Since the premium on \$1 at $1\frac{1}{4}$ % $= \$.01\frac{1}{4}$, on \$3000 it is 3000 times $\$.01\frac{1}{4}$, or \$37.50.

(*b.*) Since $1\frac{1}{4}$ % $= .01\frac{1}{4}$, $1\frac{1}{4}$ % of \$3000 $= .01\frac{1}{4}$ of \$3000, or \$37.50.

NOTE TO THE PUPIL. — In solving the following examples, as well as those in all other cases of percentage, first determine from the *conditions* of the question the essential numbers contained in it, and their arithmetical relation, independently of any rule or formula. Give a concise and correct solution, *after which*, the application of a rule or formula will be of advantage.

2. Insured my furniture for \$800 at 2 %: give the premium. *W.* At $2\frac{3}{5}$ %: give the premium.

3. Insured a cargo of wheat for \$600 at $\frac{1}{2}$ %: what was the premium? *W.* At $\frac{7}{8}$ %?

4. Paid \$10 for insurance at 1 %: what was the sum insured? *W.* Paid \$50 for insurance at $1\frac{1}{3}$ %: what was the sum insured?

5. The premium is \$5, the rate is 2 %: what is the sum insured? *W.* The premium is \$625, the rate is $1\frac{1}{4}$ %: what is the sum insured?

6. Paid \$2 for insuring \$100: what was the rate? *W.* Paid \$112.50 for insuring \$1000: what was the rate?

7. Paid \$5 for insuring \$250: what was the rate? *W.* Paid \$1200 for insuring \$80000: what was the rate?

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REVIEW. — Define rate per cent. (1.) Base. (2.) Percentage. (3.) Amount. (4.) Difference. (5.) Give the letters representing base, &c. (6.) What is the cost *of a thing?* (7.) The selling-price? When is the selling-price the amount? *When the difference?* What is profit or gain? Loss? Net profit?

8. *W.* Insured my house for $6500, furniture for $1200, library for $800, and farm-buildings for $3000: what was the premium at 1¼ %?

9. What is the premium for insuring a steam-mill for $7500 at 1 %? *W.* At 2⅝ %?

10. What is the premium on ¾ of $100 at 1 %? W. ¾ of $634 at 2½ %?

### WRITTEN EXERCISES.

11. A merchant insured 8000 barrels of pork, worth $15.50 a barrel, for ¾ of its value, at 1½ %: what was the premium?

12. Paid $40 for insuring $500 for two years: what was the annual rate? Paid $375 for insuring $5000 for 5 years: what was the annual rate?

13. I insured my factory, worth $30000, for ¾ of its value, at an annual premium of 1½ %: at the end of three years it was burned. What did I lose? What was the net loss of the insurance company?

### ANALYTICAL STEPS.

(1.) $30000 × ¾ = $22500 = *amount insured.*
(2.) $22500 × .01½ = $337.50 = *annual premium.*
(3.) $337.50 × 3 = $1012.50 = *amt. of premium paid in 3 years.*
(4.) $30000 — $22500 = $7500 = *my loss on the property.*
(5.) $7500 + $1012.50 = $8512.50 = *my total loss, including premium.*
(6.) $22500 — $1012.50 = $21487.50 = *loss of insurance company.*

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**REVIEW.** — Give the equivalents of cost price. (7.) Selling-price. Gain or loss Rate per cent. What is commission? (8.) What is a factor? A broker? An agent? A consignment? Who is the consignor? Consignee? Define net proceeds. What is an account sales? Give the formula for finding the percentage when the base and rate are given. (9.) For finding the amount when the base and rate are given. (10.) For finding the difference. For finding the rate when the base and percentage are given. (11.) For finding the base when rate and percentage are given. (12.) For finding the base when rate and amount are given. (13.) When rate and difference are given. What is a company? (14.) A corporation? (15.) A charter? *A certificate of stock?* (16.) What is par value? Market-value?

14. I insured the contents of my store, valued at \$12000, for $\frac{5}{8}$ of its value, at $\frac{3}{4}$ %. What was the premium paid?

15. What will be my loss, including premium, if the above-mentioned goods are all destroyed?

16. What will be the loss to the insurance company, deducting the amount of premium received by them?

17. A merchant insures \$15000 for $1\frac{3}{4}$ %, \$5000 for 2 %, \$5000 for $1\frac{1}{4}$ %: what amount of premium does he pay on the whole property? What % premium does he pay on the whole property?

18. A merchant insured his stock of goods for \$8000 at 2 % premium; being injured by fire, his loss was estimated at \$4000: what was the net loss of the insurance company?

**Partial Losses.**—When the amount of property covered by a policy is only partially destroyed or injured, the insurance company pays for the actual loss only.

19. A farmer insured his buildings, valued at \$4000, for $\frac{5}{8}$ of their value, at 2 %: they were injured by fire to the amount of \$3000. What did the farmer lose, including his premium? What was the net loss of the company?

20. What amount of insurance can be effected on a house, at $1\frac{1}{2}$ % premium, for \$54?

**Analysis.**—Since every $1\frac{1}{2}$ cts. will procure \$1 insurance, \$54 will procure as many dollars' insurance as $1\frac{1}{2}$ cts. are contained times in \$54, or \$3600.

21. What amount of insurance can I secure on my library, at $2\frac{1}{4}$ %, for \$9? For \$13.50?

*22.* *Insured* my house for $\frac{3}{4}$ of its value, at $1\frac{1}{4}$ %, and paid $150: what was the amount of insurance? What was the value of the house?

23. A merchant insured his goods for $7500 at 1⅕ %. His goods being destroyed by fire, what was the actual loss to the insurance company?

### LIFE INSURANCE.

**21. *Life Insurance*** is a contract made with a company, by which it agrees to pay to some person named a certain sum of money on the death of the insured, or when he has reached a specified age.

1. When the money is paid at the death of the insured, the contract is called a *life policy*.

2. When the money is paid at a specified age, or at the death of the insured if he dies before that age, the contract is called an *endowment policy*.

3. When the premium is to be paid annually during life, it is called a *continued premium life policy*.

4. When the payment of the annual premium ceases at the end of ten years, five years, or one year, it is called a *ten, five*, or *single payment policy*.

5. The premium is computed at a certain rate per $1000 insured, varying with the age of the insured at the date of issuing the policy.

**22. *A Mutual Life Assurance Company*** is one in which the profits are divided among the policy-holders, and are either applied to increase the sum insured, or to lessen the annual payment.

#### WRITTEN EXAMPLES.

1. At the age of 26, I insure my life for $10000, at the rate of $20.40 per $1000, payable quarterly: what is the amount of each quarterly payment?

2. A man aged 30 takes a continued premium policy

for $3000, at $22.70 per $1000. He lives 20 years after insuring: what will his heirs receive more than was paid?

3. A man takes out an endowment policy for $5000, payable in ten years, with annual payments at the rate of $104.75 per $1000: how much does he pay more than he receives?

4. A man insured his life on the endowment plan for $8000 for 10 years, at $103.70 per $1000. He died after being insured for 4 years. How much would have been saved had he taken out a life-policy, with annual payments of $26.38 per $1000?

5. A lady aged 40 insures for $5000, at $32 per $1000. After six years, the dividends reduce all further premiums one-half. How much will she have paid in premiums at the age of 70?

## TAXES.

**23. *A Tax*** is a certain sum of money assessed upon individuals, or their property, for any public or general purpose. Taxes are either direct or indirect.

1. When a certain sum, or rate per cent, is assessed upon individuals, or on their real or personal property, it is called a *direct tax.*

2. When a tax is levied upon merchandise, as in the case of manufacturers' or importers' duties, or licenses, or permits, it, being added to the cost of the goods, and thus ultimately paid by the purchaser, is called an *indirect tax.*

3. A tax levied directly upon persons, without regard to property, is called a *poll tax.*

4. All kinds of immovable property, such as lands, buildings, &c., are known as *real estate.*

5. All movable property, such as furniture, merchandise, cattle, notes, money, books, &c., are known as *personal property.*

6. **An Assessor** is a person appointed by proper authority, or elected by the people, whose duty it is to make a list of all the property in a special district, with the names of the several owners, and to affix to all property a taxable value. Such a list is called an *assessment roll.*

7. **A Collector** is a person appointed by law to receive the taxes.

***24. The Tax Rate*** is the rate per cent levied on the assessed valuation of the property.

EQUIVALENTS.

Total property = B.
Tax or amount to be raised = P.
Tax rate = R.

WRITTEN EXAMPLES.

In a certain village the sum of $12202.50 is to be raised by tax for the erection of a school-building. The valuation of taxable property, as shown by assessment roll, is $800000, and there are 225 polls to be assessed 90 cts. each. What will be the tax of Clarence Thompson, whose property is valued at $3426, and who pays for 1 poll?

ANALYTICAL STEPS.

1. *Find the amount raised by poll-tax.*
($0.90 × 225 = $202.50.)
2. *Find the amount to be raised on the property.*
($12202.50 — $202.50 = $12000.)
3. *Find the amount to be assessed on $1, or the rate %.*
($12000 ÷ 800000 = $0.015, *or* 1.5 %.)
4. *Find the amount of tax on Mr. Thompson's property.*
($0.015 × 3426 = $51.39.)
5. *Find the total amount of Mr. Thompson's tax.*
($51.39 + $.90 = $52.29.)

Having found the tax on $1, or the Rate of Tax, as above, assessors avoid labor by constructing a table as follows, the rate being 1¼ %: —

| Property. | Tax. | Property. | Tax. | Property. | Tax. | Property. | Tax. |
|---|---|---|---|---|---|---|---|
| $1 | $.0125 | $10 | $.125 | $100 | $1.25 | $1000 | $12.50 |
| 2 | .025 | 20 | .25 | 200 | 2.50 | 2000 | 25 00 |
| 3 | .0375 | 30 | .375 | 300 | 3.75 | 3000 | 37.50 |
| 4 | .05 | 40 | .50 | 400 | 5.00 | 4000 | 50.00 |
| 5 | .0625 | 50 | .625 | 500 | 6.25 | 5000 | 62.50 |
| 6 | .075 | 60 | .75 | 600 | 7.50 | 6000 | 75.00 |
| 7 | .0875 | 70 | .875 | 700 | 8.75 | 7000 | 87.50 |
| 8 | .10 | 80 | 1.00 | 800 | 10.00 | 8000 | 100 00 |
| 9 | .1125 | 90 | 1.125 | 900 | 11.25 | 9000 | 112.50 |

1. By the table, what is the tax on $8600 of real estate, and $3785 of personal property?

PROCESS.

(1.)
$8600
3785
$12385

(2.)
$9000 prop. = $112.50 tax.
3000 " = 37.50 "
300 " = 3.75 "
80 " = 1.00 "
5 " = .0625 "
$12385 prop. = $154.81 tax.

2. A. owns $8700 real estate, assessed at ¾ of its value, and $1258 assessed personal property: what is his tax by the above table, including 1 poll at 1.25?

3. The amount to be raised by tax in a certain town is $14000. The real estate of the town is assessed at $678000; and the personal property at $122000. What is the tax of Mr. C. Brown, whose property is valued at $4860? Of Mr. W. T. Cavenaugh, whose property is valued at $2800? Of Mr. McFadden, whose property *is valued* at $11600?

4. A schoolhouse is to be built, at an expense of $8400. The money is to be raised by tax upon property valued at $700000. What will be the rate of taxation to cover the cost of the schoolhouse and the collector's commission of 4 %?

5. The expense of erecting the county buildings at Marion Centre was $4250: this was raised upon the assessed valuation of the county of 4½ mills on a dollar. The collector's commission was 5 %: what was the valuation of the property?

6. A tax of $17250 is to be assessed upon a town. The real estate is valued at $850000, and the personal property at $250000. There are 500 polls, each of which is taxed $1.50: what is the tax on $1, or the rate of taxation? Construct a table.

7. What is B's. tax (by the table), whose property is valued at $9864, and who pays for one poll?

8. To how much will the tax of a person amount, whose property is assessed at $12500, if he pays 1½ % town tax, 1½ mills school tax, and ½ % State tax?

## CUSTOM-HOUSE BUSINESS.

**25. *Customs, or Duties,*** are taxes laid by government upon imported goods, and upon the tonnage of vessels.

1. **Tonnage** is a tax levied upon vessels, according to their capacity, for the privilege of coming into a port of entry.

2. **A Custom-House** is a building occupied by government officers appointed to collect the duties, and register the entry and clearance of vessels.

3. **A Port of Entry** is a seaport at which a custom-house is established.

4. **A Collector of the Port** is the officer charged by government with the duty of overseeing the collections of customs, and the entry and departure of vessels engaged in foreign trade.

5. **A Tariff** is a schedule fixed by Act of Congress, showing the rates of duties on all goods imported.

6. **A Clearance** is a paper given to the master of a vessel by the collector of the port, certifying that the owners of the vessel have complied with the laws regulating foreign trade.

7. **A Manifest** is an invoice of the goods forming the ship's cargo.

**26.** ***A Specific Duty*** is a definite sum fixed upon each item of merchandise, without reference to its cost; as, 40 cts. per gallon, 8 cts. per pound, &c.

**27.** ***An Ad Valorem Duty*** is estimated at a certain per cent upon the value of each article as shown by the manifest.

Before assessing specific duties, certain allowances are made; as,

1. **Draft** is an allowance for waste: it must be deducted before other allowances are made.

2. **Tare** is an allowance for the weight of whatever contains the merchandise; as, box, cask, bag, &c.

3. **Leakage** is an allowance for waste of liquids contained in barrels or casks.

4. **Breakage** is an allowance for loss of liquids contained in bottles.

5. **Gross Weight** is the weight of the goods before any allowance is made.

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REVIEW. — What is stock? (16.) What is the value of a share in bank, insurance, and railroad companies? What is a certificate of stock? What is par value? Market value? When is stock at par? At premium? At discount? What is a stock broker? What is a partnership, or firm? (17.) What is the base of percentage in stocks? How are all premiums, discounts, brokerages, dividends, and assessments computed? What is insurance? (18.) What is fire insurance? Marine insurance? What is a policy? (19.) An insurer, or underwriter? (20.) What is a mutual insurance company? Who is the policy-holder? What is the premium? *How is it* estimated?

6. **Net Weight** is the weight of the goods after all allowances are deducted.

### WRITTEN EXAMPLES.

1. What is the duty, at 2 cts. per lb., on 10 boxes of sugar, each weighing 316 lbs., tare being 5 %?

### ANALYTICAL STEPS.

1. *Find the weight of the sugar.*

316 *lbs.* × 10 = 3160 *lbs.*

2. *Find the amount of tare.*

3160 *lbs.* × .05 = 158 *lbs.*

3. *Find the net weight of sugar.*

3160 *lbs.* — 158 *lbs.* = 3002 *lbs.*

4. *Find the duty.*

$.02 × 3002 (*lbs.*) = $60.04.

2. What is the duty, at 5 cts. per lb., on 137 cases of indigo, each case weighing 194 lbs., allowing 15 % tare?

3. Imported 170 hogsheads of molasses, containing 87 gallons each. What duty must be paid, at 8 cts. per gallon, after deducting 2 % for leakage?

4. What is the duty, at 25 %, on 100 barrels of sperm oil at 87 cents a gallon, allowing 2½ % leakage?

5. Bought 3 cases of silk, valued respectively at $2350, $2185, and $3140. What was the duty, at 20 % *ad valorem?*

6. What duty must an importer pay who has an invoice of 600 tons of iron, costing $90 per ton, at $8 per ton specific, and 40 % ad valorem duty?

7. An importer receives an invoice of 420 dozen pint bottles of ale, invoiced at $.95 per dozen. If 2½ % of the bottles are found broken, what will be the duty, at 40 cts. a gallon?

## ANALYSIS OF PERCENTAGE.

### I. Definitions.

1. RATE PER CENT.
2. BASE.
3. PERCENTAGE.
4. AMOUNT.
5. DIFFERENCE.
6. SIGNS.

### II. Applications.

1. PROFIT AND LOSS.
    - (a.) *Cost.*
    - (b.) *Selling-Price.*
    - (c.) *Profit or Gain.*
    - (d.) *Loss.*
2. COMMISSIONS.
    - (a.) *Factor.*
    - (b.) *Broker.*
    - (c.) *Agent.*
    - (d.) *Net Proceeds.*
    - (e.) *Account Sales.*

### III. Propositions.

1. PROP. I. BASE AND RATE GIVEN, TO FIND PERCENTAGE.
    - (a.) *Analysis.*
    - (b.) *Formula and Rule.*
2. PROP. II. BASE AND RATE GIVEN, TO FIND THE AMOUNT OR DIFFERENCE.
    - (a.) *Analysis.*
    - (b.) *Formula and Rule.*
3. PROP. III. BASE AND PERCENTAGE GIVEN, TO FIND THE RATE.
    - (a.) *Analysis.*
    - (b.) *Formula and Rule.*
4. PROP. IV. RATE AND PERCENTAGE GIVEN, TO FIND THE BASE.
    - (a.) *Analysis.*
    - (b.) *Formula and Rule.*
5. PROP. V. RATE AND AMOUNT OR DIFFERENCE GIVEN, TO FIND THE BASE.
    - (a.) *Analysis.*
    - (b.) *Formula and Rule.*

### IV. Application,—Stocks, &c.

1. COMPANY.
2. CORPORATION.
    - *Charter.*
3. STOCK, OR JOINT STOCK.
    - (a). *Certificate of Stock or Scrip.*
    - (b). *Par Value.*
    - (c). *Market Value.*
        - Value at Par.
        - Value at Premium.
        - Value at Discount.
    - (d.) *Stock Jobbing.*
    - (e.) *Stock Broker.*
4. PARTNERSHIP OR FIRM.
5. INSURANCE.
    - (a.) *Fire Insurance.*
    - (b.) *Marine Insurance.*
6. POLICY.
7. AN INSURER OR UNDERWRITER.
    - (a.) *Insurance Company.*
    - (b.) *Mutual Insurance Company.*
    - (c.) *Policy-Holder, or Insured.*
    - (d.) *Premium.*
8. LIFE INSURANCE.
    - (a.) *Life Policy.*
    - (b.) *Endowment Policy.*
    - (c.) *Continued Premium Life Policy.*
    - (d.) *One, Five, or Ten Payment Policy.*
9. MUTUAL LIFE ASSURANCE COMPANY.
10. TAXES.
    - (a.) *Direct Tax.*
    - (b.) *Indirect Tax.*
    - (c.) *Poll-Tax.*
    - (d.) *Real Estate.*
    - (e.) *Personal Property.*
    - (f.) *An Assessor.*
    - (g.) *A Collector.*
11. THE TAX RATE.
12. CUSTOMS, OR DUTIES.
    - (a.) *Tonnage.*
    - (b.) *A Custom-House.*
    - (c.) *A Port of Entry.*
    - (d.) *A Collector of the Port.*
    - (e.) *A Tariff.*
    - (f.) *A Clearance.*
    - (g.) *A Manifest.*
13. A SPECIFIC DUTY.
14. AN AD VALOREM DUTY.
    - (a.) *Draft.*
    - (b.) *Tare.*
    - (c.) *Leakage.*
    - (d.) *Breakage.*
    - (e.) *Gross Weight.*
    - (f.) Net Weight.

Banking Room.

## INTEREST.

Mr. Edwards borrowed \$5 of Mr. Lewis, and agreed to pay him at the end of the year the \$5 he borrowed, and also 7 hundredths (7 cts.) of a dollar additional for the use of each dollar borrowed. For the \$5, this amounted to 35 cents: hence, at the end of the year, he paid Mr. Lewis \$5.35.

In the above statement, the \$5 borrowed is called the *principal*; the $\frac{7}{100}$ (7 %) paid for the use of each dollar, the *rate per cent*; the 1 year, the *time*; the 35 cents, the *interest*; and the \$5.35, the sum paid at the end of the year, the *amount*.

**28. *Interest*** is the sum paid for the use of money.

**29. *The Principal*** is the sum of money for the use of which interest is paid.

**30.** ***The Rate per Cent*** is the number of hundredths of the principal paid each year.

**31.** ***The Amount*** is the sum of the principal and interest.

**32.** ***Simple Interest*** is interest on the principal only.

**33.** ***Compound Interest***[*] is interest on both principal and interest when not paid at the end of each year.

1. **In Compound Interest,** at the end of each year the interest is added to the principal, and the sum forms a new principal for the succeeding year.
2. **Simple Interest** is not due, and cannot be collected, until the principal is due.
3. **Annual Interest**[†] is due at the end of each year. Semi-annual interest is due at the end of each half year. Either can be collected like any other debt.

**34.** ***Legal Interest*** is the rate per cent fixed by law.

**35.** ***Usury*** is a rate of interest greater than that fixed by law.

Legal interest varies in the different States. The following table shows the rate in each State or Territory: —

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[*] "It seems well settled that compound interest cannot be recovered, as such, even if it be expressly promised, . . . unless upon a promise of a debtor made *after* the interest upon which interest is demanded has accrued." — *Parsons on Contracts*, vol. iii. 153.

[†] "One holding a note on which interest is payable annually or semi-annually may sue for each instalment of interest as it becomes payable, although the note is not yet due; . . . but, if he allows the time to run by without demanding interest, he cannot afterward, in action on the note, recover compound interest." — *Parsons on Contracts*, vol. ii. 636.

TABLE.

| Name of State. | Legal. | Special. | Name of State. | Legal. | Special. |
|---|---|---|---|---|---|
| Alabama | 8 % | ...... | Minnesota | 7 % | 12 % † |
| Arkansas | 6 % | any * | Mississippi | 6 % | any * |
| Arizona | 10 % | any * | Missouri | 6 % | 10 % † |
| California | 10 % | any * | Montana | 10 % | ...... |
| Connecticut | 6 % | any * | New Hampshire | 6 % | ...... |
| Colorado | 10 % | any * | New Jersey | 6 % | 7 % |
| Canada | 6 % | ...... | New York | 7 % | ...... |
| Delaware | 6 % | ...... | North Carolina | 6 % | 8 % † |
| Dist. Columbia | 6 % | 10 % † | Nebraska | 10 % | 12 % † |
| Dakota | 7 % | 18 % | Nevada | 10 % | any * |
| Florida | 8 % | any * | Ohio | 6 % | 8 % † |
| Georgia | 7 % | 12 % † | Oregon | 10 % | 12 % † |
| Illinois | 6 % | 8 % † | Pennsylvania | 6 % | ...... |
| Indiana | 6 % | 10 % † | Rhode Island | 6 % | any * |
| Iowa | 6 % | 10 % † | South Carolina | 7 % | any * |
| Idaho | 10 % | 24 % | Tennessee | 6 % | 10 % † |
| Kansas | 7 % | 12 % † | Texas | 8 % | 12 % |
| Kentucky | 6 % | 8 % † | Utah | 10 % | any * |
| Louisiana | 5 % | 8 % † | Vermont | 6 % | ...... |
| Maine | 6 % | any * | Virginia | 6 % | any * |
| Maryland | 6 % | ...... | West Virginia | 6 % | ...... |
| Massachusetts | 6 % | any * | Wisconsin | 7 % | 10 % † |
| Michigan | 7 % | 10 % † | Wash. Territory | 10 % | any * |

N.B.—When the rate of interest is not specified in accounts, notes, or contracts, the legal rate is always understood.

**36. *Terms.***—There are five *terms* used in problems in interest; viz., *Principal*, *Interest*, *Rate per Cent*, *Amount*, and *Time*.

An additional element to those used in percentage is *time*.

EQUIVALENTS.

|  | Interest. | Percentage. |
|---|---|---|
| Principal | = P | = B. |
| Rate per cent | = R | = R. |
| Interest | = I | = P. |
| Amount | = A | = A. |
| Time | = T. |  |

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* Any per cent is legal, if agreed upon by the parties in writing.

† Any per cent *not exceeding the* per cent given is legal, if agreed upon by the *parties in writing.*

## FELTER'S

# ARITHMETICAL DRILL CARDS.

## TABLE VIII.

## INTEREST.

| No. of Exam. | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| *1* | 3 | 3 | 10 | 12½ | 25 | 41.34 | 43.60 | **Jan. 3, 1870.** |
| *2* | 5 | 6 | 15 | 6 | 140 | 68.13 | 3.93 | **Aug. 7, 1870.** |
| *3* | 6 | 7 | 27 | 4 | 37 | 9.137 | 5.20 | **Feb. 9, 1871.** |
| *4* | 4 | 4 | 24 | 5 | 96 | 41.67 | 6.50 | **Nov. 11, 1871.** |
| *5* | 7 | 8 | 12 | 8⅛ | 60 | 8.901 | 12.90 | **Mar. 6, 1872.** |
| *6* | 2 | 4 | 16 | 6⅞ | 84 | 10.01 | 80.00 | **Oct. 3, 1872.** |
| *7* | 11 | 9 | 4 | 12 | 90 | 9.031 | 25.00 | **Apr. 11, 1873.** |
| *8* | 3 | 5 | 3 | 9 | 37 | 2000 | 6.73 | **Nov. 1, 1873.** |
| *9* | 6 | 7 | 9 | 3⅓ | 212 | 8109 | 5.88 | **May 3, 1874.** |
| *10* | 8 | 10 | 17 | 1 | 310 | 41.80 | 7.44 | **Dec. 21, 1874.** |
| *11* | 7 | 11 | 11 | ½ | 168 | 7.453 | 18.60 | **June 11, 1875.** |
| *12* | 5 | 6 | 18 | ⅜ | 48 | 91.36 | 37.24 | **Sept. 3, 1875.** |
| *13* | 9 | 7 | 20 | 15 | 126 | 71.80 | 8.37 | **July 1, 1876.** |
| *14* | 5 | 4 | 24 | 3½ | 143 | 9111 | 2.63 | **Nov. 11, 1876.** |
| *15* | 8 | 9 | 21 | 4¾ | 63 | 8000 | 91.31 | **Jan. 3, 1877.** |
| *16* | 10 | 5 | 6 | 6⅛ | 99 | 10.80 | 25.20 | **Aug. 1, 1877.** |
| *17* | 6 | 7 | 26 | 11 | 105 | 9.901 | 86.30 | **Sept. 3, 1877.** |
| *18* | 9 | 4 | 13 | 9⅞ | 330 | 98.01 | 24.40 | **Dec. 30, 1877.** |
| *19* | 7 | 3 | 5 | 5¼ | 162 | 5702 | 93.34 | **Oct. 31, 1878.** |
| *20* | 11 | 8 | 7 | ⅞ | 192 | 10.38 | 72.11 | **Nov. 9, 1878.** |

## 37. To find the interest on any principal for any number of years and months, at any rate per cent.

### ORAL EXAMPLES.

1. What is the interest on $2 for 1 year at 6 per cent?

**Formula.** $2 × .06 = $.12 = interest.

**Analysis.** — Since the interest on $1, at 6 % for 1 year, is 6 cents, on $2 it is 2 times 6 cts., or 12 cts.

2. What is the interest on $5 for 1 year at 7 %? At 8 %?

3. What is the interest, at 5 %, for 1 year, of $3? Of $8? Of $50?

4. What is the interest of $1, at 5 %, for 1 year? For 3 years? For 8 years?

5. What is the interest of $8 for 3 years at 7 %?

**Formula.** $8 × .07 × 3 = $1.68.

**Analysis.** — (1.) Since the interest of $1, at 7 %. for 1 year is 7 cts., of $8 it is 8 times 7 cts., or 56 cts.

(2.) Since the interest of $8 for 1 year is 56 cts., for 3 years it is 3 times 56 cts., or $1.68.

6. What is the interest of $7 for 4 years at 9 %? At 6 %?

7. What is the interest of $5 at 4 % for 3 years? For 5 years? For 2½ years?

8. What is the interest of $20 for 3 yrs. at 5 %? For 5 years? For 10 years?

9. What is the interest of $200 at 5 % for 2 years? For 2 years and 6 mos.?

**Suggestion.** — Reduce the months to a fraction of a year; thus, 2 years 6 mos. = 2½ *years; 3 years* 7 mos. = 3$\frac{7}{12}$ years.

10. What is the interest of \$50 at 8 % for 3 years, 3 mos.? For 1 year, 9 mos.?

PROBLEM.—What is the interest of \$260 for 3 years at 7 %?

PROCESS.—\$260
.07
\$18.20 int. for 1 year.
3
\$54.60 int. for 3 years.

ANALYSIS.—Since the interest of \$1 for 1 year is 7 cents, for \$260 it is 260 times 7 cents, or \$18.20; and for 3 years it is 3 times \$18.20, or \$54.60.

***38. Principle.***—The interest of any sum of money is equal to the product of that sum multiplied by the rate per cent, multiplied by the number of years: hence the following:—

RULE.—*Multiply the principal by the rate per cent, and this product by the number of years.*

FORMULA. $P \times R \times T = I$.

### WRITTEN EXERCISES.

1. What is the interest of \$250 for 3 yrs. at 6 %?

2. What is the interest of \$1240 for 5 yrs. at 8 %?

3. What is the interest of \$372.40 for 6 yrs. at 9 %?

4. What is the interest of \$253.60 for 2 yrs. 6 mos. ($2\frac{1}{2}$ years) at 6 %? At 8 %?

5. Find the interest of \$683.40 for 2 yrs. 3 mos. ($2\frac{1}{4}$ yrs.) at 5 %. For 1 year and 11 mos. ($1\frac{11}{12}$ years) at 8 %.

6. Find the interest of \$1550 for 4 yrs. 4 mos. at 6 %. At 9 %.

7. Gave my note for \$420.30 at 6 %: what should I pay at the end of 2 years, 6 mos., interest and principal?

8. Loaned Mr. Randall $853.20 at 7 % for 3 years, 8 months: what will be the amount due me at that time?

SUGGESTION. P + I = A.

9. Loaned Mr. Ames $873.65 at 7 % for 2 yrs. 8 mos.: how much interest will be due me? How much will Mr. Ames owe me?

10. Find amount of $487.40 at 7 % for 2 yrs. 2 mos.

11. What is the amount of $83.20 at 7 % for 7 yrs. 2 mos.?

12. What is the amount of $635 for 3½ yrs. at 4½ %?

13. What is the interest on $1 for 11 years, 4 mos. at 9 %? What is the amount?

### DRILL CARD EXERCISES. — TABLE VIII.

**For brief business methods for solving problems in interest, see Key to Felter's Drill Cards.**

**Exercise I. — Find the interest of E dollars at A % for A years.**
**II. — Find the amount of F dollars at B % for A years.**
**III. — Find the interest of G dollars at D % for B years.**
**IV. — Find the amount of E dollars at D % for A years.**
**V. — Find the interest of G dollars at A % for D years.**
**VI. — Find the amount of G dollars at D % for A years.**
**VII. — Find the interest of E dollars at A % for A years and B months.**
**VIII. — Find the amount of G dollars at D % for B years and A months.**
**IX. — Find the interest of G dollars at B % for A years and D months.**

***39.*** TO FIND THE INTEREST ON ANY SUM, FOR ANY TIME, AT ANY RATE.

PROBLEM. — Find the interest of $30 for 5 yrs. 7 mos. *and 11 days, at 7 %.*

**Process.**

$30

.07

12)2.10 × 5 (yrs.) = $10.50

30) .175 × 7 (mos.) = 1.225

.0058 × 11 (da.) = .063

$11.79

**Analysis.** — (1.) Since the interest of $30 at 7 % for 1 yr. = $2.10, the interest for 5 yrs. = 5 times $2.10, or $10.50.

(2.) Since the interest of $30 at 7 % for 1 yr. (12 mos.) = $2.10, for 1 mo. it = $\frac{1}{12}$ of $2.10, or $.175; and for 7 mos. it = 7 times $.175, or $1.225.

(3.) Since the interest of $30 for 1 mo. (30 da.) is $.175, for 1 da. it = $\frac{1}{30}$ of $.175, or $.0058; and for 11 da. it = 11 times $.0058, or $.063.

(4.) Since the interest of $30 for 5 yrs. = $10.50, for 7 mos. it = $1.225, and for 11 da. it = $.063; for 5 yrs. 7 mos. 11 da. it = the sum of these numbers, or $11.79.

### ANALYTICAL STEPS.

(1.) *Find the interest for* 1 *year.*
(2.) *Find the interest for* 5 *years.*
(3.) *Find the interest for* 1 *month.*
(4.) *Find the interest for* 7 *months.*
(5.) *Find the interest for* 1 *day.*
(6.) *Find the interest for* 11 *days.*
(7.) *Find the interest for* 5 *yrs.* 7 *mos.* 11 *da.*

### WRITTEN EXERCISES.

14. Find the interest of $375 for 2 years, 3 mos. 8 days, at 8 %.

15. What is the interest of $236 for 1 year, 8 mos. 14 days, at 5 %?

16. What is the interest of $1673 for 5 years, 2 mos. 18 days, at 6 %?

17. What is the interest of $362.31 for 2 years, 7 months, 13 days, at 7 %?

18. Find the interest of $643 for 3 years 11 mos. 17 da. at 8 %.

19. Find the amount of $300.20 for 1 year, 8 months, 23 days, at 5 %.

20. Find the amount due on a note of $837.50 at 6 %, due 2 years, 21 days hence.

21. Find the amount of $4260 for 6 months, 13 days, at 7 %.

22. The principal is $413.20; the time is 11 months, 3 days; the rate is 5 %: what is the amount?

SUGGESTION. — When the number of days is a convenient fractional part of a month, use that fraction; thus, 3 days = $\frac{1}{10}$ month; 10 da. = $\frac{1}{3}$ mo., &c.

23. If a person borrow $1384 at 7 % for 9 months, 15 days, how much will he owe at the end of that time?

24. If a man loan $265 at 8 %, what will be due him at the end of 27 yrs. 7 mos. 12 days?

25. Dec. 15, 1872, I borrowed $875.20 at 12 %: what do I owe Aug. 12, 1876.

SUGGESTION. — Find the difference of time between the two dates.

| Yrs. | Mos. | Days. |
|---|---|---|
| 1876 | 8 | 12 |
| 1872 | 12 | 15 |
| 3 | 7 | 27 |

26. What was due on the following note, if settled Oct. 9, 1875?

$580. *New York, May* 3, 1874.

*On demand I promise to pay James Griffith, or order, the sum of five hundred and eighty dollars, with interest at* 7 % *for value received.* *Lemuel Dobbs.*

27. The date of a note is Aug. 13, 1869; the face of the note is $340; and the rate is 8 %. What is the interest due Oct. 5, 1871.

28. What is the amount due Sept. 6, 1874, on a note for $158.40, given Nov. 20, 1872, with interest at 7 %?

29. The date of a note is Sept. 24, 1874; the face of the note is $853.40: what is due April 16, 1876, the rate being 7 %?

30. If a man borrow $467.63 at 6 %, how much will he owe at the end of 4 yrs. 7 mos. 18 da.?

### DRILL CARD EXERCISES. — TABLE VIII.

**Exercise X.** — Find the interest of **E** dollars for **B** mo. and **C** da. at **A** %.

**XI.** — Find the amount of **F** dollars for **A** yrs. **B** da. at 6 %.

**XII.** — Find the interest of **G** dollars for **B** yrs. and **C** da. at 10 %.

**XIII.** — Find the amount of **G** dollars at **C** % for **B** months and **A** days.

**XIV.** — Find the interest of **G** dollars for **A** yrs. **B** mo. and **C** days, at **D** %.

### *40.* TO CALCULATE INTEREST BY THE SIX PER CENT METHOD.

PROBLEM. — What is the interest of $1 at 6 % for 1 year, 1 month, and 1 day?

PROCESS. — $1.00
.06

| | |
|---|---|
| 12) .06 int. for 1 year. | .06 |
| 30) .005 int. for 1 mo. | .005 |
| .000⅙ int. for 1 day. | .000⅙ |
| | .065⅙ int. for 1 yr. 1 mo. 1 day. |

ANALYSIS. — The analysis is the same as for the examples under *the previous* rule, and will not be repeated here.

**Problem.** — What is the interest of $372, at 6 %, for 2 yrs. 3 mos. and 13 days?

**Process.**
$.06 × 2 = $0.12
.005 × 3 = .015
.000⅙ × 13 = .002⅙
.137⅙

$372
.137⅙
62
2604
1116
372
$51.03

**Analysis.** — (1.) Since the interest of $1 for 1 year is 6 cents, for 2 years it will be twice as much, or 12 cents.

(2.) Since the interest of $1 for 1 month is ½ cent, for 3 months it will be three times ½ cent, or 1½ cents.

(3.) Since the interest of $1 for 1 day is ⅙ mill, for 13 days it will be 13 times ⅙ mill, or 2⅙ mills.

(4.) The interest of $1 for 2 years, 3 months, and 13 days, is the sum of these amounts, or $.137⅙.

(5.) Since the interest of $1 for the given time is $.137⅙, for $372 it is 372 times $.137⅙, or $51.03: hence the following

**Rule.** — *Multiply* 6 *cts. by the number of years,* ½ *cent by the number of months, and* ⅙ *mill by the number of days. Multiply the principal by sum of these products.*

### ORAL EXERCISES.

**Note.** — All questions, unless otherwise stated, are to be calculated at 6 %.

What is the interest of $1 for the following periods of time: —

| Model. | (1.) | (2.) | (3.) | (4.) | (5.) |
|---|---|---|---|---|---|
| 1 yr. = $.06 | 2 yrs. | 11 mos. | 6 da. | 11 da. | 3 mos. 2 da. |
| 3 yrs. = .18 | 4 yrs. | 7 mos. | 8 da. | 3 da. | 5 mos. 6 da. |
| 2 mos. = .01 | 3 yrs. | 5 mos. | 12 da. | 8 da. | 8 mos. 4 da. |
| 6 da. = .001 | 6 mos. | 3 mos. | 9 da. | 5 da. | 7 mos. 9 da. |
| 5 mos. = .025 | 8 mos. | 1 mo. | 27 da. | 13 da. | 11 mos. 13 da. |
| 9 da. = .001½ | 5 mos. | 9 mos. | 17 da. | 19 da. | 7 mos. 29 da. |

6. For 2 yrs. 4 mos. 6 days? For 1 yr. 2 mos. 7 da.?
7. For 5 yrs. 3 mos. 11 da.? For 1 yr. 5 mos. 9 da.?
8. For 7 yrs. 2 mos. 9 da.? For 3 yrs. 9 da.?
9. For 2 yrs. 3 mos. 11 da.? For 2 yrs. 23 da.?
10. Int. of $5 for 2 yrs. 5 mos.?

SUGGESTION. — Int. of $1 for given time, $.14½.
" " $5 = $.14½ × 5 = $.72½.

11. Int. of $8 for 3 yrs. 2 mos.? For 2 yrs. 7 mos.?
12. Int. of $10 for 5 yrs. 7 mos.? For 4 yrs. 4 mos.?
13. Int. of $100 for 1 yr. 8 mos.? For 2 yrs. 6 mos.?

### WRITTEN EXERCISES.

31. What is the interest of $427.35 for 3 yrs. 8 mos. 12 days?

ANALYTICAL STEPS. — (1.) *Find the interest of* $1 *for the given time.*
(2.) *Find the interest of* $427.35.

32. What is the interest of $485.36 for 2 yrs. 7 mos.?

33. What is the interest of $736.62 for 8 yrs. 3 mos. 18 days?

34. What is the interest of $1575 for 4 yrs. 9 mos. 15 days?

35. Find the amount of $783 for 2 yrs. 4 mos. 24 days.

36. Amount of $175.20 for 2 yrs. 12 days.

37. Amount of $82.40 for 3 yrs. 5 mos. 10 days.

38. Interest of $832.50 for 9 mos. 28 days.

39. Amount of $75.62½ for 11 mos. 9 days.

40. Amount of $304.08 for 4 yrs. 4 mos.

41. Interest of $175.25 for 9 mos. 12 days.

42. Interest of $1825.41 for 28 days.
43. Amount of $3264.30 for 2 yrs. 7 mos. 19 days.
44. Amount of $104.33 for 11 mos. 29 days.
45. Interest of $10805 for 2 yrs. 18 days.
46. Amount of $639 for 11 yrs. 11 mos. 11 days.
47. Amount of $375.75 for 2 yrs. 20 days.
48. Interest of $1976 for 7 yrs. 8 mos. 9 days.
49. Interest of $287.40 for 27 days.
50. Amount of $4385.70 for 3 days.

### DRILL CARD EXERCISES. — TABLE VIII.

**Exercise XV.** — Find the interest (by 6 % method) of **E** dollars for **B** months and **C** days.

**XVI.** — Find the interest of **G** dollars for **B** years, **A** months, and **C** days.

**XVII.** — Find the amount of **G** dollars for **C** months and **B** days.

## *41.* TO FIND INTEREST AT ANY RATE BY THE 6 % METHOD.

PROBLEM. — What is the interest of $854 for 2 yrs. 3 mos. 12 days at 7 %?

**PROCESS.**

$854
.137 int. at 6 % for 2 yrs. 3 mos. 12 da.
5978
2562
854
6)$116.998 int. of $854 at 6 %.
19.499⅔ int. at 1 %.
7
$136.50 int. at 7 %.

ANALYSIS. — (1.) The interest of $854 at 6 % for the given time is $116.998.

(2.) The interest at 1 % is ⅙ of $116.998, and at 7 % is 7 times ⅙ of $116.998 (or ⅙ of 7 times $116.998), or $136.50.

6. For 2 yrs. 4 mos. 6 days? For 1 yr.
7. For 5 yrs. 3 mos. 11 da.? For 1 yr. 5
8. For 7 yrs. 2 mos. 9 da.? For 3 yrs. 9
9. For 2 yrs. 3 mos. 11 da.? For 2 yrs.
10. Int. of $5 for 2 yrs. 5 mos.?

SUGGESTION.—Int. of $1 for given time
" " $5 = $.14½ × 5 =

11. Int. of $8 for 3 yrs. 2 mos.? Fo
12. Int. of $10 for 5 yrs. 7 mos.? F
13. Int. of $100 for 1 yr. 8 mos.? F

WRITTEN EXERCISES.

31. What is the interest of $427.3
12 days?

ANALYTICAL STEPS.—(1.) Find the int
sum.
(2.) Find

32. What is the inte
33. What is the i
15 days?
34. What

### WRITTEN EXERCISES.

51. What is the interest of $385 for 2 yrs. 4 mos. 9 days, at 8 %?

52. Find the interest of $112.30 for 3 yrs. 4 mos. 8 days, at 9 %.

53. What is the amount of $175.50 for 6 yrs. 2 mos. 12 days, at 7 %?

54. Find the amount of $128.30 for 3 yrs. 6 mos. 9 days, at 7 %.

55. A man gave a note dated June 17, 1871, for $180.40 at 7 %: what was due Aug. 5, 1875?

56. What is the amount of $60.75, at $4\frac{1}{2}$ %, for 3 yrs. 7 mos. 13 days?

57. Find the interest of $95.63, at 5 %, for 7 mos. 19 days.

### DRILL CARD EXERCISES. — TABLE VIII.

Exercise XVIII. — Find the interest of **F** dollars for **B** mos. **C** days, at **A** %.

XIX. — What is the amount of **G** dollars, at **D** %, for **A** years, **B** mos.?

XX. — What is the interest of **E** dollars, at **B** %, for **A** mos. **C** days?

## ***42.*** TO FIND INTEREST FOR ANY NUMBER OF DAYS AT 6 %.

Many business-men and bankers, when the time is short, reckon interest for the actual number of days included in the time, calling each day $\frac{1}{360}$ of a year.*

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* The true interest of any sum for any number of days will be found by taking one day's interest as $\frac{1}{365}$ of a year's interest; thus the interest of $1 at 7 % for 87 days would be $\frac{87}{365}$ of $.07. This method is used by the U. S Government, by *bankers*, and by a few business-men.

**PROBLEM.**—What is the interest of $184 from June 17 to Oct. 10, at 6 %?

| PROCESS. | | (a.) | (b.) |
|---|---|---|---|
| In June, | 13 days | $184 | 184 |
| " July, | 31 " | .019⅙ | .115 |
| " August, | 31 " | | |
| " September, | 30 " | 30 | 920 |
| " October, | 10 " | 1656 | 184 |
| | | 184 | 184 |
| | 6)115 days | $3.53 | 6)21160 |
| | 19⅙ | | $3.53 |

**ANALYSIS.**—(a.) The time from June 17 to Oct. 10 is 115 days. Since the interest of $1 for 1 day at 6 % is ⅙ of a mill, the interest of $1 for 115 days will be $\frac{115}{6}$ of a mill, or $.019⅙; and the interest of $184 for the same time will be 184 times $.019⅙, or $3.53.

(b.) If the interest of $1 for 115 days is $\frac{115}{6}$ mills, the interest of $184 will be 184 times $\frac{115}{6}$ mills, or ⅙ of $.115 × 184 = $\frac{\$21.160}{6}$ = $3.53: hence

**RULE.**—I. *Multiply the principal by the number of days taken as mills, and divide the product by* 6; *or,*

II. *Divide the number of days by* 6, *and multiply the principal by the quotient taken as mills.*

### WRITTEN EXERCISES.

58. What is the interest of $84 for 63 days, at 8 %?
59. What is the interest of $122 for 48 days, at 9 %?
60. Give the interest of $84.24 for 50 days, at 12 %.
61. Give the interest of $38.80 for 36 days, at 9 %.

---

**REVIEW.**—What is stock? (16.) What is the value of a share in bank, insurance, and railroad companies? What is a certificate of stock? What is par value? Market value? When is stock at par? At premium? At discount? What is *stock-jobbing? A stock broker?* What is a partnership, or firm? (17.)

To find the exact number of days between two dates is given the following

### TABLE.

| Jan. | Feb. | Mar. | Apr. | May. | June. | July. | Aug. | Sept. | Oct. | Nov. | Dec. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 32 | 60 | 91 | 121 | 152 | 182 | 213 | 244 | 274 | 305 | 335 |
| **2** | 33 | 61 | 92 | 122 | 153 | 183 | 214 | 245 | 275 | 306 | 336 |
| **3** | 34 | 62 | 93 | 123 | 154 | 184 | 215 | 246 | 276 | 307 | 337 |
| **4** | 35 | 63 | 94 | 124 | 155 | 185 | 216 | 247 | 277 | 308 | 338 |
| **5** | 36 | 64 | 95 | 125 | 156 | 186 | 217 | 248 | 278 | 309 | 339 |
| **6** | 37 | 65 | 96 | 126 | 157 | 187 | 218 | 249 | 279 | 310 | 340 |
| **7** | 38 | 66 | 97 | 127 | 158 | 188 | 219 | 250 | 280 | 311 | 341 |
| **8** | 39 | 67 | 98 | 128 | 159 | 189 | 220 | 251 | 281 | 312 | 342 |
| **9** | 40 | 68 | 99 | 129 | 160 | 190 | 221 | 252 | 282 | 313 | 343 |
| **10** | 41 | 69 | 100 | 130 | 161 | 191 | 222 | 253 | 283 | 314 | 344 |
| **11** | 42 | 70 | 101 | 131 | 162 | 192 | 223 | 254 | 284 | 315 | 345 |
| **12** | 43 | 71 | 102 | 132 | 163 | 193 | 224 | 255 | 285 | 316 | 346 |
| **13** | 44 | 72 | 103 | 133 | 164 | 194 | 225 | 256 | 286 | 317 | 347 |
| **14** | 45 | 73 | 104 | 134 | 165 | 195 | 226 | 257 | 287 | 318 | 348 |
| **15** | 46 | 74 | 105 | 135 | 166 | 196 | 227 | 258 | 288 | 319 | 349 |
| **16** | 47 | 75 | 106 | 136 | 167 | 197 | 228 | 259 | 289 | 320 | 350 |
| **17** | 48 | 76 | 107 | 137 | 168 | 198 | 229 | 260 | 290 | 321 | 351 |
| **18** | 49 | 77 | 108 | 138 | 169 | 199 | 230 | 261 | 291 | 322 | 352 |
| **19** | 50 | 78 | 109 | 139 | 170 | 200 | 231 | 262 | 292 | 323 | 353 |
| **20** | 51 | 79 | 110 | 140 | 171 | 201 | 232 | 263 | 293 | 324 | 354 |
| **21** | 52 | 80 | 111 | 141 | 172 | 202 | 233 | 264 | 294 | 325 | 355 |
| **22** | 53 | 81 | 112 | 142 | 173 | 203 | 234 | 265 | 295 | 326 | 356 |
| **23** | 54 | 82 | 113 | 143 | 174 | 204 | 235 | 266 | 296 | 327 | 357 |
| **24** | 55 | 83 | 114 | 144 | 175 | 205 | 236 | 267 | 297 | 328 | 358 |
| **25** | 56 | 84 | 115 | 145 | 176 | 206 | 237 | 268 | 298 | 329 | 359 |
| **26** | 57 | 85 | 116 | 146 | 177 | 207 | 238 | 269 | 299 | 330 | 360 |
| **27** | 58 | 86 | 117 | 147 | 178 | 208 | 239 | 270 | 300 | 331 | 361 |
| **28** | 59 | 87 | 118 | 148 | 179 | 209 | 240 | 271 | 301 | 332 | 362 |
| **29** |  | 88 | 119 | 149 | 180 | 210 | 241 | 272 | 302 | 333 | 363 |
| **30** |  | 89 | 120 | 150 | 181 | 211 | 242 | 273 | 303 | 334 | 364 |
| **31** |  | 90 |  | 151 |  | 212 | 243 |  | 304 |  | 365 |

NOTE. — To find from the above table the number of days between two dates, the following rules are given: —

RULE I. — *When the dates are in the same year, subtract the number of days in the place of the earlier date from the number of days of the later date: the result will be the number of days required.*

II. — *When the dates are in consecutive years, subtract the number of days in the earlier date from 365, and add to the remainder the number of days in the place of the later date: the result will be the number of days required.*

III. — *When the year is leap-year, add one day to the result.*

IV. — *To find the date at which a certain number of days will expire, add the given number of days to those of the given date, and then find the date corresponding to the number in the table. 87 days after April 19 will expire on July 15 of the same year. April 19 = 109 da.; 109 + 87 da. = 196 da.; 196 da. = July 15.*

### WRITTEN PROBLEMS.

**PROBLEM.** — (*a*). How many days were there between Aug. 9, 1874, and Sept. 11, 1875? (*b*). Between July 31, 1874, and Dec. 2, 1874.

PROCESSES.

(*a.*)

365 = da. in a year.
221 = Aug. 9, 1874.
144 = difference.
254 = Sept. 11, 1875.
398 = No. days.

(*b.*)

336 da. Dec. 2, 1874.
212 = July 31, 1874.
124 days.

NOTE. — The reason for the above processes will be readily seen by referring to the Table.

Find the number of days between the following dates: —

62. Jan. 4, 1873, and Oct. 11, 1873.
63. April 9, 1875, and Nov. 28, 1875.
64. May 30, 1876, and Nov. 30, 1876.
65. Feb. 28, 1870, and Nov. 22, 1870.
66. July 4, 1859, and Sept. 3, 1860.
67. Sept. 25, 1874, and Sept. 23, 1875.
68. Aug. 3, 1872, and April 1, 1873.

69. What is the interest of $125.80 from April 6 to July 20, at 6 %?

70. What is the interest of $235.30 from Sept. 10 to Nov. 8, at 6 %?

71. What is the amount of \$85.40 from March 5 to July 1, at 6 %?

72. What is the amount of \$96.36 from Aug. 15 to Nov. 12, at 6 %?

73. What is the interest of \$435.20, at 6 %, from Nov. 10, 1875, to April 1, 1876?

74. What is the amount of \$80.35 from Dec. 10, 1874, to Feb. 26, 1875, at 6 %?

75. What is the interest of \$185.50 from May 18 to Aug. 24, at 6 %? At 7 %?

SUGGESTION. — The interest at 7 % will be $\frac{7}{6}$, or $\frac{1}{6}$ of 7 times, the interest at 6 %.

76. A note given July 10 for \$871.25 was paid Sept. 12, with interest at 6 %: what was the amount paid?

77. What is the interest of \$710.56 for 69 days, at 6 %? At 7 %? At 5 %?

78. What is the amount of \$351.25 for 102 days, at 6 %? At 5 %? At 8 %?

### DRILL CARD EXERCISES. — TABLE VIII.

**Exercise XXI.** — Give the numbers of days between the dates (column **H**) opposite the following numbers: ***1*** and ***2***; ***2*** and ***3***; ***3*** and ***4***; ***4*** and ***5***; &c.

**XXII.** — At what dates will **E** days after **H** dates expire?

---

REVIEW.— What is the base of percentage in stocks? (17.) How are all premiums, discounts, brokerages, dividends, and assessments computed? What is insurance? (18.) What is fire insurance? Marine insurance? What is a policy? (19.) An insurer, or underwriter? (20.) What is a mutual insurance company? Who is the policy-holder? What is the premium? How is it estimated? Upon how much of the value of the insured property is insurance generally taken? Give the equivalent of the amount insured. Of the premium. Of the rate per cent. What is life-insurance? (21.) What is a life policy? An endowment policy? A continued premium life policy? A ten, five, or single payment policy? How is the premium computed?

# PROPOSITIONS IN INTEREST.

## PROPOSITION I.

**43.** TO FIND THE INTEREST, WHEN THE PRINCIPAL, RATE PER CENT, AND TIME ARE GIVEN.

[This case has been fully treated in the discussion of Interest, ¶ **36** to **41.**]

RULE.—I. *To find the interest, multiply the principal by the* INTEREST *of* $1 *for the given time, at the given rate.*

II. *To find the amount, add the interest to the principal, or multiply the principal by the* AMOUNT *of* $1 *for the given time, at the given rate.*

FORMULAS. $P \times R \times T = I.$
$P + I = A.$

## PROPOSITION II.

**44.** TO FIND THE PRINCIPAL, THE INTEREST, RATE PER CENT, AND TIME BEING GIVEN.

PROBLEM.—What principal, at 6 %, will gain $22.19 in 2 yrs. 4 mos.?

PROCESS. $\$1 \times .06 \times 2\frac{1}{3} = \$.14.$
$\$22.19 \div .14 = \$158.50.$

ANALYSIS.—(1.) The interest of $1 for 2 yrs. 4 mos. at 6 % is 14 cents.

(2.) Since 14 cents is the interest of $1 for the given time, it will take as many dollars to earn $22.19 interest as 14 cents is contained times in $22.19, or $158.50.

RULE.—*Divide the given interest by the interest of one dollar for the given time and rate.*

FORMULA. $\frac{I}{R \times T} = P.$

### ORAL EXERCISES.

1. What principal will earn $12 in 2 years, at 6 %?

2. What principal will earn $30 in 2 years, at 5 %?

3. What principal will gain $3 in 1 year, at 5 %?

4. What principal will earn $24 in 3 years, at 4 %? In 1 yr. 6 mos. at 8 %?

### WRITTEN EXERCISES.

79. Find the principal, when the interest is $33.50, the rate 6 %, and the time 2 yrs. 8 mos.

80. What principal will earn $152.20, at 6 %, in 1 year, 9 mos. 15 days?

81. What principal will earn $167.40, at 8 %, in 1 yr. 6 mos. 18 days?

SUGGESTION. — $.093 × $\frac{8}{6}$ = int. of $1, at 8 %, for given time.

82. What principal will earn $145 in 3 yrs. 2 mos. 12 days, at 7 %?

83. What must I invest, at 6 %, for 1 year, 9 mos. 10 days, to earn $450?

84. What principal will earn $450 in 1 year, 9 mos. and 10 days, at 8 %?

### DRILL CARD EXERCISES. — TABLE VIII.

Exercise XXIII. — What principal will earn **E** dollars' interest in **A** yrs. **B** mo. **C** da. at **D** %?

XXIV. — What principal will earn **G** dollars' interest in **F** days, at **C** %?

---

REVIEW. — What is a mutual life-assurance company? (22.) What is a tax? (23.) What is a direct tax? An indirect tax? A poll-tax? What is real estate? Personal property? What is an assessor? A collector? What is the tax rate? (24.) What are customs, or duties? (25.) What is tonnage? A custom-house? A port of entry? A collector of the port? A tariff? A clearance? A manifest? What is a specific duty? (26.) An ad valorem duty? (27.) What is draft? Tare? Leakage? Breakage? Gross weight? Net weight? What is interest? (28.) What is *the principal*? (29.) The rate per cent? (30.) The amount? (31.)

### PROPOSITION III.

**45.** TO FIND THE PRINCIPAL, WHEN THE AMOUNT, TIME, AND RATE ARE GIVEN.

PROBLEM. — What principal, at 8 %, will amount to $28 in 2 years, 8 mos. 12 days?

PROCESS.

Int. of $1 at 6 % = $.162.
" " $1 " 8 % = .216.
Amount " $1 " 8 % = 1.216.
$28 ÷ 1.216 = $23.03

ANALYSIS. — (1.) The amount of $1 for the given time, at the given rate, is $1.216.

(2.) Since the amount of $1 is $1.216, it will take as many dollars of principal to yield the amount of $28 as $1.216 is contained times in $28, or $23.03.

RULE. — *Divide the amount by the amount of $1 at the given rate for the given time.*

FORMULA. $\frac{A}{1+(R\times T)} = P.$

#### ORAL EXERCISES.

1. What sum of money will amount to $106 in 1 yr. at 6 %? To $214 in 1 yr. at 7 %? To $230 in 3 years, at 5 %?

2. What principal will amount to $428 in 4 years, at 7 %? To $672 in 6 years, at 12 %?

#### WRITTEN EXERCISES.

85. Find the principal which will amount to $73.84 in 8 yrs. 4 mos. at 9 %.

86. What sum must I put at interest to amount to $636 in 3 yrs. 4 mos. at 6 %?

---

REVIEW. — What is simple interest? (32.) Compound interest? (33.) Legal interest? (34.) Usury? (35.) Name the terms used in interest. (36.) Give their equivalents in percentage. What term is used in interest which is not used in percentage? Give the rule for finding the interest of any sum, at any rate per cent, for any number of years and months. (38.) Give the rule for calculating interest by the six per cent method. (40.)

87. What sum of money, put at interest at 5 %, will, in 2 yrs. 4 mos. 24 days, amount to $86.24?

88. What principal will amount to $132.75 in 124 days, at 18 %?

89. How much money must I invest, at 8 %, for 3 yrs. 8 mos., to receive at that time $4656?

### DRILL CARD EXERCISES. — TABLE VIII.

Exercise XXV. — What principal, at **A** %, will amount to **F** dollars in **B** mo. and **C** days?

XXVI. — How much must I invest at **B** % for **D** years to amount to **E** dollars?

### PROPOSITION IV.

***46.*** TO FIND THE RATE PER CENT WHEN THE INTEREST, PRINCIPAL, AND TIME ARE GIVEN.

What rate of interest must I charge on a loan of $125 for 1 yr. 6 mos. to earn $7.50?

PROCESS. $1.25 × 1½ (yrs.) = $1.875.
$7.50 ÷ $1.875 = 4 = 4 %.

ANALYSIS. — (1.) The interest on $125, at 1 %, for 1 yr. 6 mos. (1½ yrs.) is $1.875.

(2.) Since $125 in 1 yr. 6 mos. requires a rate of 1 % to earn $1.875, to earn $7.50 it will require a rate as many times 1 % as $1.875 is contained times in $7.50, or 4 = 4 %.

RULE. — *Divide the given interest by the interest of the principal, at 1 %, for the given time.*

FORMULA. $\frac{\mathrm{I}}{\mathrm{P} \times 1\% \times \mathrm{T}} = \mathrm{R}.$

### ORAL EXERCISES.

At what rate will

1. \$400 gain \$40 in 1 year? In 2 years?
2. \$500 gain \$70 in 1 year? In 2 years?
3. \$300 gain \$50 in 1 year? In 5 years?
4. \$600 gain \$90 in 1 year? In 6 years?

### WRITTEN EXERCISES.

90. At what rate will \$455 gain \$100.10 in 2 yrs. 9 mos.?

91. What rate must I charge to gain \$52.50 on the loan of \$875, for 1 year, 4 mos.?

92. A man receives \$65 interest for the use of \$600 for 3 yrs. 7 mos. 10 da.: what is the rate %?

93. At what % must \$950 be loaned to gain \$126.66⅔ in 3 yrs. 4 mos.?

94. At what % will \$36, in 4 yrs. 5 mos. and 26 da., earn \$8.08 interest?

95. Mr. L. T. Gage, July 22, 1870, loaned \$700; Jan. 22, 1871, he received as principal and interest \$735: what rate of interest did he charge?

96. I invested \$2400 as a silent partner in a grocery-store: at the end of 2½ years I found I had made \$1600. What rate of interest did my investment pay annually?

### DRILL CARD EXERCISES. — TABLE VIII.

**Exercise XXVII.** — At what rate % will **F** dollars earn **E** interest in **A** yrs. **B** mo.?

**XXVIII.** — At what rate % will **F** dollars earn **G** interest in **B** mo. **C** da.?

### PROPOSITION V.

**47.** TO FIND THE TIME, WHEN THE PRINCIPAL, INTEREST, AND RATE PER CENT ARE GIVEN.

PROBLEM. — The interest of $900, at 7% was $170.10: what was the time for which it was loaned?

PROCESS (*a*).
$.07 × 900 = $63.
$170.10 ÷ 63 = 2.7 yrs.
2.7 yrs. = 2 yrs. 8 mos. 12 days.

PROCESS (*b*).

```
$63.00) $170.10 (2 yrs.
          126.00
          ------
           44.10
              12 (mos.)
          ------
  63.00) 529 20 (8 mos.
          504.00
          ------
           25.20
               30 (da.)
          -------
  63.00) 756.00 (12 days.
           63.00
          ------
          126.00
          126.00
```

ANALYSIS. — (1.) *a*. The interest of $900 for 1 year, at 7 %, is $63.

(2.) Since $63 is the interest of $900 for 1 year, it will take as many years for $900 to earn $107.10 interest as $63 is contained times in $107.10., or 2.7 years, or 2 yrs. 8 mos. 12 days.

(*b*.) This process is introduced to give a brief method of finding the months and days without reducing the remainder to the decimal fraction of a year.

RULE. — *Divide the given interest by the interest of the principal at the given rate for* 1 *year.*

FORMULA. $\frac{I}{P \times R} = T.$

NOTE. — When the remainder equals or exceeds one-half a day, add one day to the time; if less, drop it.

### ORAL EXERCISES.

1. In what time will \$50 gain \$10, at 4 %? At 5 %?

2. In what time will \$200 earn \$30, at 5 %? At 6 %?

3. In what time, at 3 %, will \$600 earn \$36? In what time will it earn \$72?

4. In what time, at 4 %, will \$500 earn \$20? In what time will it earn 100?

5. How long, at 7 %, must I invest \$300 to earn \$42? To earn \$105?

6. How long, at 10 %, must \$400 be loaned to earn \$90? How long must \$600 be loaned?

### WRITTEN EXERCISES.

97. In what time will \$375 earn \$94.50, at 7 %?

98. How long will it take \$720 to gain \$76.32, at 6 %?

99. At 8 %, how long must I invest \$635 to gain \$58.85?

100. At 4½ %, how long must I loan \$425.50 to gain \$76.59?

101. At 7 %, how long will it take \$425 to double itself?

102. At 3 %, how long will it take \$500 to double itself?

103. How long will it take \$600 to double itself, at 5 %? At 8 %? At 9½ %? At 15 %?

104. How long will it take any sum of money to double itself at 7 %? At 9 %? At 20 %? At $9\frac{1}{11}$ %?

---

**Review.** — What does rate per cent show? (1.) What is the denominator of the fraction in rate per cent? What is the numerator of the fraction? Define base. (2.) Percentage. (3.) Amount. (4.) Difference. (5.) Give the letters representing base, &c. (6.) What is the cost of a thing? (7.) The selling-price? When is the selling-price the amount? When the difference? What is profit, or gain? Loss? *Net profit?*

## DRILL CARD EXERCISES. — TABLE VIII.

**Exercise XXIX. — At B %, how long will it take E dollars to earn G dollars' interest?**

**XXX. — At A %, how long will it take G dollars to earn F dollars' interest?**

**XXXI. — At D %, how long will it take E dollars to double?**

## COMPOUND INTEREST.

**48.** ***Compound Interest**** **is the sum paid as interest on both principal and interest; the two being united at stated intervals to form a new principal.**

Interest is usually compounded annually, semi-annually, or quarterly, as may be agreed upon.

Find the amount of $350 compounded annually, at 6 %, for 3 years, 6 mos. 12 days. What is the compound interest?

PROCESS.

$350  
1.06  
371.00 = amt. for 1st yr.  
1.06  
393.26 = amt. for 2d yr.  
1.06  
416.86 = amt. for 3d yr.  
1.032  
430.20 = amt. 6 mos. 12 da.  
350. = principal.  
80.20 = comp. interest.

ANALYSIS. — (1.) Since the amount of $1 for 1 year, at 6 %, is $1.06, the amount of $350 for the same time is 350 times $1 06, or $371. This amount, due at the end of the first year, becomes the principal for the second year.

NOTE. — Since the analysis for each year, or fractional part of a year, is essentially the same, it will not be repeated.

(2.) Since the final amount at the close of the fractional part of the fourth year is $430.20, the compound interest will equal this amount less the principal, $350, or $80.20.

RULE. I. *Find the amount of the given principal for the first interval of time, which amount is a new principal for the next interval. Proceed in the same way for each*

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* See note, ¶ 33.

*consecutive interval. The last amount will be the amount at compound interest.**

II. *Subtract the given principal from the last amount; and the difference will be the compound interest.*

WRITTEN EXAMPLES.

105. What is the amount of $300 for 3 years, at 7 %, compounded annually? What is the compound interest?

106. What is the compound interest of $6200, at 9 %, for 4 years, compounded annually?

107. What is the amount of $325, at 7½ %, for 3 years, compounded annually?

108. What is the compound interest on $250 for 2 yrs. 4 mos. 18 da., at 8 %, compounded annually?

109. What is the compound interest of $325.75, at 6 %, for 1 yr. 3 mos., compounded semi-annually?

110. What is the compound interest on $500, at 8 %, for 2 years, compounded quarterly?

### ANNUAL OR SEMI-ANNUAL INTEREST.

**49. *Annual or Semi-Annual Interest*** is interest on the principal, payable at the end of each year or half-year.

Notes, bonds, or mortgages bearing annual, semi-annual, or quarterly interest, must contain the words, "with interest annually," or "semi-annually," or "quarterly," as the case may be. Unless a *demand* is made for the interest installments when due, or a "coupon" note or other instrument is given to or held by the creditor for the same, he can not, upon the maturity of the note, bond, or mortgage, collect interest on the unpaid interest installments. (See note, ¶ *33*, 3.)

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* In the same manner, find the amount for the last interval, when less than the specified time.

PROBLEM. — Find the final value of a note for $600, interest payable annually, at 6 %, with interest unpaid for 4 years, 4 months, and 12 days.

PROCESS.

(1.) Int. of $600 for 1 year, at 6 % = $600 × .06 = $36.00.
" " 33 " 3 " 4 mos. 12 da.
" " 36 " 2 " 4 " 12 "
" " 36 " 1 " 4 " 12 "
" " 36 " 4 " 12 "
" " 36 " 7 " 5 " 18 " = $36 × .448 = $16.13
" " 600 " 4 " 4 " 12 " = 600 × .262 = 157.20

(2.) Annual interest = $173.33
600 = prin.
$773.33 = final val.

ANALYSIS. (1.) — The interest of the principal for the given time is $157.20. The interest of the principal for the 1st year remaining unpaid draws interest for 3 yrs. 4 mos. 12 da.; the 2d year's interest, for 2 yrs. 4 mos. 12 da.; the 3d year's interest, for 1 yr. 4 mos. 12 da.; and the 4th year's interest, for 4 mos. 12. da., making an equivalent of $36 (one year's interest) on interest for 7 yrs. 5 mos. 18 da., which equals $16.13.

(2.) Since the annual interest is $173.33 ($16.13 + $157.20), and the principal is $600, the final value is $173.33 + $600 = $773.33.

RULE. — *Compute the interest on the principal for the specified time; then compute the interest on one year's interest of the principal for the sum of the times for which interest is due and unpaid. Add the sum of this interest to the principal for the required amount.*

WRITTEN EXERCISES.

111. I borrowed of Prescott & Prescott $1000 for 5 years, at 12 %, interest payable annually. I gave for *said interest* annual interest-notes at 12 %: what will

be due at the maturity of the note, if I fail to pay the interest-notes?

112. What will be due on a coupon bond of $5000, running 5 years, interest payable semi-annually, at 7 %, if the coupons are not collected until the maturity of the bond?

113. The interest at 6 %, payable annually, on a note for $250, remained unpaid for 3 years, 8 mos., 18 da. How much interest was then due?

114. What is the amount of a note, interest payable annually, for $880, at 6 %, for 2 yrs. 7 mos. 10 da., interest unpaid?

115. Find interest due, promised annually, on $378, at 5 %, for 4 yrs. 6 mo. 9 da.

116. What is the interest due on a note of $1000 for 3 years, at 6 %, simple interest? At 6 %, annual interest, interest unpaid? At 6 %, compound interest?

## PARTIAL PAYMENTS.

***50. A Partial Payment*** is the payment of part of the amount due on a note or obligation.

***51. A Promissory Note*** is a written promise to pay a specified sum of money.

1. The **Maker** or **Drawer** of the note is the person who promises to pay.

2. The **Payee** is the person to whom the money is to be paid.

***52. The Face of the Note*** is the amount promised to be paid.

1. To make a note valid in law, the words "value received" must be inserted.

2. If drawn "to order," the payee must write his name upon the back of the note when payment is made.

3. If the note is drawn "or bearer," it does not require indorsement, and may be paid to any one holding it.

4. If neither "to order" nor "or bearer" is inserted in the note, it is payable only to the payee. Such notes are called *Non-negotiable.*

5. Notes given to merchants for goods are usually drawn payable to the order of the maker. Such notes may be negotiated without the indorsement of the "owner" or "holder:" consequently no one becomes responsible except the maker.

6. A note does not draw interest before maturity, unless the words "with interest" are inserted: in which case, interest is reckoned from date, unless it is expressly stated when it is to commence.

***53. An Indorsement*** is the acknowledgment of a partial payment written upon the back of the note.

1. The term *indorsement* is also applied to the writing of the name of the payee upon the back of a note drawn "to order." If the name only is written, it is called a *general indorsement,* and makes the note payable to bearer.

2. A *special indorsement* directs the note to be paid to some person named, or to his order.

3. The person who signs his name upon the back of the note is called the *indorser,* and is made responsible for the payment of the note, if the maker fails to pay it.

***54. Supreme Court Rule.*** — Interest on partial payments is reckoned either by the *Supreme Court Rule* or by the *Mercantile Rule.*

The following rule for computing interest on notes, bonds, or mortgages, &c., running for more than one year, has been laid down by the United States Supreme Court, and is adopted by all the States, excepting New Hampshire and Connecticut.

RULE. — I. *Compute the interest on the principal to the time of the first payment: if the payment equals or exceeds this interest, add the interest to the principal, and subtract the payment from the sum: the remainder forms a new principal.*

II. *If the payment is less than this interest, compute the interest on the principal to the time of that payment,* *which, with all preceding payments, will exceed the interest on the principal to the time of that payment; then add the interest to the principal, and subtract the sum of the payments: the remainder will form a new principal, with which proceed as before.*

### WRITTEN EXERCISES.

**$500.** *Trenton, N. J., Aug.* 10, 1871.

*Two months after date, for value received, I promise to pay J. C. Armstrong, or order, the sum of five hundred dollars, with interest at* 6 *per cent.*

*Alfred Peyton.*

Indorsed as follows: Dec. 1, 1871, $40; June 1, 1872, $12; Aug. 1, 1872, $30; May 7, 1873 $63: what was due Oct. 21, 1874.

### PROCESS.

**Note.** — Arrange the details of the note for solution as follows: —

| | | | | Intervals. | | | Interest | |
|---|---|---|---|---|---|---|---|---|
| | Yrs. | Mos. | Da. | Yrs. | Mos. | Da. | of $1, at 6 %. | Payments. |
| Date of note, | 1871 | 8 | 10 | | | | | |
| " 1st paym't, | 1871 | 12 | 1 | 0 | 3 | 21 | $.0185 | $40 |
| " 2d paym't, | 1872 | 6 | 1 | 0 | 6 | 0 | .03 | 12 |
| " 3d paym't, | 1872 | 8 | 1 | 0 | 2 | 0 | .01 | 30 |
| " 4th paym't, | 1873 | 5 | 7 | 0 | 9 | 6 | .046 | 63 |
| Settlement, | 1874 | 10 | 21 | 1 | 5 | 14 | .087⅓ | |

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**Review.** — Give the equivalents of cost price. (7.) Selling-price. Gain or loss. Rate per cent. What is commission? (8.) What is a factor? A broker? An agent? A consignment? Who is the consignor? Consignee? Define net proceeds. What is an account sales? Give the formula for finding the percentage when the base and rate are given. (9.) For finding the amount when the base and rate are given. (10.) For finding the difference. For finding the rate when the base and percentage are given. (11.) For finding the base when rate and percentage are given. (12.) For finding the base when rate and amount are given. (13.) When rate and difference are given. What is a company? (14.) A corporation? (15.) A charter? What is stock? (16.) What is the usual value of shares in banks, insurance, and railroad companies? A certificate of stock? (16.) What is par value? Market-value?

**ANALYSIS.**

| | | | |
|---|---|---|---|
| Face of the note, or 1st principal, | | | $500.00 |
| Interest for 1st interval (less than pay't), | | | 9.25 |
| Amount, | | | $509.25 |
| 1st payment, | | | 40.00 |
| 2d principal, | | | $469.25 |
| Int. for 2d interval, | $14.08 (more than pay't) | | |
| 2d payment, | | $12.00 | |
| Int. for 2d interval, | 4.69 | | |
| 3d payment, | | 30.00 | |
| Sum of interests less than sum of payments, | $18.77 | $42.00 | 18.77 |
| Amount, | | | $488.02 |
| Sum of payments, | | | 42.00 |
| 3d principal, | | | $446.02 |
| Int. for 4th interval, | | | 20.52 |
| Amount, | | | $466.54 |
| 4th payment, | | | 63.00 |
| 4th principal, | | | $403.54 |
| Int. for 5th interval (to settlement), | | | 35.24 |
| Amount due, | | | $438.78 |

117. What is due on the following note, July 18, 1876?

$840. *New York*, *Sept.* 6, 1874.

*For value received, I promise to pay to J. C. Castle, or order, eight hundred and forty dollars, with interest at* 7 %. *Amos Frazee.*

Indorsements: Oct. 1, 1875, $43.75; Dec. 19, 1875, $125; May 6, 1876, $396.40.

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REVIEW. — When is stock at par? (16.) When at premium? When at discount? What is stock-jobbing? A stock broker? What is a partnership? (17.) What is the base of percentage in stocks? How are all premiums, discounts, &c., computed? What is insurance? (18.) Name the kinds of insurance. What is fire *insurance? Marine insurance?* What is a policy? (19.) An insurer, or *underwriter? (20.)*

118. What is due at settlement, Feb. 20, 1876, on the following note, indorsed as follows: Nov. 6, 1873, $163.20.; June 24, 1874, $137.50; Oct. 12, 1875, $15.25?

$694. *Philadelphia, Oct.* 21, 1873.

*For value received I promise to pay on demand to H. G. Parker & Co., or order, the sum of six hundred and ninety-four dollars, with interest at* 6 %.

*E. G. Patton.*

119. A note for $650, dated May 20, 1868, is indorsed as follows: May 20, 1869, $165; July 14, 1870, $18; June 30, 1871, $265.13. What is due Aug. 30, 1878, interest at 7 %?

120. Upon a note for $250, dated June 1, 1869, due two years after date, there were the following indorsements: July 20, 1871, $9.50; Oct. 22, 1872, $160; Jan. 16, 1874, $75. What is due Nov. 16, 1875, interest 6 %, after maturity?

121. A note, at 6 % interest, dated Nov. 20, 1873, for $283.50, is indorsed as follows: June 26, 1874, $83.50; Feb. 14, 1875, $7.30; Nov. 10, 1875, $186. If this note commenced to draw interest three months after date, what was due April 16, 1876?

122. A note, at 7 %, for $1500, dated April 9, 1875, was indorsed as follows: Aug. 12, 1875, $327.40; June 30, 1876, $26.50; Oct. 30, 1876, $265. What is due at settlement, Jan. 1, 1877?

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**REVIEW.** — What is a mutual insurance company? Who is the policy-holder? What is the premium? How is it estimated? What part of the value of property is usually insured? Give the equivalents.

The annexed problems will illustrate the several forms of notes given in mercantile transactions.

123. DEMAND NOTE (*not negotiable*).

$500. *Chicago, Ill., Sept.* 11, 1874.

*On demand I promise to pay Hadley Brothers five hundred dollars, with interest at* 6 %, *value received.*

*Thos. Scholes.*

What was due Oct. 1, 1876, at which time the demand was made?

NOTE.—Had no mention of interest been made in the above note, it would have drawn interest only from the date on which the *demand* was made.

124. NEGOTIABLE NOTE.

$380. *Cincinnati, O., April* 1, 1870.

*One year after date I promise to pay Alonzo Reed, or order, three hundred and eighty dollars, with interest, at* 6 %, *value received.* *O. H. Hall & Co.*

Rec'd on the within note:—

Aug. 7, 1870, $35.
Nov. 12, 1871, $40.
Jan. 13, 1872, $100.

What was due Nov. 11, 1872?

NOTE.—Had no mention of interest been made in the above note, interest would have commenced only at the expiration of the year, or on the maturity of the note. Payments made before a note is due draw interest until the maturity of the note.

125. NOTE PAYABLE TO BEARER.

$250. *New York, July* 7, 1874.

*Six months from date, for value received, I promise to pay C. M. Harrison, or bearer, two hundred and fifty dollars.* *Isaac Vanness.*

What is due on the above note July 4, 1876, interest being 7 %?

NOTE.—It will be observed that the above note is not due until six months *after date, at* which time it commences to draw interest.

126.

### NOTE PAYABLE IN BANK.

$280.20. *Topeka, Kan., April* 6, 1876.

*Ninety days after date, I promise to pay J. K. Hudson, or order, two hundred eighty* $\frac{20}{100}$ *dollars at the Topeka Savings Institution, value received.* *C. W. Cone.*

What was due Nov. 11, 1876; interest at 7 %?

### MERCANTILE RULE.

***55. The Mercantile Rule*** is generally adopted in the settlements of accounts past due, and of notes for not more than one year.

RULE. — *From the amount of the debt at the date of settlement, subtract the sums of the amounts of the payments at the same date.*

PROBLEM. — What is due, at the end of one year, on the following note?

$610. *Harrisburg, Penn., Oct.* 1, 1876.

*One year from date, I promise to pay to the order of John Collins the sum of six hundred and ten dollars, with interest at* 6 %, *value received.* *James Dupont.*

The indorsements are as follows: Nov. 10, 1876, $110; Jan. 20, 1877, $63; May 25, 1877, $112.50; July 5, 1877, $85.

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REVIEW. – What is life-insurance? (21.) A life policy? An endowment policy? A continued premium life policy? A ten, five, or single payment policy? How is the premium calculated? What is a mutual life-assurance company? (22.) What is a tax? (23.) A direct tax? An indirect tax? A poll-tax? Real estate? Personal property? An assessor? A collector? What is the tax rate? (24.) What are customs, or duties? (25.) What is tonnage? A custom-house? A port of entry? A collector of the port? A tariff? A clearance? A manifest? What is a specific duty? (26.) An ad valorem duty? (27.) What is draft? Tare? Leakage? Breakage? Gross weight? Net weight? What is interest? (28.) Define principal. (29.) Rate per cent. (30.) The amount. (31.) Simple interest. (32.) Compound interest. (33.) When is simple interest due? Annual interest?

## BANKING.

***56. A Bank*** is an incorporated company, authorized by law to receive deposits, loan money, and issue drafts, bills of exchange, and promissory notes, which circulate as money.

1. A **Bank Note** is a printed note issued by the bank, payable on demand, without interest, used as money.

2. A **Bankable Note** is a note of an individual, or firm, made payable at a bank.

***57. The Days of Grace*** are three days added by law to the time specified in a note.

***58. The Maturity*** of a note is the time when it becomes due, and includes the three days of grace.

***59. Bank Discount*** is the simple interest on the face of a note to maturity.

1. Banks require the payment of interest when the money is borrowed, instead of waiting until the money is due; consequently they *discount*, or *deduct* the interest from the *face* of the note: hence the name *bank discount*. The balance, after deducting the interest, is called the PROCEEDS, or AVAILS.

2. In computing bank interest or discount, the three *days of grace* must always be added to the time mentioned in the note; thus, upon a note drawn for 90 days, interest must be calculated for 93 days.

DIFFERENCE BETWEEN BANK DISCOUNT AND TRUE DISCOUNT. — *Bank Discount* is simple interest upon the face of the note, paid in advance, for 3 days more than the time.

*True Discount* is the interest of a sum of money, which, put at interest, will amount to the face of the note in the specified time, or it is the difference between the principal and the amount.

REVIEW. — What is legal interest? (34.) Usury? (35.) Name the terms used in problems in interest, and their corresponding terms in percentage. (36.) What element is used in interest that is not used in percentage? Give the formula for finding the interest when the principal, rate per cent, and time are given. (43.) For finding the principal when the interest, rate per cent, and time are given. (44.) For finding the principal when the amount, time, and rate are given. (45.) For finding the rate when the interest, principal, and time are given. (46.) For finding the time when the principal, interest, and rate are given. (47.) What is a partial payment? (50.) A promissory note? (51.) Who is the maker, or drawer, of a note?

### *60.* — TO FIND THE BANK DISCOUNT AND THE PROCEEDS OF A NOTE.

**PROBLEM.** — What shall I receive from a bank for a note for $280, due in 60 days, discounted at 6 %?

PROCESS. — $.0105 × 280 = $2.94 = bank discount.
$280 — 2.94 = $277.06 = proceeds.

ANALYSIS. — (1.) Since the bank charges interest for three days more than the time specified, the bank discount will be the interest of $280 for 63 days, at 6 %, which equals $.0105 × 280 = $2.94.

(2.) If the bank charges $2.94 for the use of the money, I shall receive the face of the note less the bank discount, or $280 — $2.94 = $277.06.

ANALYTICAL STEPS. — (1.) *Find the interest, or bank discount.*
(2.) *Find the proceeds.*

**RULE.** — I. *To find bank discount, compute the interest on the face of the note for three days more than the specified time.*

II. *To find the proceeds, subtract the bank discount from the face of the note.*

#### WRITTEN EXERCISES.

Find the bank discount of: —

132. $433.39 for 2 yrs. 4 mos. 3 days, at 6 %.
133. $387 for 11 mos. 15 days, at 8 %.
134. $184 for 5 mos. 8 days, at 6 %.
135. $1292.50 for 30 days, at 8 %
136. $4518.10 for 108 days, at 10 %.

137. Find the proceeds of the following notes, discounted at a bank: —

(1.) $350 for 30 days, at 6 %.
(2.) $200 for 60 days, at 7 %.
(3.) $444 for 15 days, at 12 %.
(4.) $500 for 46 days, at 9 %.
(5.) $800 for 66 days, at 18 %.
(6.) $360 for 42 days, at 10 %.
(7.) $400 for 55 days, at 8 %.
(8.) $1000 for 3 mos. at 6 %.

Note. — To find the maturity of a note, when the time is specified in days, count forward in days from the date. When the time is specified in months, count forward in months; thus, 93 days from May 30 is Aug. 31; 4 months from May 30 is Sept. 30. The bank discount is computed on the actual number of days.

138. What is the bank discount, at 8 %, on a note for $445.70, dated July 31, 1876, due in 90 days?

139. What will be the proceeds of a note for $581.95, due in 4 months, and discounted at 9 %?

140. What is the interest on a note for $5897.50, due 4 months from date, at 5 %?

141. What were the avails of a note for $163.36, dated Aug. 13, 1875, due Sept. 11, 1875, and discounted at 9 %?

142. What were the proceeds of a note for $156.36, dated Aug. 1, 1874, due Dec. 11, 1874, and discounted Sept. 5, 1874, at 9 %?

143. What were the proceeds of a note for $245, dated Jan. 11, 1875, due July 4, 1875, and discounted April 11, 1875, at 18 %?

144. What was the bank discount of a note for $729, dated May 10, 1876, due Nov. 25, 1876, and discounted July 7, 1876, at 7 %?

145. What are the proceeds of a note for $217.40, at 90 days, dated May 30, and discounted June 10, at 6 %?

146. Bought a horse for $275, and sold him for a note at 60 days, for $350, which I had discounted at 6 %. What was the gain?

147. Discounted a note of $304.72 for 90 days, at 6 %, and invested the proceeds in flour at $10 per bbl. How *many barrels did* I purchase?

**148.** What are the proceeds of a note of $500, due in 90 days, bearing interest at 9 %, which I discounted at the same rate at a bank?

NOTE. — When a note bearing interest is discounted at a bank, the discount is computed on the amount of the note at maturity: thus the interest of $500 for 93 days, at 9 %, equals $11.625; the amount = $511.625. The bank discount is the interest on this amount for the same time.

**149.** I have a note for $704.70, dated June 30, 1876. It is due Oct. 7, 1876, and bears interest, at 7 %, from date. What will be the proceeds, if discounted at a bank, Aug. 15, 1876, at 18 %?

**150.** A note of 3 months, dated July 16, 1873, for $632, is discounted at 6 %, Aug. 10. What are the proceeds?

**151.** Bought a horse for $270 cash, and sold him for a note at 60 days, for $355, which I had discounted at 6 %. What did I gain?

**152.** Discounted a note of $309.59 for 90 days, at 12 %, and invested the proceeds in flour at $12 per barrel. How many barrels of flour did I purchase?

### *61.* TO FIND THE FACE OF A NOTE WHEN THE TIME, RATE, AND PROCEEDS ARE GIVEN.

PROBLEM. — For what sum shall I draw a note at 60 days, discounted at 6 %, to realize $600?

PROCESS. — $1 — $.0105 = $.9895 = proceeds of $1.
$600 ÷ $.9895 = $606.37 = face of the note.

ANALYSIS. — (1.) Since the interest of $1 for the given time, at the given rate, equals $.0105, the proceeds of $1 will equal the difference of $1 and $.0105, or $.9895.

(2.) If $.9895 is the proceeds of $1, then $600 will be the proceeds of *as many dollars* as $.9895 is contained times in $600, or $606.37.

RULE. — *Divide the proceeds, or given sum, by the proceeds of* $1, *at the given rate.*

Find the face of a note, so that the proceeds may be as follows: —

153. $612 for 2 mos. 18 da., discounted at 6 %.

154. $354 for 9 mos. 3 da., discounted at 8 %.

155. $683.50 for 4 mos., discounted at 7 %.

156. $325.75 for 60 da., discounted at 5 %.

157. $37.50 for 90 da., discounted at 9 %.

158. What should be the face of a note at 60 da. to realize $450, discount at 10 %?

159. What must be the face of a note at 90 da., discount at 8 %, to realize $95.75?

160. For what sum must I draw a note for 90 days, so that the proceeds, at 8 %, may be $500?

161. What must be the face of a note at 60 days, to realize $560 when discounted at 4 %?

162. For what sum shall I draw my note for 60 days, to realize $350, at 6 % discount?

163. What must be the face of a note at 90 days, to realize $625 when discounted at 7 %?

164. What must be the face of a note payable in 3 mos., that I may realize $280 when discounted at 8 %?

165. What must be the face of a note payable in 30 days, to realize $1500 when discounted at 6 %?

166. Bought merchandise for $3625 cash. For what sum must I give my note at 3 mos. to obtain that sum from the bank, discount at 6 %?

167. A man bought 320 bbls. of apples at $2.75 per barrel, on 15 days' time: he sold them the same day at $3.25 for a note at 90 days: he discounted the note at 12 % to meet his own obligation. What did he make *by the transaction?*

### PRESENT WORTH.

**62. *The Present Worth*** of any note or debt due at some future time, without interest, is such a sum as, placed at interest for the time at a given rate, would amount to the debt, or face of the note.

**63. *Discount*** is an allowance made for the payment of the debt or note before it is due. It is the difference between the face of the note or debt, and the present worth.

1. **Commercial Discount** is an allowance made to the purchaser on the asking or list price of any article.

2. **Bank Discount.** When the interest on a note or debt due in a given time is paid in advance, the amount paid is called "bank discount."

**64.** TO FIND THE PRESENT WORTH AND DISCOUNT OF A NOTE OR DEBT DUE AT A FUTURE TIME WITHOUT INTEREST.

PROBLEM. — What discount should be made for the immediate payment of a debt of $275, payable in 9 months at 6 %?

PROCESS. $275 ÷ $1.045 = $263 16 P W.
$275 — $263.16 = $11.84 D.

ANALYSIS. — (1.) The amount of $1 for 9 mos. at 6 % is $1.045.

(2.) Since $1.045, due 9 mos. hence, has a present worth of $1, $275, due in 9 months, will have a present worth of as many dollars as $1.045 is contained times in $275, or $263.16.

If the present worth of $275 is $263.16, the discount will be $275 — $263.16, which is $11.84.

RULE. — I. *Divide the face of the debt by the amount of* $1 *for the given rate and time: the quotient will be the present worth.*

II. *Subtract the present worth from the face of the debt, and the difference will be the discount.*

FORMULAS. — (1.) $\frac{F}{1+(R \times T)} = P.W.$

(2.) $F - P.W = D.$

168. Find the present worth, at 6 %, of $185, due in 4 mos.

169. Find the present worth, at 6 %, of $364, due in 8 mos.

170. Find the discount, at 7 %, of $298.91, due in 9 mos.

171. Find the discount, at 6 %, of $702.65, due in 4 mos. 6 da.

172. What is the present worth, at 10 %, of $366.40, due in 1 yr. 3 mos.? What is the discount?

173. What is the present value, at 8 %, of $72.68, due 8 mos. 11 da. hence?

174. What is the discount, at 6 %, on a note for $995.76, due in 2 yrs. 6 mos. 15 da.?

NOTE. — Unless otherwise mentioned, true or real discount is understood.

175. What is the bank discount on the same note, at the same rate and time?

176. What is the present worth, at 6 %, of $395.31, due in 2 yrs. 4 mos. 15 da.?

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REVIEW. — What is the face of a note? (52.) What is an indorsement? (53.) Give the supreme court rule. (54.) The mercantile rule. (55.) What is a bank. (56.) A bank note? A bankable note? What are days of grace? (57.) What is the maturity of a note? (58.) What is bank discount? (59.) What is the difference between bank discount and true discount? Give the rules for finding the bank discount, and the proceeds of a note. (60.) For finding the face of a note when the time, rate, and proceeds are given. (61.) What is present worth? (62.) *Discount? (63.) Commercial discount?* Give the rules for finding the present worth *and discount of a note? (64.)*

177. B owes A $291.33, payable in 6 mos. without interest. B, wishing to pay A immediately, discounts his note at a bank, at 7 %, for an amount, the proceeds of which shall be sufficient to pay the present worth of his debt at the same rate. What is the face of the note? How much will he lose by so doing?

178. What is the difference between the discount and interest of $800 for 90 da. at 7 %? Between the discount and bank discount?

179. I have two offers for my house; one of $6200 cash, and the other of a note for $6800 for 2 yrs. 6 mos. without interest. Would I gain, or lose, and how much, by accepting the latter offer, money being worth 7 %?

### DRILL CARD EXERCISES. — TABLE VIII.

Exercise XXXII. — Find the present worth of **E** dollars for **A** months, **B** days, at **C** %.

XXXIII. — Find the discount of **G** dollars for **B** months and **C** days, at **A** %.

XXXIV. — Find the bank discount of **F** dollars for **A** months, including days of grace, at **B** %.

XXXV. — Find the bank discount of **G** dollars for **F** days, including grace, at **D** %.

## ANALYSIS OF INTEREST.

### I. Definitions.

1. INTEREST.
2. PRINCIPAL.
3. RATE PER CENT.
4. AMOUNT.
5. SIMPLE INTEREST.
6. COMPOUND INTEREST.
7. ANNUAL INTEREST.
8. LEGAL INTEREST.
9. USURY.
10. EQUIVALENT TERMS.
    (*a.*) *Principal.*
    (*b.*) *Rate.*
    (*c.*) *Interest.*
    (*d.*) *Amount.*
    (*e.*) *Time.*

## II. Propositions in Interest.

1. **Proposition I.**
   *Formula.*
2. **Proposition II.**
   *Formula.*
3. **Proposition III.**
   *Formula.*
4. **Proposition IV.**
   *Formula.*
5. **Proposition V.**
   *Formula.*

## III. Compound Interest.

1. **Analysis.**
2. **Rule.**

## IV. Annual Interest.

1. **Analysis.**
2. **Rule.**

## V. Promissory Notes.

1. **Partial Payments.**
2. **Maker or Drawer.**
3. **Payee.**
4. **Face.**
5. **Indorsements.**
6. **Supreme Court Rule.**
7. **Mercantile Rule.**

## VI. Banking.

1. **Bank Note.**
2. **Bankable Note.**
3. **Days of Grace.**
4. **Maturity of Note.**
5. **Bank Discount.**
   (*a.*) ***Analysis.***
   (*b.*) ***Rule.***

## VII. Discount.

1. **Commercial Discount.**
2. **Bank Discount.**
3. **Present Worth.**
   (*a.*) ***Analysis.***
   (*b.*) ***Rule.***

### TEST QUESTIONS FOR EXAMINATION.

**1.** What does the rate per cent show? What is meant by one hundredth of a thing? By 5 hundredths? By 5 per cent? How are the words *per cent* represented?

**2.** When I say, "3 % of $5 is 15 cts.," which is the base? which the rate? which the percentage? What is the base?

**3.** What is the percentage? To what is the term "percentage" applied?

**4.** What is the sum of the base and percentage called? What is the amount?

**5.** What is the difference? In the statement, "$202 is 2 % more than the cost of my horse," which is the base? which the rate? which the amount? In the statement, "$198 is 2 % less than the cost of my horse," which is the base? which is the rate? which is the difference? If the base is concrete, what is the percentage? Of what denomination is the rate?

**6.** What often stands for the term "base"? Percentage? Rate per cent? Amount? *Difference?*

**7.** What is meant by the cost? what by the selling price? When is the selling price the amount? When is it the difference? What is usually the base in profit and loss? What is meant by profit or gain? What is meant by loss? What is *net profit?*

**8.** Define commission. What is the base in commission? What is meant by a factor? By what other name is a factor sometimes called? What is a broker? How does a broker differ from a factor or agent? What is a consignment? What is understood by the term "consignor"? Which is the consignee? What are net proceeds? What is an account sales? How is commission reckoned when a purchase is made? when a sale is made?

**9.** When the base and rate are given, how is the percentage found? In the statement, "4 % of 200 is 8," which is the base? which the rate? which the percentage?

**10.** When the base and rate are given, how is the amount found? How is the difference found? Illustrate by examples.

**11.** When the base and percentage are given, how can the rate be found? Illustrate by an example.

**12.** When the rate and percentage are given, how can the base be found? Illustrate by an example.

**13.** When the rate and amount are given, how can the base be found? Illustrate by an example. When the rate and difference are given, how can the base be found?

**14.** What is a company?

**15.** What is a corporation? What is a charter?

**16.** What is a stock or joint-stock company? How is the stock divided? What is the usual value of a share? What is meant by a certificate of stock, or scrip? How many shares may scrip represent? What is meant by the par value of a share? by the market value of a share? When is stock said to be at par? when at premium? when at discount? What is stock jobbing? What is a stock broker?

**17.** What is a partnership, or firm? What is the difference between a partnership and a corporation? What is the base of percentage in the computation of stocks? Premiums, discounts, brokerages, dividends, and assessments, are computed upon what value of stocks? Is stock sold at a discount necessarily sold at a loss? Is stock sold at a premium necessarily sold at a profit?

**18.** What is insurance? How many kinds of insurance are there? Name them. What is meant by fire insurance? by marine insurance? What is hull insurance? cargo insurance?

**19.** What is a policy?

**20.** What is an insurer, or underwriter? What is usually the insurer? What is a mutual insurance company? What is the policy-holder, or the insured? What is the premium? How is the premium estimated? What part of the value of property is usually insured? In percentage, what represents the base? what the percentage? what the rate?

**21.** What is life insurance? What is meant by a life policy? by an endowment policy? by a continued premium life policy? When the payment of the annual premium ceases after one, five, or ten payments, what is the policy called? How is the premium computed? How does it vary?

**22.** What is a mutual life assurance company?

**23.** What is a tax? What is a direct tax? An indirect tax? What is a poll tax? What property is included in real estate? What property is known as personal property? What is an assessor? What is an assessment roll? What is a collector?

**24.** What is a tax rate? In computing the assessment of taxes, which is the base? Which is the percentage? Which the rate?

**25.** What are customs, or duties? What is meant by tonnage? What is a custom-house? What is meant by a port of entry? What is a collector of a port? What is a tariff? A clearance? A manifest?

**26.** What is a specific duty?

**27.** What is an advalorem duty? What is draft? tare? leakage? breakage? gross weight? net weight? In what are duties payable?

## INTEREST.

**28.** What is interest?

**29.** What is the principal?

**30.** What is the rate per cent?

**31.** What is the amount?

**32.** What is simple interest?

**33.** What is compound interest? In computing compound interest, what is done with the interest at the end of each year? When is simple interest due? When can it be collected? Can compound interest be collected? When? When is annual interest due? Semi-annual interest? When can annual or semi-annual interest be collected?

**34.** What is legal interest?

**35.** What is usury?

**36.** What and how many terms are used in interest? Which term is equivalent to base? Which to rate? Which to percentage? Which to amount? What element is found in percentage that is not found in interest?

**37.** How can interest be found on any principal for any time at any per cent? Illustrate by an example.

**38.** The interest of any sum of money is equal to what?

**39.** Give the steps for finding the interest on any sum for any number of years, months, and days, at any per cent.

**40.** How is interest calculated by the 6 % method?

**41.** How is interest calculated for any rate by the 6 % method?

**42.** How is the interest found for any number of days by the 6 % method?

**43.** How can the interest be found when the principal, rate, and time, are given? Illustrate by an example.

**44.** How can the principal be found when the interest, rate, and time are given? Illustrate by an example.

**45.** How can the principal be found when the amount, time, and rate are given? Illustrate by an example.

**46.** How can the rate per cent be found when the interest, principal, and time are given? Illustrate by an example.

**47.** How can the time be found when the principal, interest, and rate are given? Illustrate by an example.

**48.** Define compound interest. How is compound interest calculated? Illustrate by an example.

**49.** Define annual or semi-annual interest. Give the method of computing annual or semi-annual interest. Illustrate by an example.

**50.** What is meant by a partial payment?

**51.** Describe a promissory note. Who is the maker or drawer? Who is the payee? Who may be the holder or owner?

**52.** What is the face of the note? What is necessary to make a note valid in law? When a note is drawn to order, what must be done at the time of payment? Is the same necessary with a note drawn to bearer? To whom must a note be paid if written without, "to order," or "to bearer"? What are such notes called? Who is responsible when notes drawn to the order of the maker are negotiated or sold? What is necessary in a note to make interest collectible? When a note is written, "with interest," when does interest commence?

**53.** What is an indorsement? To what else does the term "indorsement" apply? What is the effect of a general indorsement on the back of a note? What is the effect of a special indorsement? When a person signs his name on the back of a note, what is he called? When does he become responsible?

**54.** Write out the Supreme Court rule for computing interest. Illustrate by an example.

**55.** Give the mercantile rule. Illustrate by an example.

**56.** What is a bank? What is meant by a bank note? What is meant by a bankable note?

**57.** What are days of grace?

**58.** What is meant by the maturity of a note?

**59.** What is bank discount? How does bank discount differ from simple interest? Whence the name bank discount? After the interest is paid, what is the balance of the money borrowed on a note called? In computing bank discount, what is added to the time mentioned in the note? What is the difference between bank discount and true discount?

**60.** How are the bank discount and the proceeds of a note found? Illustrate by an example.

**61.** How is the face of a note found when the time, rate, and proceeds are given? Illustrate by an example.

**62.** What is the present worth of any note or debt, due at a future time, without interest?

**63.** What is discount? What is commercial discount? What is bank discount?

**64.** How are the present worth and discount of a note or debt, due at a future time without interest, found? Illustrate by an example.

## BONDS.

***65. A Bond*** is a written instrument pledging the payment of a sum of money after a specified time, bearing interest from date of issue.

**Bonds** are evidences of debt incurred by governments, States, counties, cities, school districts, railroad companies, &c.

***66. Coupons*** are checks attached to the bond, assuring payment of the interest on it at specified times. U. S. bonds are of two kinds, registered and coupon bonds.

1. **Registered Bonds** are entered upon the books of the Treasury Department at Washington in the name of the person purchasing or owning the same, by whose written order only, or that of his legal representatives, can the interest be collected. When sold, the owner must notify the Treasury Department, that the registration may be changed.

2. **Coupon Bonds** are made payable to bearer, and are transferable without an indorsement. The interest is collectible by means of checks or coupons, payable to bearer, which may be detached from the bond. If not paid when presented, they draw interest like a promissory note.

3. **Consols** is a term applied to a portion of the public debt of Great Britain. More recently it has been applied to the bonds issued by Act of Congress for refunding a portion of the public debt. The English *consols* bear 3 % interest.

4. **U. S. 6's** and U. S. 5's are designated by the rate of interest they bear; the latter bearing 5 %, the former 6 %.

5. **U. S. 5-20's,** or U. S. 6's 5-20's, mean bonds of the U. S. bearing 6 % interest, payable in 5 or 20 years, as the Government may choose.

6. **Bonds** are usually designated thus: U. S. 6's 5-20 of '84 (U. S. 6 % bonds payable in 5 or 20 years, and due in 1884); U. S. 6's 5-20 of '85. Government securities and the coupons are payable in gold.

7. **The Market-Price** of U. S. bonds is quoted at a certain per cent of their par value: thus a bond selling at 10 % above par is *worth in currency 10 %* more than its face, or 110 % of its face.

**67. *Currency*** is the paper money used as a substitute for gold and silver, and consists of treasury notes and national bank notes.

NOTE.—Since the first edition of this book was printed, the premium on gold has disappeared, and some issues of United States bonds have been retired, and others at lower rates have been issued in their places. The examples in this article are changed to correspond with these circumstances.

### WRITTEN EXERCISES.

1. What cost eight \$500 U. S. 5 20's, quoted at $108\frac{1}{2}$?

PROCESS.—(1.) \$500 × 8 = \$4000, face of bonds.
(2.) \$4000 × $1.08\frac{1}{2}$ = \$4340, cost of bonds.

2. When U. S. 10–40's are quoted at $112\frac{1}{2}$, what will six \$500 bonds cost?

3. What is the value of sixteen \$100 U. S. bonds, when quoted at 112?

4. What is the value of \$8700 in U. S. fives, when quoted at $111\frac{3}{4}$?

5. What shall I pay for \$3500 in New York Central Railroad bonds, when quoted at $109\frac{7}{8}$?

6. What will be the cost of twenty \$1000 New York State 7's, purchased at 115?

7. How many \$100 U. S. 4's at 106 can be bought for \$2862?

8. How many \$100 U. S. 5's at $112\frac{1}{2}$ can be bought for \$5400?

9. How many \$500 U. S. $4\frac{1}{2}$ % bonds at $107\frac{1}{2}$ can be bought for \$5375?

10. How many \$1000 U. S. 5's at $116\frac{5}{8}$ can be bought for \$4665?

11. How many \$500 Ohio 6 % bonds at $112\frac{1}{2}$ can be bought for \$45000?

12. How many U. S. 4's at $98\frac{7}{8}$ can I purchase for $7910?

13. Bought $25000 of U. S. 6's at 104. What is my annual income? What rate per cent is that on my investment?

PROCESS. — (1.) $\$25000 \times .06 = \$1500 =$ income.
(2.) $\$25000 \times 1.04 = \$26000 =$ cost of bonds.
(3.) $\$1500 \div \$26000 = .05\frac{10}{13} = 5\frac{10}{13}\,\% =$ rate.

14. Bought $18000 U. S. 4's at 103. What is my annual income? What rate per cent is that on my investment?

15. What is the rate % on my investment, when I buy U. S. 6's at 110?

16. When bought at 109, what is the % of income from money invested in U. S. 6's? In U. S. 5's?

17. When bought at $112\frac{1}{2}$, what per cent will U. S. 4's pay?

18. What rate of interest on the investment will U. S. $4\frac{1}{2}$'s pay, when purchased at 108?

19. What rate of interest on the investment will U. S. 5's quoted at 110 pay?

20. A gentleman purchased $45000 U. S. $4\frac{1}{2}$ % at $115\frac{1}{2}$. What was his annual income from them? What per cent of interest did he receive on his investment?

21. When U. S. 5's are 102, what rate of interest will they pay?

22. When U. S. 4's are at 95, what rate of interest will they pay?

23. When U. S. 5's are at 97, what interest will they pay? When at $112\frac{1}{2}$, what interest will they pay? What interest will they pay at $97\frac{1}{2}$?

## EXCHANGE.

**68. *Exchange*** is the method of making payments at distant places by sending written orders, or drafts, instead of the money.

**69. *A Bill of Exchange or Draft*** is an order on a person or firm in a distant place to pay money to a third person on his order.

1. The **Drawer** or **Maker** is the person making the draft.

2. The **Drawee** is the person on whom the bill of exchange is drawn.

3. The **Payee** is the one to whose order the draft is to be paid.

4. The **Buyer** or **Remitter** is the person who sends the bill of exchange.

5. The **Holder** is the person who has possession, or who owns the bill.

6. **Acceptance.** — A bill of exchange is called an *acceptance*, when the drawee writes the word "accepted," with his name and the date, across the face. The drawee by accepting a bill becomes responsible for the payment of the bill. If not "accepted," a draft cannot be collected of the drawee.

7. **Time.** — Bills may be drawn at *sight*, or at a certain number of days *after sight*, or *after date*. Time drafts are usually allowed three days' grace.

8. **Form of a Draft.** — The following is the usual form of a domestic bill of exchange, or draft: —

$500. *St. Louis, Mo., Sept.* 30, 1876.

*Twenty days after sight, pay to the order of Noble L. Prentiss five hundred dollars, and charge the same to the account of* *Chas. Sheffield & Co.*

*To Messrs. F. B. Hunt & Co.,*
*Boston, Mass.*

**70. *An Inland or Domestic Bill*** is a draft *upon* some person or firm in the same country.

**71.** ***A Foreign Bill*** is a draft on a person residing in another country.

1. **Par.**—Exchange is at *par*, when the cost equals the face of the draft.
2. **Premium.**—Exchange is at *premium*, when the draft sells for more than its face.
3. **Discount.**—Exchange is at *discount*, when the draft sells for less than its face.

**72.** ***The Course or Rate of Exchange*** is the cost of $1 of exchange at the current premium or discount at the place drawn.

## DOMESTIC EXCHANGE.

### 73. TO FIND THE COST OF AN INLAND DRAFT OR BILL.

What is the cost of the following draft, exchange being at 3 % discount?

$600. *New York*, *June* 10, 1876.

*At sight*, *pay John Morrell*, *or order*, *six hundred dollars*, *and place the same to the account of*

*Fisk & Evans.*

*To Day*, *Swift*, *& Co.*,
*Chicago*, *Ill.*

PROCESS. (1.) $1—$.03 = $.97, rate of exchange.
(2.) $.97 × 600 = $582, cost of draft.

ANALYSIS.—(1.) Since each $1 of the draft is purchased at 3% discount, $1 of the draft will cost $1—$.03, or $.97.

(2.) If $1 of the draft costs $.97, the whole draft will cost 600 times $.97, or $582.

Rule. — *Multiply the face of the draft by the cost of $1 of exchange.*

### WRITTEN EXERCISES.

13. What must I give for a sight draft of $800 on Boston, at 2½ % discount?

14. What must I give for a sight draft of $900 on New Orleans, at 5 % premium? At 4⅔ % premium? At 2¾ % discount?

Problem. — What is the cost of the following draft, at 3½ % premium of exchange?

$450. *St. Louis, Mo., Oct. 2, 1876.*

*Thirty days after sight, pay to the order of James Otis the sum of four hundred and fifty dollars, and place the same to my account.* *Amos C. Park.*

*To Albert Zell & Co.,*
*New York City.*

Process. — (1.) $1 + $.035 = $1.035, rate or course of exchange.
(2.) $1.035 — $.0055 = $1.0295, cost of $1 of the draft.
(3.) $1.0295 × 450 = $463.28, cost of the draft.

Analysis. — (2.) If the rate of exchange is $1.035, and the bank discount of $1 for 33 days is $.0055, the cost of each dollar of the draft is the difference of these numbers, or $1.0295.

(3.) If the cost of $1 of the draft is $1.0295, the whole draft will cost 450 times $1.0295, or $463.28.

Note. — Since the draft is not due until 33 days *after sight*, the seller or maker, receiving the cash on the delivery of the draft, has the use of the money until its maturity, and hence should deduct the bank discount for the time from the cost of the draft.

15. Find the cost of a draft on Chicago for $480, at 2½ % discount.

16. I wish to pay a merchant in St. Louis $758: what *will the* remittance cost, exchange being at 2¼ % dis-

17. What will a draft for \$680 cost, at 5 % premium?

18. Bought a bill on Boston for \$350, exchange being at $\frac{1}{2}$ % premium: what was its cost?

19. Cook Brothers of Boston sold a draft on J. Tomlinson of Cincinnati for \$600, at 30 days' sight. Exchange was at $2\frac{1}{2}$ % premium, and interest 6 %: what did they receive?

20. What will a draft for \$1200, at 60 days' sight, cost, exchange being 5 % premium, and interest 7 %?

21. Bought a draft on Philadelphia for \$800, at 60 days' sight, at $2\frac{1}{2}$ % discount, interest 7 %: what was the cost?

22. If exchange is 6 % premium, and interest is 8 %, what is the cost of a draft for \$280, at 60 days?

### 74. TO FIND THE FACE OF AN INLAND DRAFT.

23. What is the face of a draft on Boston for 60 days, at $1\frac{1}{2}$ % premium, which can be bought for \$800, interest being 6 %?

PROCESS. — (1.) $\$1 + .015 = \$1.015$ = course or rate of exchange.
(2.) $\$1 \times .0105 = \$.0105$ = bank disc. of \$1 for 63 days.
(3.) $\$1.015 - \$.0105 = \$1.0045$ = cost of \$1 of draft.
(4.) $\$800 \div \$1.0045 = 796.42$ = no. dollars in face of draft.

ANALYSIS. — (4.) Since \$1.0045 will buy \$1 of the draft, \$800 will buy as many dollars as \$1.0045 is contained times in \$800, or \$796.42.

NOTE. — The first three steps in the above analysis being like the corresponding steps in the previous case, they are not repeated. The full analysis should, however, be required of the pupil.

RULE. — *Divide the given sum by the cost of \$1 of exchange.*

### WRITTEN EXERCISES.

24. Find the face of a draft on Buffalo, at 90 days, which can be bought for $1200; interest being 7 %, and exchange at 2 % premium.

25. What is the face of a draft that may be purchased for $1600, exchange being at 2 % discount?

26. I owe a merchant in Chicago $2300, who directs me to remit a draft at 30 days. What will be the face of the draft, exchange being at 4 % premium, and interest 6 %?

27. What will be the face of a sight draft purchased for $600, exchange being at $\frac{1}{2}$ % discount?

28. A draft on New York for $625 was bought in Topeka, Kan., at a premium of 1%; what did it cost? How large a draft could have been bought for $1500?

29. Suppose Rand, Avery, & Co., Boston, owe you $450.50. Write out a draft on them for that sum at 5 days' sight, in favor of John Stevens. If you sell the draft at a discount of $\frac{7}{8}$ %, what will you realize?

30. My agent in New Orleans owes me $4960. He bought a draft for that amount, which cost $5059.20; what was the rate of exchange?

31. A person living in Philadelphia has $1000 owing him in Topeka: will he make or lose by drawing on the debtor, and selling the draft at $1\frac{1}{4}$ % discount, when exchange in Topeka is at a premium of $1\frac{1}{2}$ %?

32. A man in Chicago has $4400 due him in Albany:

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REVIEW.—Give the formula for finding the percentage when the base and rate are given. (9.) For finding the amount when the base and rate are given (10.) For finding the difference. For finding the rate when the base and percentage are given. (11.) For finding the base when rate and percentage are given. (12.) For finding the base when rate and amount are given. (13.) When rate and *difference are given.* What is a company? (14.) A corporation? (15.) A charter?

how much more will he realize by selling a draft on his debtor for this sum, at ½ % discount, than by having his debtor remit him a draft on Chicago, purchased in Albany for this sum at ¾ % premium?

## FOREIGN EXCHANGE.

**75.** Foreign exchange is computed by the same method as domestic exchange, except that the currency of one nation must be reduced to that of the other.

Foreign bills are usually drawn in sets of three of the same tenor and date, each containing the condition that it shall be payable only when the others are unpaid.

**Par of Exchange.** — In commercial transactions the value of the several monetary units are fixed as in the following

### TABLE.

| | |
|---|---|
| LONDON. | BERLIN. |
| 1 pound sterling = $4.8665. | 1 thaler = $.714. |
| AMSTERDAM. | FRANCE. |
| 1 florin = $.405. | 1 franc = $.193. |
| $1 United States currency = 5.18 francs. | |

*Quotations.* — When quotations are made above these rates, exchange is at a premium; when below, exchange is at a discount.

In sterling or English exchange, the value of the pound is usually quoted in United States money; but in French exchange the value of the dollar is given in francs.

Thus, when exchange on London is quoted at 485¼, it is understood that £1 of exchange is worth $4.85¼, and, when exchange on Paris is quoted at 5.26⅜, it is understood that $1 of exchange is worth 5.26⅜ francs.

Exchange on England quoted at 485¼ would be at a discount; while French exchange at 5.26⅜ francs would be at a premium.

In sterling exchange, the fractions of the cent are quoted in halves or quarters; while in exchange on France the fractions of the centime (the one-hundredth part of a franc) are quoted in halves, quarters, or eighths.

### **76.** TO FIND THE VALUE OF A FOREIGN BILL OF EXCHANGE.

PROBLEM. — What is the cost, at par, of the following bill of exchange on London?

£253 8*s*. 6*d*. *New York, Nov.* 12, 1876.

*At sight of this our first of exchange (second and third of the same tenor and date remaining unpaid), pay to the order of Donnell, Lawson, & Co., two hundred and fifty-three pounds, eight shillings, and sixpence, value received, and place the same to our account.*

*Duncan, Platt, & Co.*

*To Messrs. Rothschild & Co., London, Eng.*

PROCESS.

(1.) £253 8*s*. 6*d*. = £253.425.

(2.) \$4.8665 × £253.425 = \$1233.29 value in U. S. money.

ANALYTICAL STEPS. — (1.) *Find the value of the given sum in pounds and decimals of a pound.*

(2.) *Find the value in U. S. money.*

PROBLEM. — What is the cost of a bill of exchange on Paris for 3364 francs, exchange on Paris being at 5.25⅝?

PROCESS. — 3364 fr. ÷ 5.25⅝ = \$640, value in U. S. gold.

WRITTEN EXERCISES.

33. What will be the cost in U. S. money, of a bill of exchange on Paris of 4120 francs, exchange being at 5.28½?

34. I wish to make a remittance of £850 to my agent in Bristol: what is the cost of a bill of exchange covering that amount, at 488½?

35. What will be the cost of a bill of exchange on London of £321 2*s*., the rate of exchange being 482¼?

36. What must I give for a bill of exchange on Paris for 1480 francs, with exchange quoted at 5.26?

37. London exchange is quoted at 490½ cts. What is the cost of a bill of exchange for £50?

PROCESS. — \$4.905 × 50 = \$245.25, cost.

38. Paris exchange is quoted at 5.22 francs: what must I give for a bill of exchange of 2500 francs?

39. Berlin exchange is quoted at 72⅕ cts.: what must I pay for a bill of exchange for 300 thalers?

### 77. TO FIND THE FACE OF A BILL OF FOREIGN EXCHANGE.

**PROBLEM.** — What is the amount of a bill of exchange on London that I can purchase for \$4000, when exchange on London is quoted 480?

PROCESS. — \$4000 ÷ \$4.80 = £833.33 = £833 6*s.* 8*d.*, face of draft.

#### WRITTEN EXERCISES.

40. What will be the face of a bill of exchange on London that can be purchased for \$6700, with exchange at 486½?

41. What will be the amount of a bill of exchange on London that can be bought for \$8200, exchange being at 485?

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**REVIEW.** — What is stock? (16.) What is the value of a share in bank, insurance, and railroad companies? What is a certificate of stock? What is par value? Market value? When is stock at par? At premium? At discount? What is stock-jobbing? A stock broker? What is a partnership, or firm? (17.) What is the base of percentage in stocks? How are all premiums, discounts, brokerages, dividends, and assessments computed? What is insurance? (18.) What is fire insurance? Marine insurance? What is a policy? (19.) An insurer, or underwriter? (20.) What is a mutual insurance company? Who is the policy-holder? What is the premium? How is it estimated? Upon how much of the value of the insured property is insurance generally taken? Give the equivalent of the amount insured. Of the premium. Of the rate per cent. What is life-insurance? (21.) What is a life *policy? An* endowment policy? A continued premium life policy? *How is the premium computed?*

42. What is the amount, in francs, of a bill of exchange on Paris that can be purchased for $2000, exchange being at 5.27?

43. Exchange on Amsterdam is quoted at 41.5 cents: what will be the face, in florins, of a bill of exchange that can be bought for $500?

44. When exchange is quoted at 482, what will be the amount of a draft on London, that 3000 will buy?

## EQUATION OF PAYMENTS.

***78. Equation of Payments*** is the process of finding the time at which several sums of money, due at different times, may be paid at once, without loss to either debtor or creditor.

NOTE.—The principle on which the process is based is that the use of a certain sum of money for a given time is worth the same as the use of *half* the money for *twice* the time, or *twice* the money for *half* the time, &c.

***79. The Equated Time, or Average Time,*** is the time at which the several sums may be paid in one payment without loss.

***80. The Term of Credit*** is the time from incurring the debt to the date at which it becomes due.

***81. The Average Term of Credit*** is the equated time at the end of which all the sums may be paid.

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REVIEW.—What is a mutual life-assurance company? (22.) What is a tax? (23.) What is a direct tax? An indirect tax? A poll-tax? What is real estate? Personal property? What is an assessor? A collector? What is the tax rate? (24.) What are customs, or duties? (25.) What is tonnage? A custom-house? A port of entry? A collector of the port? A tariff? A clearance? A manifest? What is a specific duty? (26.) An ad valorem duty? (27.) What is draft? Tare? Leakage? Breakage? Gross weight? Net weight? What is interest? (28.) What is the *principal*? (29.) The rate per cent? (30.) The amount? (31.)

**82.** ***An Account*** is a written statement of debit and credit transactions, with their respective dates.

1. A **Balance** is the difference between the debits and credits of an account.

2. A **Cash Balance** is the difference between the debits and credits of an account when interest is considered.

3. **Averaging an Account** is the process of finding the equated time or term of credit of the balance of an account.

4. The **Focal Date** is any assumed time from which the terms of credit are computed.

### 83. TO FIND THE AVERAGE TERM OF CREDIT WHEN ALL THE TERMS OF CREDIT BEGIN AT THE SAME TIME.

PROBLEM.—I owe, April 10, 1875, \$3, to be paid in 3 months, \$5 to be paid in 8 months, and \$12 payable in 9 months without interest: what is the average term of credit? When will the whole debt become due?

PROCESS.

$$\begin{array}{lrrr}
(1.) & 3 \text{ mos.} \times & 3 \text{ (dol.)} = & 9 \text{ mos.} \\
(2.) & 8 \text{ mos.} \times & 5 \text{ (dol.)} = & 40 \text{ mos.} \\
(3.) & 9 \text{ mos.} \times & 12 \text{ (dol.)} = & 108 \text{ mos.} \\
\hline
 & & 20 & 157
\end{array}$$

(4.) 157 mos. ÷ 20 = 7.85 mos.
(5.) 7.85 mos. = 7 mos. 26 da.
(6.) Apr. 10 + 7 mos. 26 da. = Dec. 6.

ANALYSIS.—(1.) The interest of \$3 for 3 mos. is equal to the interest of \$1 for 9 mos.

(2.) The interest of \$5 for 8 mos. is equal to the interest of \$1 for 40 mos.

(3.) The interest of \$12 for 9 mos. is equal to the interest of \$1 for 108 mos. The interest of the sum of the items of the debt is equal to the interest of \$1 for 157 mos.

4. If \$1 is entitled to a credit of 157 mos., \$20, the sum of the debts, is entitled to a credit of $\frac{1}{20}$ of 157 mos., or 7.85 mos. = 7 mos. 26 da.

6. As the several items of credit begin at April 10, the equated time is 7 mos. 26 da. from that date, or Dec. 6, 1875.

RULE. — I. *Multiply each item by the term of credit, and divide the sum of the products by the sum of the items; the quotient will be the average term of credit.*

II. *Add the average term of credit to the date at which the credits begin, and the result will be the equated time.*

NOTE. — The above rule may be applied to notes as well as accounts. It is founded on the principle of making the bank discount equal to the interest. Though this principle is not strictly accurate, the rule is in general use.

### WRITTEN EXERCISES.

45. Find the average term of credit of $300 due in 4 mos., $600 in 5 mos., and $850 in 8 mos.

46. A merchant bought goods as follows: $600 worth at 6 mos., $300 at 4 mos., and $800 at 9 mos. credit: what is the average term of credit?

47. A dealer bought goods as follows: —

150 prs. coarse boots at 3 mos. @ $4.50.
75 prs. calf boots at 4 mos. @ 6.50.
100 prs. morocco gaiters at 6 mos. @ 3.35.

What is the average term of credit?

48. Bought merchandise Feb. 1, 1876, on the following terms: $600 worth at 4 mos., $500 at 8 mos., $700 at 9 mos., and $1200 at 12 mos. What was the average term of credit? What is the equated time of payment?

49. A merchant bought a bill of goods June 5, 1876, amounting to $7250, on the following terms: $1600 in 3 mos., $1500 in 5 mos., $2000 in 6 mos., and the remainder in 8 mos. What is the equated time of payment?

50. Bought a farm June 1, 1875, for $8000, on the following conditions: $3000 cash, $3000 in 4 mos., and the remainder in 8 mos. What would have been the equated time for paying the whole amount?

SUGGESTION. — The sum paid in cash has no credit, but should be included in the sum of the items.

51. Bought 4000 barrels of lime, at $3.50 per barrel; giving in payment notes of $6000 for 3 mos., and $600 for 6 mos., and cash to balance. What would have been the equated time for the whole bill?

52. Bought 300 yds. of broadcloth at $2.50 per yd., on 30 days' credit; 100 yds. cassimere at $1.50 per yd., on 60 days; and 240 yds. flannel at 75 cts. per yd., cash. For what time should a note be drawn to pay the whole amount?

53. A broker sold a farm for $4000, on the following terms: $1000 cash, $250 in 4 mos., $750 in 6 mos., and the balance in one year. He afterward agreed to take a note for the whole amount. For what time must the note be drawn?

### **84.** TO FIND THE EQUATED TIME WHEN THE TERMS OF CREDIT BEGIN AT DIFFERENT DATES.

NEW YORK, Jan. 1, 1877.

J. T. Janeway,

To Edgar W. Holmes, DR.

| | | |
|---|---|---|
| 1876. Aug. 3, | To Cash | $500. |
| " 19, | " Mdse. on 3 mos. | 375. |
| " Sept. 11, | " Cash | 590. |
| " Dec. 25, | " Mdse. on 9 mos. | 100. |

What is the equated time of settlement?

**PROCESS.**

| 1876 | da. | $ | da. |
|---|---|---|---|
| Due Aug. 3, | 0 × | 500 = | 0 |
| Nov. 19, | 108 × | 375 = | 40500 |
| Sept. 11, | 39 × | 590 = | 23010 |
| 1877. | | | |
| Sept. 25, | 419 × | 100 = | 41900 |
| | | 1565 | 105410 |

(da.) ($)

105410 ÷ 1565 = 67 da. +.

Aug. 3, 1876 + 67 da. = Oct. 9, 1876.

ANALYSIS. — (1.) For convenience we select, as the focal date, the earliest date at which any of the items become due, which is Aug. 3, 1876: hence the first item, $500, has no credit.

(2.) If $375 is payable 3 mos. from Aug. 19, it becomes due Nov. 19, and the term of credit from the focal date is 108 da.: hence the credit is equal to the credit of $1 for (108 da. × 375) 40500 days.

(3.) If $590 is due Sept. 11, the credit from the *focal date* is 39 days: hence the credit is equal to the credit of $1 for (39 da. × 590) 23010 days.

(4.) If $100 is payable 9 mos. from Dec. 25, 1876, it becomes due Sept. 25, 1877, and the term of credit from the focal date is 419 days: hence the credit is equal to the credit of $1 for (419 da. × 100) 41900 days.

(5.) The credit of the sum of the payments, $1565, is equal to the credit of $1 for the sum of the credits of $1, or 105410 days.

(6.) If the credit of $1 requires 105410 days to be worth a certain sum, then for the credit of $1565 to equal the same sum will require $\frac{1}{1565}$ of 105410 da., or 67 days, the average term of credit.

(7.) The focal date, Aug. 3, 1876 + 67 days = Oct. 9, 1876, the equated time of payment, from which time the $1565 should be at interest until paid.

RULE. — I. *Find the time at which each item becomes due.*

II. *Assume the earliest date as the focal date, and find the term of credit from the focal date to the date when each item becomes due.*

III. *Find the equated time of the several terms of credit, by the rule under* ¶***83.***

WRITTEN EXERCISES.

54. What is the equated time for the payment of the following account?

| | | | |
|---|---|---|---|
| 1876. | Jan. 18, | Mdse. at 3 mos. | $460. |
| " | Feb. 17, | Cash | 180. |
| " | Mar. 10, | Mdse. at 6 mos. | 420. |
| " | Aug. 10, | Cash | 140. |
| " | Sept. 5, | Mdse. at 4 mos. | 300. |

55. What is the equated time for the payment of the following bill?

| | |
|---|---|
| 1876. | |
| Jan. 10, | To 214 yds. carpet @ $1.50 at 3 mos. |
| Feb. 21, | " 450 yds. oil-cloth @ 80 cts. at 6 mos. |
| Mar. 14, | " 10 doz. cane-seat chairs @ $20 at 4 mos. |
| Apr. 1, | " 15 bedroom suits @ $25 at 3 mos. |

56. Stuart and Co. sell to A. L. Williams the following bill of goods: Mar. 1, 1874, $450 at 4 mos.; June 4, $250 at 6 mos.; Aug. 13, $415 at 3 mos. What is the equated time of payment?

57. Loaned to A. B. Perkins, Mar. 5, $900; June 6, $450; Oct. 12, $375. What is the equated time at which the whole should have been paid?

58. A dealer sold me, March 15, $1600 worth of goods, to be paid for as follows: $\frac{1}{4}$ in cash, $\frac{1}{4}$ in 3 months, and the remainder in 6 months. When in equity should the whole be paid for in one payment?

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**Review.**—What is the base of percentage in stocks? (17.) How are all premiums, discounts, brokerages, dividends, and assessments computed? What is insurance? (18.) What is fire insurance? Marine insurance? What is a policy? (19.) An insurer or underwriter? (20.) What is a mutual insurance company? Who is the policy-holder? What is the premium? How is it estimated? Upon how much of the value of the insured property is insurance generally taken? Give the equivalent of the amount insured. Of the premium. Of the rate per cent. What is life-insurance? (21.) What is a life policy? An endowment policy? A continued premium life policy? A ten, five, or single payment policy? How is the *premium computed*?

59. I gave a note for $1400, to be paid in 12 mos., but in 6 months I paid $400, and in 3 mos. thereafter $400 more. When in equity should I pay the balance?

PROCESS.

6 mos. × $400 = 2400 mos.
3 mos. × 400 = 1200 mos.
800 3600 mos.

$1400 − 800 = $600
3600 mos. ÷ 600 = 6 mos.
12 mos. + 6 mos. = 18 mos. *Ans.*

ANALYSIS.—(1.) If I pay $400 6 mos. before it is due, I am entitled to a credit of $1 for (6 mos. × 400) 2400 mos.; and, if I pay $400 3 mos. before it is due, I am entitled to an additional credit of $1 for (3 mos. × 400) 1200 mos., or in all 3600 mos.

(2.) A credit of $1 for 3600 mos. is equal to a credit of $600 for $\frac{1}{600}$ of 3600 mos, or 6 mos., making the balance due 6 mos. after the original 12 mos., or 18 mos. from date.

60. A man borrowed $3600, to be paid in 9 months. At the end of 2 mos. he paid $600, at the end of 4 mos. thereafter he paid $800, and at the end of the 8th month he paid $1000. For how long after the expiration of the 9 mos. should he have credit on the balance?

61. On a note given July 5, 1875, for $930, payable in 12 mos., there was paid Aug. 5, $200; Oct. 5, $200; Dec. 5, $230. What is the equated time for paying the balance?

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REVIEW.—What is a mutual life-assurance company? (22.) What is a tax? (23.) What is a direct tax? An indirect tax? A poll-tax? What is real estate? Personal property? What is an assessor? A collector? What is the tax-rate? (24.) What are customs, or duties? (25.) What is tonnage? A custom-house? A port of entry? A collector of the port? A tariff? A clearance? A manifest? What is a specific duty? (26.) An ad valorem duty? (27.) What is draft? Tare? Leakage? Breakage? Gross weight? Net weight? What is interest? (28.) What is the principal? (29.) The rate per cent? (30.) The amount? (31.)

* The pupil should remember that 1876 was a leap year.

## RATIO.

**85. *Ratio*** is the relation which one number bears to another of the same kind, and is found by dividing the given number by the one with which it is compared. Thus the ratio of 12 to $6 = 2$; the ratio of 5 to $10 = \frac{5}{10} = \frac{1}{2}$.

**86. *The Terms*** of a Ratio are the numbers compared; and when taken together they are called a *couplet*.

**87. *The Antecedent*** is the first term of the couplet, and is the dividend or numerator of the fraction expressing the value of the ratio.

**88. *The Consequent*** is the second term of the couplet, and is the divisor or denominator of the fraction expressing the value of the ratio.

Some teachers prefer the French method of expressing the ratio, in which the antecedent is the measure or divisor of the consequent, and make the ratio of 8 to $12 = \frac{12}{8}$ or $\frac{3}{2}$. Either treatment will produce the same result in all questions involving the use of the ratio.

**89.** The ***Sign*** of ratio is a colon (:) or the ratio may be written as a fraction: thus, the ratio of 7 to 9 may be expressed either as $7:9$, or $\frac{7}{9}$.

**90. *Simple Ratio*** is a comparison of two numbers.

**91. *Compound Ratio*** is a comparison of the products of the corresponding terms of two ratios.

Thus the ratio of $\left.\begin{matrix}3:4\\6:8\\7:9\end{matrix}\right\} = \frac{\cancel{3}\times\overset{\cancel{2}}{\cancel{6}}\times 7}{4\times\underset{4}{\cancel{8}}\times\underset{\cancel{3}}{\cancel{9}}} = \frac{7}{16}$, and is formed by dividing the products of the antecedents by the product of the consequents.

**92.** ***The Value of the Ratio*** is the quotient of the antecedent divided by the consequent.

The numbers compared may be concrete numbers, but the ratio is always an abstract number.

Thus in the ratio, 14 tons to 7 tons, 14 tons is the antecedent, and 7 tons the consequent, and 2 the ratio.

**93.** Since the value of the ratio is a fraction, it follows that its terms may be treated like those of a fraction: hence the following principles: —

1. *Multiplying the antecedent multiplies the ratio.*
2. *Multiplying the consequent divides the ratio.*
3. *Dividing the antecedent divides the ratio.*
4. *Dividing the consequent multiplies the ratio.*
5. *Multiplying or dividing both antecedent and consequent by the same number does not affect the ratio.*
6. *The product of two or more simple ratios is the ratio of their products: as,* $3:4=\frac{3}{4}$, *and* $5:6=\frac{5}{6}$; *then* $(5:6)\times(3:4)=15:24=\frac{5}{8}=\frac{3}{4}\times\frac{5}{6}$.

### ORAL EXERCISES.

1. What is the ratio of 4 to 7? of 6 to 12? of 8 to 14? of 25 to 100? of 100 to 25?

2. What is the ratio of 6 to 18? of 18 to 6? of 7 to 7? of 25 to 6?

3. What is the ratio of $3 to $30? of 4 gal.: 12 gal.? Of 9 lbs.: 27 lbs.?

4. What is the ratio of 7 to 15? of 12:15? of 8 yds.: 12 yds.?

5. What part of 7 is 2? of 9 is 5? of $8 is $3? of 11 is 2?

6. Two is what part of 5? 6 is what part of 11? 27 *is what* part of 50?

### WRITTEN EXERCISES.

1. What is the value of the ratio of 85 to 34? of 34 to 85?

2. What is the ratio of 25 cents to \$2? of 75 lbs. to 1 cwt.? of 3 *s.* 6 *d.* to 1£?

3. What is the ratio of $3\frac{1}{2}$ to 7? of $6\frac{1}{4}$ to 25? of 39 to 1728?

4. What is the ratio of $2\frac{3}{4}$ to $3\frac{3}{5}$?

Suggestion.—$2\frac{3}{4} : 3\frac{3}{5} = \frac{11}{4} : \frac{18}{5} = \frac{11}{4} \div \frac{18}{5} = \frac{11}{4} \times \frac{5}{18} = ?$
or, $\frac{11}{4} = \frac{55}{20}$; $\frac{18}{5} = \frac{72}{20}$; $55 : 72 = ?$

5. What is the value of the compound ratio $\left.\begin{array}{r} 7 : 4 \\ 3 : 6 \\ 8 : 14 \end{array}\right\}$?

6. What is the value of the compound ratio $\left.\begin{array}{l} 1\frac{1}{2} : 4 \\ 2 \ \ : 3 \\ 5 \ \ : 6 \end{array}\right\}$?

7. What is the value of $6\frac{1}{2}$ to $7\frac{3}{4}$? of $2\frac{3}{4}$ to $\frac{1}{2}$?

8. What is the ratio of 3 bu. 2 pks. to 3 pks. 2 qts.?

9. If the consequent of the ratio is 18, and the antecedent 24, what is the ratio?

10. If the ratio is $2\frac{1}{2}$, and the antecedent is 15, what is the consequent?

## PROPORTION.

**94. *Proportion*** is an equality between two ratios, and is expressed by the sign =, or the double colon (::).

Thus the ratio of 5 : 10 is equal to the ratio 6 : 12; and the proportion may be expressed thus: 5 : 10 = 6 : 12; or, 5 : 10 :: 6 : 12. In either form, the proportion is to be read as follows: The ratio of *5 to 10 is equal to the ratio of* 6 *to* 12; or, 5 is to 10 as 6 is to 12.

TERMS. — Every proportion consists of two couplets, or four terms.

1. The **Antecedents** are the first and third terms.
2. The **Consequents** are the second and last terms.
3. The **Extremes** are the first and last terms.
4. The **Means** are the terms between the extremes.
5. **Mean Proportional.** — When, in a proportion, the consequent of the first couplet is the same as the antecedent of the second couplet, this number is called a *mean proportional.* Thus in the proportion 4 : 8 :: 8 : 16, 8 is the mean proportional between 4 and 16.

In the proportion 3 : 7 :: 9 : 21, the ratios $\frac{3}{7}$ and $\frac{9}{21}$ are equal; for, reducing these fractions to a common denominator $\frac{3}{7} \times \frac{21}{21} = \frac{63}{147}$, and $\frac{9}{21} \times \frac{7}{7} = \frac{63}{147}$; that is, multiplying each numerator by the other denominator, the products are equal. Hence, in every proportion,

PRINCIPLE I. — *The product of the means is equal to the product of the extremes.*

If multiplying either the means or extremes together produces the same product, dividing this product by one of the terms of a couplet will give the other term as a factor. Hence,

PRINCIPLE II. — *The product of the extremes divided by either mean will give the other mean.*

PRINCIPLE III. — *The product of the means divided by either extreme will give the other extreme.*

### WRITTEN EXERCISES.

1. 16 is to 2 as 40 is to what number?

2. If the extremes are 3 and 12, and one mean is 4, what is the other mean?

3. If the means are 15 and 8, and one extreme is 6, what is the other extreme?

4. What number has the same ratio to 15 that 5 has *to 12?*

5. Express in a proportion the ratios $\frac{5}{6}$, $\frac{15}{18}$.

What is the missing term in each of the following proportions?

6. 6 : — :: 3 : 4.
7. — : 12 :: 4 : 16.
8. 16 : 4 :: 20 : —.
9. 84 : 14 :: 66 : —.
10. 896 : 112 :: — : 72.

## SIMPLE PROPORTION.

**95. *Simple Proportion*** is an equality between two simple ratios.

Simple proportion is used to find the fourth term of a proportion where the other three terms are given. The proper arrangement of these terms is called a *statement*.

In every statement, two of the terms must be of the same kind, and the third term of the same kind as the answer, or fourth term.

PROBLEM.—If 6 bushels of wheat are worth $8, what are 9 bushels worth?

STATEMENT.—$\overset{\text{bu.}}{6} : \overset{\text{bu.}}{9} :: \overset{\$}{8} : ?$

PROCESSES.

(*a.*) $9 \times 8 = 72.$
$72 \div 6 = 12.$

(*b.*) $$\frac{\overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}}}{\underset{2}{\cancel{6}}} = 12.$$

ANALYSIS.—(1.) Since like things only can be compared, the 6 bu. must be compared with the 9 bu., and the value of 6 bu. with the value of 9 bu. Hence as 6 bu. : 9 bu., so must $8, the value of 6 bu., be to the value of 9 bu.; or, 6 : 9 :: 8 : ?

By Principle III., the missing term is found by dividing the product of the means (9 × 8) by the given extreme (6); and the quotient is the required term, or answer. Hence the following

RULE.—I. *Write for the third term of the proportion the number that is of the same kind as the required answer.*

II. *Consider from the conditions of the question, whether the answer should be greater or less than the term written: if greater, write the greater of the remaining terms for* *the second term; if less, write the greater of the remaining terms for the first term.*

III. *Divide the product of the second and third terms by the first term.*

### WRITTEN EXERCISES.

11. If 19 bushels of apples cost $57, what will 36 bushels cost?

12. What will 33 yds. of cloth cost, if 18 yds. can be bought for $45?

13. If 35 acres cost $7700, what will 112 acres cost.

14. If a pole 12 feet high cast a shadow 16 feet long, what must be the length of a pole to cast a shadow 96 feet long.

15. If 15 coats are cut from 48 yds. of cloth, how many coats may be cut from 240 yds.?

16. If 63 gallons of cider cost $13.86, what will 132 gallons cost?

17. What will 137 bushels of wheat cost, if 9 bushels cost $12?

18. If 17 bushels of wheat make 4 barrels of flour, how many bushels will 200 barrels require?

19. How long will it take me to travel 288 miles, if I can travel 12 miles in 42 minutes?

20. If 6 men can dig a trench in 15 days, how long will it take 9 men to do the same work?

SUGGESTION. — More men will do the work in less time.

---

REVIEW. — What is simple interest? (32.) Compound interest? (33.) Legal interest? (34.) Usury? (35.) Name the terms used in interest. (36.) Give their equivalents in percentage. What term is used in interest which is not used in percentage? Give the rule for finding the interest of any sum, at any rate per cent, for any number of years and months. (38.)

21. How long will it take 40 men to build a wall, if 12 men can do it in 20 days?

22. When 10 barrels of apples cost \$27.50, what will 387 barrels cost?

23. If $3\frac{1}{2}$ acres of land cost \$735, what will 8 acres cost?

24. If $\frac{3}{5}$ of a barrel of syrup cost \$$43\frac{1}{2}$, what will $\frac{3}{4}$ of a barrel cost?

25. What will $\frac{7}{8}$ of a vessel cost, if $\frac{2}{3}$ of it is worth \$4800?

26. What will $9\frac{1}{2}$ bushels of wheat cost, if 1 bushel is worth \$2.30?

27. If it takes 8 men to mow 13 acres in a day, how many men will it take to mow $45\frac{1}{2}$ acres in the same time?

28. How many men will it take to do a piece of work in 18 days, if 60 men do it in 15 days?

29. If \$60 earn \$15 in 6 mos. and 6 days, how much will the same sum earn in 8 mos. and 12 days?

SUGGESTION. — 6 mos. and 6 days = 186 days: 8 mos. and 12 days = 252 days.

30. A man working eight hours a day builds a cistern in 20 days: how long will it take him to do the same work, if he labors 10 hours a day?

31. If a cistern containing 3000 gallons leak 1 gal. 2 qts. a minute, how long will it take to empty it?

32. If $\frac{4}{5}$ yard of cloth cost \$$\frac{12}{15}$, what will $1\frac{7}{8}$ yds. cost?

33. 36 men can build a wall in $12\frac{1}{2}$ days: how many men would it take to build it in $2\frac{1}{2}$ days?

34. If a certain quantity of provision will last a garrison of 350 men 7 months, how long will it feed 800 men?

## COMPOUND PROPORTION.

**96.** ***Compound Proportion*** is a proportion in which either or both the ratios are compound.

In compound proportion, the several couplets must each be of the same denomination, while the third term will be of the same kind as the answer required.

PROBLEM. — If 8 men can reap 10 acres of wheat in 4 days, how many men will reap 25 acres in 8 days?

STATEMENT.

$$\left.\begin{array}{cc}\text{Acres.} & \text{Acres.}\\ 10\ : & 25\\ \text{Days.} & \text{Days.}\\ 8\ : & 4\end{array}\right\}\begin{array}{cc}\text{men.} & \text{men.}\\ ::8 & (\ )\end{array}$$

SOLUTION. — This problem resolves itself into two simple proportions, with the same third term, 8 days.

If 8 men can reap 10 acres, how many men can reap 25 acres? As it will require more men, the ratio will be 10 : 25; and the proportion, 10:25 :: 8: ( ).

PROCESS.

$$\frac{\overset{5}{\cancel{25}} \times \overset{2}{\cancel{4}} \times \cancel{8}}{\underset{2}{\cancel{10}} \times \cancel{8}} = 10$$

If 8 men can do the work in 4 days, how many men will it require to do the work in 8 days? As it will require less men, the ratio will be 8:4; and the proportion, 8: 4::8: ( ).

Uniting the statements, we have the compound proportion $\left.\begin{array}{r}10:25\\ 8:\ 4\end{array}\right\} :: 8 : (\ )$, or 10 × 8 : 25 × 4::8: ( ).

Dividing the products of the second and third terms by the products of the first, we have 25 × 4 × 8 ÷ 10 × 8 = 10 men, the answer required.

RULE. I. — *Place that number for the third term which is of the same kind or denomination as the answer required.*

II. — *Take the other numbers in pairs of the same kind, and arrange each pair without reference to the other couplets, as in simple proportion.*

III. — *Multiply the products of the second and of the third terms together, and divide the product by the product of the first terms: the quotient will be the fourth term, or answer required.*

This problem may also be solved by analysis: —

ANALYSIS. — If 8 men can reap 10 acres in 4 days, 1 man can reap $\frac{1}{8}$ of 10 acres or $1\frac{1}{4}$ acres in 4 days, and in 1 day will reap $\frac{1}{4}$ of $1\frac{1}{4}$ acres or $\frac{5}{16}$ acres.

If 1 man can reap $\frac{5}{16}$ acres in 1 day, in 8 days he can reap 8 times $\frac{5}{16}$ or $2\frac{1}{2}$ acres; and it will take as many men to reap 25 acres in 8 days as $2\frac{1}{2}$ acres is contained times in 25 acres, or 10 men.

### WRITTEN EXERCISES.

35. If 12 men can mow 11 acres of grass in 5 days, how many men can mow 33 acres in 18 days?

36. 480 lbs. of flour will last 24 men 40 days: at the same rate, how long will 300 lbs. last 48 men?

37. 144 men, in 6 days of 12 hours each, can dig a ditch 200 ft. long, 3 ft. deep, and 2 ft. wide: in how many days of 7 hours each, can 30 men dig a ditch 350 ft. long, 6 ft. deep, and 3 ft. wide?

38. If 20 men, in $16\frac{1}{2}$ days of 8 hours each, dig a ditch 88 rods long, 8 feet deep, and 3 feet wide, how many men will be required to dig a ditch 360 rods long, 12 feet deep, and 8 ft. wide, in 18 days, by working 12 hours each day?

39. 4 men are paid $32 for 6 days' labor: how many men may be employed 21 days for $84?

40. 16 men can gather 44 acres of grain in 12 days, by working 10 hours a day: how many men will be required to gather 440 acres in 10 days, working 16 hours a day?

41. A piece of coal 2 feet long, 1 foot wide, and 6 inches thick, weighs 55 pounds: what is the weight of a block 110 feet long, 92 feet wide, and 24 feet thick?

42. A stone 4 inches wide, 8 inches long, 3 inches thick, weighs $2\frac{1}{2}$ lbs: what will be the weight of a shaft, of the same kind of stone, 30 feet long, 12 feet wide, and 12 feet thick?

43. If $500 will gain $16.50 in 4 mos. 12 days, at 9 %, how much will $750 gain in 2 yrs. 9 mos. 8 days, at 6 %?

## CAUSE AND EFFECT.

The terms of a problem involving a proportion may be compared not only in *magnitude*, but also in regard to cause and effect.

***97. A Cause*** is any thing that produces an *effect;* as, men at work, goods purchased, money loaned, &c.*

***98. An Effect*** is the result of a cause; as, work done, goods sold, interest on money loaned, food consumed, &c.*

***99. A Compound Cause,*** or ***Effect,*** is composed of two or more simple elements combined; as, 6 men working 8 hours (compound cause) can load 16 cars, each carrying 3000 lbs. (compound effect).

Since *like causes* produce *like effects*, the ratio between two like causes will equal the ratio between their effects; thus, —

1st cause : 2d cause : : 1st effect : 2d effect.
Or, 1st effect : 2d effect : : 1st cause : 2d cause.

---

* This use of these terms is not strictly accurate. We adopt them because they are better adapted to the pupils' understanding than those more logical and intricate.

### SIMPLE CAUSE AND EFFECT.

**PROBLEM.** — If 28 men earn $35 in one day, how much will 42 men earn?

#### STATEMENTS.

1st cause : 2d cause :: 1st effect : 2d effect.
28 (men) : 42 men :: $35 : ( ).

#### PROCESS.

$$\frac{\overset{3}{\cancel{42}} \times \$35}{\underset{2}{\cancel{28}}} = \frac{\$105}{2} = \$52.50.$$

**ANALYSIS.** — The first cause is 28 men; and the second cause, 42 men. Since they are like causes, they may be compared, and will form the first couplet.

The effect of the first cause is $35, and the effect of the second cause is also money earned: these effects form the second couplet.

Find the missing term by Principle III.

**PROBLEM.** — If 12 horses eat 48 bushels of grain in a week, how many horses will it take to consume 40 bushels?

#### STATEMENT. (*a.*)

1st cause. 2d cause. 1st effect. 2d effect.
12 horses : ( ) horses :: 48 bu. : 40 bu.

#### PROCESS. (*a.*)

$$\frac{\cancel{12} \times \overset{10}{\cancel{40}}}{\underset{\cancel{4}}{\cancel{48}}} = 10 \text{ horses.}$$

#### STATEMENT. (*b.*)

1st effect. 2d effect. 1st cause. 2d cause.
48 bu. : 40 bu. :: 12 horses : ( ) horses.

### WRITTEN EXERCISES.

44. If 8 yards of cloth cost $13.60, how much will 31 yards cost?

45. If $360 earn $16.20 interest, how much will $876 earn?

46. If 63 bushels of wheat are bought for $78.75, how much can be bought for $110?

47. What will 37.5 yds. of cloth cost, if $1\frac{1}{2}$ yards cost $1.36?

48. If $\frac{5}{9}$ of a vessel cost $14400, what will $\frac{3}{16}$ of the vessel cost?

49. If a horse travel 30 miles in 3 hours and 20 minutes, how long will it take him, at the same rate, to travel 48 miles?

50. How many horses can be fed from 720 bushels of oats for a given time, if 30 bushels last 12 horses the same time?

51. If 96 horses eat a certain quantity of hay in $7\frac{1}{2}$ weeks, how many horses will eat as much in 90 weeks?

### COMPOUND CAUSE AND EFFECT.

**Problem.** — If a pasture of 16 acres will feed 4 horses for 6 months, how many acres will feed 12 horses for 8 months?

STATEMENTS.

$$\begin{array}{lll} \text{1st cause} : \text{2d cause} & & :: \text{1st effect} : \text{2d effect.} \\ \left.\begin{array}{l} \text{4 horses} \;\; : \text{12 horses} \\ \text{6 months} : \;\text{8 months} \end{array}\right\} & & :: \text{16 acres} : (\quad)\text{ acres.} \end{array}$$

PROCESS.

$$\frac{\overset{2}{\cancel{12}} \times \overset{2}{\cancel{8}} \times 16}{\cancel{4} \times \cancel{6}} = 64 \overset{\text{acres}}{\text{ horses.}}$$

**Analysis.** — The first cause is 4 horses feeding for 6 mos., and the second cause 12 horses feeding for 8 mos., forming the compound ratio of the first couplets.

The first effect is 16 acres, and the second effect is the required number of acres.

*Find the missing* term by Principle III.

**Problem.**—If 9 men can do a piece of work in 14 days, how long will the same work employ 12 men?

STATEMENTS.

$$\begin{array}{lll} \text{1st cause} : \text{2d cause} & :: & \text{1st effect} : \text{2d effect.} \\ \left.\begin{array}{ll} \text{9 men} & : \text{12 men} \\ \text{14 days} & : (\quad) \text{ days} \end{array}\right\} & :: & 1 \text{ (work)} : 1 \text{ (work).} \end{array}$$

PROCESS.

$$\frac{\overset{3}{\cancel{9}} \times \overset{7}{\cancel{14}} \times 1}{\underset{\underset{2}{\cancel{4}}}{\cancel{12}} \times 1} = 10\tfrac{1}{2} \text{ days.}$$

**Analysis.**—In this problem, 9 men working 14 days is the first cause; and 12 men working a required number of days, the second cause. The effect of the first cause is the work done, represented by unity or 1; and the effect of the second cause will be the same work or 1.

The required term is an element of the second cause, and may be found by dividing the product of the extremes by the product of the given means.

**Problem.**—If 82 men build a wall 36 feet long, 8 feet high, and 4 feet thick, in 4 days, in what time will 48 men build a wall 864 feet long, 6 feet high, and 3 feet thick?

STATEMENTS.

$$\begin{array}{lll} \text{1st cause} : \text{2d cause} & :: & \text{1st effect} : \text{2d effect.} \\ \left.\begin{array}{ll} \text{82 men} & : \text{48 men} \\ \text{4 days} & : (\quad) \text{ days} \end{array}\right\} & :: & \left\{\begin{array}{ll} \text{36 ft. long} & : \text{864 ft. long.} \\ \text{8 ft. high} & : \text{6 ft. high.} \\ \text{4 ft. thick} & : \text{3 ft. thick.} \end{array}\right. \end{array}$$

**Analysis.**—In this problem, both ratios are compound, and the required number of days is an element of the second term.

The second term may be completed by dividing the continued product of the first and last terms by the product of the remaining terms of the means.

PROCESS.

$$\frac{\overset{41}{\cancel{82}} \times \cancel{4} \times \overset{\overset{\cancel{36}}{\cancel{72}}}{\cancel{864}} \times \overset{3}{\cancel{6}} \times 3}{\underset{\underset{\cancel{2}}{\cancel{24}}}{\cancel{48}} \times \cancel{36} \times \underset{4}{\cancel{8}} \times \cancel{4}} = \frac{369}{4} = 92\tfrac{1}{4} \text{ days.}$$

### WRITTEN EXERCISES.

52. If 20 men earn \$180 in 8 days of 10 hours, how much will 36 men earn in 12 days of 8 hours?

53. If a garrison of 900 men use 1250 barrels of flour in 3 mos., how long will 5000 barrels of flour last a garrison of 2700 men?

54. If \$850 earn \$150 in 8 mos., what sum will it take to earn \$62.50 in 1 year?

55. How long will it take 60 men to build a wall 660 ft. long, 10 ft. high, and 3 ft. thick, if 80 men can build a wall 1000 ft. long, 8 ft. high, and 2 ft. thick in 20 days?

The above example may be solved by analysis, as follows: —

In a wall 1000 ft. long, 8 ft. high, and 2 ft. thick there are 16000 cubic feet; and if 80 men build 16000 cu. ft. of wall in 20 days, 1 man can build in the same time $\frac{1}{80}$ as much, or 200 cu. ft., and in one day he can build $\frac{1}{20}$ of 200 cu. ft., or 10 cu. ft.

In a wall 660 ft. long, 10 ft. high, and 3 ft. thick there are 19800 cu. ft.

If one man can build 10 cu. ft. of wall in one day, 60 men can build 60 times as much, or 600 cu. ft., and it will take 60 men as many days to build the entire wall as 600 is contained times in 19800, or 33 days.

## ANALYSIS.

It will be a profitable exercise for the pupil to work over again all the examples given in Simple and Compound Proportion, solving them by Analysis.

REVIEW. — Recite the rule for calculating interest by the six per cent method. (40.) For finding the interest at any rate by the six per cent method. (41.) Give the formula for finding the interest when the principal, rate per cent, and time are given. (43.) For finding the principal when the interest, rate per cent, and time are given. (44.) For finding the principal when the amount, time, and rate are given. (45.) For finding the rate when the interest, principal, and time are given. (46.) For finding the time when the principal, interest, and rate are given. (47.) What is annual or semi-annual interest? (49.) What is a partial payment? (50.) A promissory note? (51.) Who is the maker, or drawer, of a note?

## PARTNERSHIP.

***100. A Partnership*** or ***Firm*** is an association of two or more persons for the purpose of carrying on some business. The association adopts a name, and the members agree to share the profits and losses.

Partnerships are of two kinds.

1. A **General Partnership** is one in which each partner is responsible to the amount of his entire property.

2. A **Limited Partnership** is one in which each partner is liable for a certain specified amount.

Partnerships to exist for a certain specified time are sometimes called *Limited Partnerships.*

***101. The Partners*** are the individuals whose property is invested.

Partners whose names do not appear in the title of the firm are sometimes called *silent partners.*

In many firms, some of the partners are general partners; while one or more, usually silent partners, are liable for only a specified sum: such are called *special partners.*

***102. The Capital*** or ***Stock*** is the money, or property, used by the firm in their business.

***103. A Dividend*** is the gain divided between the partners.

***104. An Assessment*** is a tax levied to meet losses incurred in the business.

***105.*** TO FIND EACH PARTNER'S SHARE OF THE GAIN OR LOSS, WHEN THE CAPITAL OF EACH HAS BEEN USED FOR THE SAME TIME.

PROBLEM. — A, B, and C engage in trade. A furnishes $400, B $300, and C $500: at the end of one year, they have gained $420. What is each one's share?

PROCESS (*a*).

$400 + $300 + $500 = $1200.
$1200 : 400 :: 420 : $140, A's share of gain.
$1200 : 300 :: 420 : $105, B's share of gain.
$1200 : 500 :: 420 : $175, C's gain.

PROCESS (*b*).

A's share $\frac{400}{1200} = \frac{1}{3}$. $420 × $\frac{1}{3}$ = $140, A's gain.
B's share $\frac{300}{1200} = \frac{1}{4}$. $420 × $\frac{1}{4}$ = $105, B's gain.
C's share $\frac{500}{1200} = \frac{5}{12}$. $420 × $\frac{5}{12}$ = $175, C's gain.

ANALYSIS. — (*a*.) The whole capital has the same relation to A's capital, that the whole gain has to A's share of the gain: hence $1200 (total capital) : $400 (A's capital) :: $420 (total gain): $140 (A's gain).

ANALYSIS. — (*b*.) If A invests $400 in a business of $1200 capital, his share of the capital is $\frac{400}{1200} = \frac{1}{3}$; and his share of the gain will be $\frac{1}{3}$ of $420, or $140. B invests $300, or $\frac{1}{4}$ of the capital, and is entitled to $\frac{1}{4}$ of the gain, or $105; and C, whose share is $\frac{5}{12}$ of the capital, will have $\frac{5}{12}$ of the gain, or $175

RULE I. — *Make the gain or loss the third term of as many proportions as there are partners, of which the partners' shares respectively shall be the second terms, and the total capital the first term.*

II. *The fourth term of each proportion will be the respective partner's share of the gain.*

Or, *multiply the entire gain by each partner's share of the capital.*

### WRITTEN EXERCISES.

1. A, B, and C entered into partnership: A put into the business $600, B $450, and C $350: they gain $350. What is the share of gain to which each is entitled?

2. In a joint-stock company, with a capital of $4800, of which A owns $2000, and B the remainder, the annual dividend is $1680. What is each man's share?

3. In a certain business, the capital was $18000, of which A contributed $\frac{1}{3}$, B $\frac{1}{2}$, and C the remainder: the profits for one year were $4530. What was each man's share of the gain?

4. Messrs. Jones and Taylor buy a farm, Jones paying $3500, and Taylor $4200. The farm rents for $924. What part of the rent should each receive?

5. A, C, and D united in the purchase of a mine, A paying $3200, C twice as much as A, and D as much as A and C. They sold the mine at a loss of $540. What was each man's share of the loss?

6. A, B, and C hire a pasture for a year for $350. A puts in 80 cows, B 40 cows, and C 20 cows. How much should each man pay?

7. B and H shipped grain from Chicago to New York, B sending 3600 bushels, and H 3200 bushels: on the

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**Review.**—What is the face of a note? (52.) What is an indorsement? (53.) Give the supreme court rule. (54.) The mercantile rule. (55.) What is a bank? (56.) A bank note? A bankable note? What are days of grace? (57.) What is the maturity of a note? (58.) What is bank discount? (59.) What is the difference between bank discount and true discount? Give the rules for finding the bank discount, and the proceeds of a note. (60.) For finding the face of a note when the time, rate, and proceeds are given. (61.) What is present worth? (62.) Discount? (63.) Commercial discount? Give the rules for finding the present worth and discount of a note. (64.)

voyage, 1326 bushels were spoiled by leakage. What was each man's share of the loss?

8. A, B, and C gained $520 in a business to which A contributed $1500, B $2500, and C $2000. What was each partner's share?

9. A house was damaged by fire to the amount of $4000. It was insured in the Home Insurance Company for $3000, in the Continental for $5600, and in the Merchants' for $4000. What share of the loss should each company sustain?

10. Two men, A and B, agree to build a bridge for $26000. A is to advance money for materials, and B to hire and pay the workmen. At settlement, A had advanced $8000, and B had furnished 4000 days' labor, which cost $2 per day. B was to receive $1500 for taking charge of the work, before the division of the profits. What were the profits? What did each man receive?

### *106.* TO FIND THE PARTNERS' SHARES OF THE GAIN OR LOSS, WHEN THEIR AMOUNTS OF CAPITAL HAVE BEEN USED FOR DIFFERENT TIMES.

PROBLEM.—A, B, and C engage in a business. A put in $400 for 5 mos., B $350 for 4 mos., and C $500 for 2 mos: they gained $540. What was each man's share of the gain?

PROCESS.

$\$400 \times 3 = \$1200$ for 1 mo. $\$540 \times \frac{1200}{3600}$ or $\frac{1}{3} = \$180$, A's gain.
$\$350 \times 4 = \$1400$ for 1 mo. $\$540 \times \frac{1400}{3600}$ or $\frac{7}{18} = \$210$, B's gain.
$\$500 \times 2 = \$1000$ for 1 mo. $\$540 \times \frac{1000}{3600}$ or $\frac{5}{18} = \$150$, C's gain.
$\$3600$ for 1 month.

**Analysis.** — (1.) The use of \$400 for 3 mos. is equal to the use of \$1200 for 1 mo. The use of \$350 for 4 months is equal to the use of \$1400 for 1 month. The use of \$500 for 2 mos. is equal to the use of \$1000 for 1 mo.

(2.) Hence the use of the entire capital for the specified times is equal to the use of \$3600 for 1 month.

(3.) As A's share of the capital is $\frac{1200}{3600}$ or $\frac{1}{3}$, he must have $\frac{1}{3}$ of the gain, or \$180. In the same way, B's share is $\frac{1400}{3600}$ or $\frac{7}{18}$, or \$210, and C's share is $\frac{1000}{3600}$ or $\frac{5}{18}$, or \$150.

**Rule.** — *Multiply each partner's share by the time for which it was employed. Each of these products divided by the sum of the products will be the respective partner's part of the capital, and his corresponding part of the loss or gain.*

### WRITTEN EXERCISES.

11. Brown and Butler engaged in business. Brown put in \$600 for 8 mos., and Butler \$800 for 6 mos. Their gain was \$1682.50. What was each one's share?

12. A, B, and C formed a partnership. A furnished \$600 for 5 mos., B \$500 for 4 months, and C \$2500 for 3 mos. The gain was \$3750. What was each man's share?

13. A and B united in a partnership. A contributed \$240 for 8 mos., and B \$560 for 5 mos. They lost \$118. How much did each man lose?

14. Messrs. Jones, Fuller, and Nash entered into partnership. Jones contributed \$5400 for 3 mos., Fuller \$3500 for 6 mos., and Nash \$4000 for 8 mos. The gain in trade was \$3633. What was each man's share?

## ALLIGATION.

**107. *Alligation*** treats of the mixing of articles of different values and qualities.

**108. *Alligation Medial*** is the process of finding the medium price of a mixture when the price and quantity of each ingredient are given.

**109. *Alligation Alternate*** is the process of finding the quantities of different values or qualities required to make a mixture at a fixed price or quality.

**110.** TO FIND THE MEAN OR AVERAGE VALUE.

PROBLEM. — A grocer mixed 60 lbs. of sugar at 15½ cts. with 30 lbs. at 18 cts., and 20 lbs. at 20 cts. What was a lb. of the mixture worth?

PROCESS.

$.155 × 60 lbs. = $9.30
.18 × 30 lbs. = 5.40
.20 × 20 lbs. = 4.00
110 )18.70
.17 per lb.

ANALYSIS. — (1.) The cost of 60 lbs. of sugar at 15½ cts. is $9.30; of 30 lbs. at 18 cts. is $5.40; of 20 lbs. at 20 cts. is $4.00; and the entire cost is $18.70.

(2.) If 110 lbs. cost $18.70, 1 lb. will cost $\frac{1}{110}$ of $18.70, or 17 cts.

RULE. — *Divide the cost of all the ingredients by the sum of the quantities mixed, and the quotient will be the average value.*

### WRITTEN EXERCISES.

1. A farmer mixed 10 bushels of wheat at $1.40, 15 bu. of oats at 60 cts., and 25 bu. of corn at 40 cts.: *what is a bushel of the mixture worth?*

2. A grocer mixed 10 gallons of water with 40 gallons of brandy worth $1.25 a gallon: what is a gallon of the mixture worth?

3. A grocer mixed 5½ lbs. of tea at $1.20 per lb., 2½ lbs. at 70 cts., and 2 lbs. at 60 cts.: how much is 1 lb. of the mixture worth?

4. A man mixed 25 lbs. of coffee worth 50 cts. per lb., with 10 lbs. of chicory worth 15 cts. per lb.: what is the value of 1 lb. of the mixture?

5. If 18 gal. of wine at $2.00 per gal., 26 gal. at $3.20, and 20 gal. at $4.80, be mixed with 16 gal. of water, what is a gallon of the mixture worth?

### *111.* TO FIND THE QUANTITY OF EACH INGREDIENT IN A MIXTURE OF A GIVEN VALUE.

PRINCIPLE. — *In a mixture the total loss on the ingredients used of the several prices or qualities must equal the total gain.*

PROBLEM. — A grocer wished to mix four kinds of tea, worth respectively $.80, $.90, $1.00, and $1.20 per lb. How many lbs. of each shall he take to make a mixture worth $.95 per lb.?

PROCESS.

$$
\begin{array}{rl}
 & \begin{array}{r|rcll} \phantom{\$.80} & \text{cts.} & & \text{lbs.} & \quad\text{cts.} \end{array} \\
\$.95 & \left\{ \begin{array}{r|rcll}
\$.80 & +15 & \times & 1 & = 15 \\
.90 & +5 & \times & (1+2) & = 15 \\
 & & & & \text{— 30 cts. gain.} \\
1.00 & -5 & \times & 1 & = 5 \\
1.20 & -25 & \times & 1 & = 25 \\
 & & & \overline{\phantom{0}} & \text{— 30 cts. loss.} \\
 & & & 6 &
\end{array} \right.
\end{array}
$$

ANALYSIS. — (1.) For convenience, arrange the prices of the simples in a column in the order of their values, with the given mean at the left. Mark the gains *plus*, and the losses *minus*.

(2.) By using 1 lb. of tea worth 80 cts. in a mixture to be worth *95 cts., the gain is 15 cts.*: by using 1 lb. at 90 cts. the gain is 5 cts.

(3.) By using 1 lb. tea worth $1.00 in a mixture to be worth 95 cts., the loss is 5 cts.; and by using 1 lb. at $1.20, the loss is 25 cts.

(4.) By using one pound of each (the smallest integral quantity), the loss is 10 cents more than the gain; and, since the gain and loss must be equal, take an additional quantity of *any* of the simples below the mean price sufficient to gain 10 cts., or 2 lbs. at 90 cts.: hence the mixture will consist of 1 lb. at 80 cts., 3 lbs. at 90 cts., 1 lb. at $1.00, and 1 lb. at $1.20, — a total of 6 lbs.

(5.) Prove by Alligation Medial.

RULE. — I. *Arrange the prices or qualities of the ingredients in a vertical column, with the mean price or quality at the left.*

II. *Find the gain or loss on* 1 *unit of each. Take such an additional portion of any as will make the losses balance the gains.*

NOTE. — (1.) There can be an *indefinite* number of mixtures made, each having different proportional quantities of the several ingredients. The only condition to be observed is, that the losses and gains shall be equal.

(2.) Pupils should be required to exercise their ingenuity in compounding mixtures differing in the proportional quantities of the several ingredients. Each example should be proved by Alligation Medial.

PROBLEM. — A grocer mixed sugars at 16, 18, and 22 cents, to sell the mixture at 20 cents per lb.: how much was taken of each?

PROCESS.

$$\begin{array}{c}\text{cts.}\\20\end{array}\left\{\begin{array}{r|lllll}
\text{cts.} & \text{cts.} & & \text{lbs.} & & \text{cts.}\\
16 & +4 & \times & 1 & = & 4\\
18 & +2 & \times & 1 & = & 2\\
 & & & & & \text{—}\ 6\text{ cts. gain.}\\
22 & -2 & \times & (1+2) & = & 6\\
 & & & & & \text{—}\ 6\text{ cts. loss.}
\end{array}\right.$$

| | | |
|---|---|---|
| **Proof.** — | 1 lb. | $.16 |
| | 1 lb. | .18 |
| | 3 lbs. at $.22 = | .66 |
| | | 5)$1.00 |
| | | $.20 |

###### WRITTEN EXERCISES.

6. What quantities of coffee, worth respectively 28, 32, 40, and 48 cents, must be mixed to sell at 36 cents per lb.?

7. A farmer has sheep worth $3.50, $6, and $7 each: how many of each price must be taken to average $5 per head?

8. In what proportion shall oats at $.60 per bushel be mixed with corn at $.90 per bushel to sell the mixture at 80 cents.

9. A jeweler melted together gold of 16 carats, 12 carats, and 18 carats, to make an alloy of 15 carats: how many oz. of each did he take?

***112.*** TO FIND THE QUANTITY OF EACH INGREDIENT, WHERE THE QUANTITY OF ONE INGREDIENT, OR OF THE MIXTURE, IS GIVEN.

**Problem.** — A merchant having wheat at $1.50, $1.75, $1.55, and $1.80, wished to make a mixture of 150 bushels, which could be sold at $1.60 per bushel: how many bushels must he take of each?

###### PROCESS.

| | | cts. | bush. | cts. | |
|---|---|---|---|---|---|
| $1.60 | $1.50 | + 10 | × (1 + 2) | = 30 | |
| | 1.55 | + 5 | × 1 | = 5 | |
| | | | | — | 35 cts. gain. |
| | 1.75 | − 15 | × 1 | = 15 | |
| | 1.80 | − 20 | × 1 | = 20 | |
| | | | —<br>6 | — | 35 cts. loss. |

bush.
150 ÷ 6 = 25

bush. bush.
3 × 25 = 75 at $1.50
1 × 25 = 25 at 1.55
1 × 25 = 25 at 1.75
1 × 25 = 25 at 1.80

**Analysis.** — Since, to make a mixture containing 6 bush., it requires 3 bush. at $1.50, 1 bush. at $1.55, 1 bush. at $1.75, and 1 bush. at $1.80, to make a mixture containing 150 bush. it will take as many times the amounts of the several ingredients as 6 bush. are contained times in 150 bush., or 25 times: hence it will require 75 bush. at $1.50, 25 bush. at $1.55, 25 bush. at $1.75, and 25 bush. at $1.80.

**Note.** — To avoid fractions, the amount of the mixture containing the smallest integral quantities of the several ingredients should, if possible, be an aliquot part of the given amount.

### WRITTEN EXERCISES.

10. How much sugar at 8, 10, and 12 cents per lb. must be mixed with 72 pounds at 16 cents, so that the mixture may be worth 14 cents per lb.?

11. A farmer has oats at 40, 60, and 70 cents a bushel: what quantity of each must he take to make a mixture of 40 bushels, worth 50 cents a bushel?

12. How much wool at 35 and at 40 cents a pound must be mixed with 100 lbs. at 60 cents, to make the mixture worth 45 cents per lb.?

13. A broker has an order for 1000 bushels of wheat at $1.25 per bushel: how shall he mix wheat which he values at $1.15, $1.20, $1.35, and $1.50, to fill the order?

14. How many gallons of alcohol of 90 % and of 85 % strong must be mixed with 100 gallons of water to make the mixture 80 % strong?

## POWERS AND ROOTS.

***113. A Power*** of any number is the number itself, or the product of the number, taken as a factor a certain number of times.

***114. Involution*** is the process of finding a power.

1. **The first power** is the number itself.
2. **The second power** or the **square** is the product of two equal factors.
3. **The third power** or the **cube** is the product of three equal factors.

NOTE.—The area of a geometrical square is the product of two equal factors (the length by the breadth): hence the product of two equal factors is called a square. For a similar reason, the product of three equal factors is called a cube.

4. **The exponent or index** is a small figure placed above and at the right of the number, to show how many times it is taken as a factor; thus,

$5 = 5^1 = 5$, 1st power.
$5 \times 5 = 5^2 = 25$, 2d power.
$5 \times 5 \times 5 = 5^3 = 125$, 3d power.
$5 \times 5 \times 5 \times 5 = 5^4 = 625$, 4th power.

PROBLEM.—What is the square of 23?

### ILLUSTRATION AND PROCESS.

| (*a.*) | (*b.*) | (*c.*) | (*d.*) |
|---|---|---|---|
| $23 =$ | $20 + 3$ | $\overset{t}{2} + \overset{u}{3} =$ | $t + u$ |
| $23 =$ | $20 + 3$ | $2 + 3 =$ | $t + u$ |
| $69 =$ | $60 + 9$ | $1\,(2 \text{ tens} \times 3) + 3^2 =$ | $tu + u^2$ |
| $46 =$ | $400 + 60$ | $(2 \text{ tens})^2 + 1\,(2 \text{ tens} \times 3)$ | $t^2 + tu$ |
| $529 =$ | $400 + 120 + 9 =$ | $(2 \text{ tens})^2 + 2\,(2 \text{ tens} \times 3) + 3^2 =$ | $t^2 + 2tu + u^2$ |

**ANALYSIS.**—It is evident that 23 is composed of 20 units + 3 units (*b.*), or of 2 tens + 3 units (*c.*). When each part of the number is multiplied separately, the result in (*b.*) = 400 + 120 + 9 units; in (*c*) = the square of 2 tens + 2 times (2 tens × 3) + the square of 3 units. Substituting the letters $t$, for the 2 tens, and $u$ for the 3 units, and omitting the signs of multiplication, in (*d.*), the result is $t^2 + 2\,t\,u + u^2$: hence we have the following

**FORMULA.**— *The square of a number expressed by two figures equals the square of the tens, plus twice the product of the tens by the units, plus the square of the units; or,* $(t + u)^2 = t^2 + 2\,t\,u + u^2$.

### WRITTEN EXERCISES.

**PROBLEM.**— Write the square of 85.

$$\begin{array}{rr}
\text{PROCESS.} — 80^2 = & 6400 \\
2(80 \times 5) = & 800 \\
5^2 = & 25 \\ \hline
 & 7225
\end{array}$$

Write in the same manner the squares of each of the following numbers:—

1. 87; 76; 57; 21.
2. 49; 85; 64; 54.
3. 34; 93; 32; 67.
4. 68; 55; 24; 38.
5. $\frac{15}{18}$; $\frac{14}{15}$; $\frac{21}{25}$; $\frac{27}{30}$.
6. .17; .025; .031; .0025.

**PROBLEM.**— What is the cube of 23?

Applying the formula for the square, we have,

$$\begin{array}{lr}
(20 + 3)^2 = & 20^2 + 2\,(20 \times 3) + 3^2 \\
\text{Multiplying by } 20 + 3 & 20 + 3 \\ \hline
 & (20^2 \times 3) + 2\,(20 \times 3^2) + 3^3 \\
 & 20^3 + 2\,(20^2 \times 3) + \quad (20 \times 3^2) \\ \hline
\end{array}$$

Substituting letters for figures, and omitting the signs of multiplication, we have,

$$\begin{array}{lr}
(t + u)^2 = & t^2 + 2\,t\,u + u^2 \\
\text{Multiply by } t + u & t + u \\ \hline
 & t^2\,u + 2\,t\,u^2 + u^3 \\
 & t^3 + 2\,t^2\,u + \quad t\,u^2 \\ \hline
\end{array}$$

$$(20 + 3)^3 = 20^3 + 3\,(20^2 \times 3) + 3\,(20 \times 3^2) + 3^3 = (t + u)^3 = t^3 + 3\,t^2\,u + 3\,t\,u^2 + u^3$$

*Hence* we have the following

**FORMULA.** — *The cube of a number expressed by two figures equals the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units; or,* $(t+u)^3 = t^3 + 3\,t^2\,u + 3\,t\,u^2 + u^3$.

**PROBLEM.** — Write the cube of 43.

$$\begin{array}{rr} \text{PROCESS.} — 40^3 = & 64000 \\ 3 \times (40^2 \times 3) = & 14400 \\ 3 \times (40 \times 3^2) = & 1080 \\ 3^3 & 27 \\ \hline & 79507 \end{array}$$

In the same manner write the cubes of each of the following numbers: —

7. 23; 45; 24; 53.
8. 47; 83; 96.
9. 27; 84; 63; 54.
10. 37; 29; 98.
11. $\frac{12}{16}$; $\frac{25}{32}$; $1\frac{11}{13}$; $2\frac{3}{24}$.
12. 4.5; .25; .018; .0036.

## EXTRACTION OF ROOTS.

**115. *The Root*** of a number is one of the equal factors that will produce it.

1. The **First Root** is the number itself.

2. The **Square Root** is one of the two equal factors of the number.

3. The **Cube Root** is one of the three equal factors of the number.

4. A **Perfect Power** is the product of equal factors. It has an exact root.

5. An **Imperfect Power** has no *exact* root.

6. The **Radical Sign** is a character placed before the number to show that its root is to be taken; thus $\sqrt{\phantom{x}}$ denotes the *square root;* $\sqrt[3]{\phantom{x}}$ denotes the *cube root;* $\sqrt[5]{\phantom{x}}$ denotes the 5th root, etc.

**NOTE.** — The root of a number is also indicated by a fractional exponent: thus $8^{\frac{1}{3}}$ denotes the cube root of 8; $16^{\frac{1}{2}}$ denotes the square root of 16; $8^{\frac{2}{3}}$ *denotes* the cube root of the square of 8.

***116. Evolution*** is the process of finding the root of a number.

## *117.* TO FIND THE NUMBER OF FIGURES IN THE ROOT OF ANY NUMBER.

TABLE.

| SQUARE. | | | CUBE. | | |
|---|---|---|---|---|---|
| $1^2 =$ | $1$ | 1 period. | $1^3 =$ | $\dot{1}$ | 1 period. |
| $9^2 =$ | $81$ | | $9^3 =$ | $72\dot{9}$ | |
| $10^2 =$ | $\dot{1}0\dot{0}$ | 2 periods. | $10^3 =$ | $\dot{1}00\dot{0}$ | 2 periods. |
| $99^2 =$ | $9\dot{8}0\dot{1}$ | | $99^3 =$ | $97\dot{0}29\dot{9}$ | |
| $100^2 =$ | $\dot{1}0\dot{0}0\dot{0}$ | 3 periods. | $100^3 =$ | $\dot{1}00\dot{0}00\dot{0}$ | 3 periods. |
| $999^2 =$ | $9\dot{9}8\dot{0}0\dot{1}$ | | $999^3 =$ | $99\dot{7}00\dot{2}99\dot{9}$ | |
| $1000^2 =$ | $\dot{1}0\dot{0}0\dot{0}0\dot{0}$ | 4 periods. | $1000^3 =$ | $\dot{1}00\dot{0}00\dot{0}00\dot{0}$ | 4 periods. |
| $9999^2 =$ | $9\dot{9}9\dot{8}0\dot{0}0\dot{1}$ | | $9999^3 =$ | $99\dot{9}70\dot{0}02\dot{9}99\dot{9}$ | |

From the above table it will be seen, by inspection, that the *square* of any number contains twice as many, or one less than twice as many, figures as the root.

It will also be seen that the *cube* of any number does not contain more than three times as many figures as the root.

For **Square Root,**

I. *Separate the given number into periods of two places each, beginning* AT *units in pointing integers, and counting* FROM *units in pointing decimals.*

For **Cube Root,**

II. *Separate the given number into periods of three places each, beginning* AT *units in pointing integers, and counting* FROM *units in pointing decimals.*

---

REVIEW.—What is a bond? (65.) A coupon? (66) A registered bond? A coupon bond? What are consols? What is currency? (67.) What is exchange? (68.) A bill of exchange? (69.) An acceptance? An inland bill? (70.) A foreign bill? (71) Define equation of payments. (78) Equated time or average time. (79.) *The term* of credit. (80.) Average term of credit. (81.) What is an account? (82.) *A balance?* A cash balance? The focal date?

## SQUARE ROOT.

**PROBLEM.** — What is the square root of 4225?

PROCESS AND ILLUSTRATION.

$$\begin{array}{lcr}
t^2 + 2tu + u^2 & = & 4\dot{2}2\dot{5}(65 \\
t^2 & = & \underline{36} \\
\hline
2tu + u^2 & = & 625 \\
2tu & = (120 \times 5) & \underline{600} \\
\hline
u^2 & = & 25 \\
u^2 & = & 25 \\
\hline
\end{array}$$

**ANALYSIS.** — (1.) Since the number has two periods, the root has two figures, tens and units.

(2.) The square of the tens is part of the first period. The largest square in 42 is 36, the root of which is 6, the tens' figure of the required root.

(3.) The square of the tens taken from the power leaves a remainder of 625, which contains twice the product of the tens by the units, plus the square of the units.

(4.) Twice 6 tens are 12 tens, or 120, and 120 is contained 5 times in 625: 5 is therefore *probably* the unit figure of the root: 5 times 120 = 600, which deducted from 625 leaves 25.

(5.) But $u^2$ remains; and, as $u = 5$, $u^2 = 25$, the exact number of units remaining: hence the given number is a perfect square, of which 65 is the square root.

CONTRACTED PROCESS.

$$\begin{array}{rr}
 & 4225(65 \\
 & \underline{36} \\
125) & 625 \\
 & \underline{625} \\
\end{array}$$

**ANALYSIS.** — (1.) Deduct the square of the tens' figure from the first period.

(2.) Twice 6 tens = 12 tens, the trial or partial divisor. 12 tens is contained in 62 tens 5 times. As 5 is the probable unit figure, add it to 12 tens as a complete divisor. Multiply this divisor by the unit figure, and deduct the product from the remainder of the power.

NOTE. — If the complete divisor should prove too large, the unit figure of the root is too large.

---

**REVIEW.** — What is ratio? (85.) What are the terms of a ratio? (86.) What is the antecedent? (87.) The consequent? (88.) What is the sign of ratio? (89.) Define simple ratio. (90.) Compound ratio. (91.) What is the value of the ratio? (92.) What is the effect of multiplying the antecedent? (93.) Of multiplying the consequent? Of dividing the antecedent? Of dividing the consequent? Of multiplying or dividing both antecedent and consequent by the same number?

**PROBLEM.** — What is the square root of 784?

### GEOMETRICAL ILLUSTRATION.

**PROBLEM.** — What is the length of each side of a square piece of zinc containing 784 sq. in.?

Fig. 1.

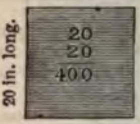


Fig. 2.

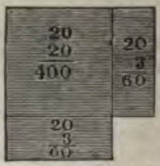
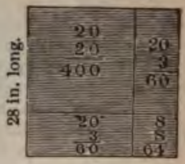


Fig. 3.



PROCESS.

$$\begin{array}{lrl} & \dot{7}8\dot{4} & \underline{|20+8}=28 \\ 20\times 20= & \underline{400} & \\ & 384 & \\ 20\times 2=40 & & \\ 40\times 8= & \underline{320} & \\ & 64 & \\ 8^2= & \underline{64} & \end{array}$$

PROOF.

sq. in.

$$\begin{array}{r} 20\times 20=400 \\ 20\times\ 8=160 \\ 20\times\ 8=160 \\ 8\times\ 8=\underline{\ 64} \\ 784 \end{array}$$

ANALYSIS. — (1.) Since the length of the zinc is equal to the breadth, and the product of the length by the breadth gives the area, the square root of 784 sq. in. will give the length of each side of the zinc.

(2.) Since the number contains two periods, the root will contain two figures.

(3.) By trial, the largest square in the left-hand period is 20 inches, and the product of the length by the breadth = 400 square inches (Fig. 1), which subtracted from 784 sq. in. leaves 384 sq. in.

(4.) In order to preserve the form of the square, the remainder must be added to two of the sides. (Fig 2.) Since the length of each side is 20 inches, the length of two sides must be twice 20 inches, or 40 inches.

(5.) Since there are 40 sq. in. in one row of the addition to the two sides, the addition will be as many rows of sq. in. in width as 40 sq. in. are contained times in 384 sq. in., or 8. If the length of the entire addition is 40 in., and the width is 8 in., the addition to the two sides will contain 8 times 40 sq. in. = (20 sq. in. × 8) + (20 sq. in. × 8) or 320 sq. in. (Fig. 2): 320 sq. in. subtracted from 384 sq. in. leave 64 sq. in.

(6.) To complete the square (Fig. 2), it is necessary to add the small square 8 in. long and 8 in. wide (Fig. 3), containing 64 sq. in., which subtract from 64 sq. inches: hence the zinc is 28 inches long and 28 inches wide.

**RULE I.** — *Separate the number into periods of two figures each.*

**II.** — *Find the root of the left-hand period; subtract its square from the period, and to the remainder annex the next period for a dividend.*

III.—*Multiply the root found by* 20 *for a trial or partial divisor; divide, and write the quotient for the next figure in the root, also add it to the partial divisor.*

IV.—*Multiply and subtract, as in simple division, and to the remainder annex the next period for a new dividend. Proceed as before until all the periods are exhausted.*

NOTES.—(1.) If the dividend at any time does not contain the complete divisor, place a cipher in the root, and bring down the next period for a new dividend.

(2.) If the given number contains an uneven number of decimal places, make them even by annexing a cipher.

(3.) If an integral number is not a perfect power, annex as many periods of ciphers as there are to be decimal places in the root.

(4.) When the exact root of the terms of a fraction cannot be found, reduce the fraction to an equal fraction of higher or lower terms, that shall have a denominator which is a perfect square; thus $\sqrt{\frac{3}{4}} = \sqrt{\frac{12}{16}} = \sqrt{\frac{1}{16}} \times \sqrt{12} = \frac{1}{4}\sqrt{12} = \frac{1}{4} \times 3.46 + = .86+$, or $\sqrt{\frac{3}{4}} = \sqrt{\frac{12}{16}} = \frac{3.46}{4} = .86+$. Or reduce the fraction to a decimal, and then extract the root.

### WRITTEN EXERCISES.

PROBLEM.—Find the square root of 584926.42.

PROCESS.

$$\begin{array}{rl}
 & 5\dot{8}4\dot{9}2\dot{6}.4\dot{2}(764.804+ \\
7^2 = & \underline{49} \\
7 \times 20 + 6 = & 146)949 \\
146 \times 6 = & \underline{876} \\
76 \times 20 + 4 = & 1524)7326 \\
1524 \times 4 = & \underline{6096} \\
764 \times 20 + 8 = & 15288)1230.42 \\
\&c. & \underline{1223.04} \\
 & 1529604)7.380000 \\
 & \underline{6.118416}
\end{array}$$

ANALYTICAL STEPS.

(1.) *Find the greatest square of the left-hand period.*

(2.) *Multiply the root found by* 20 *for a trial divisor.*

(3.) *Find the next figure in the root, and add it to the trial divisor for a complete divisor.*

(4.) *Multiply the divisor by the last root figure, subtract the product, and to the remainder annex the next period as a new dividend.*

(5.) *Multiply the root now found by* 20, &c.

Find the square root of the following numbers:—

13. 625; 576; 1444; 3025; 4225.

14. 1296; 5476; 400689.
15. 516961; 182329; 23804641.
16. 56.25; 306.25; 680625.
17. 75.364; 6.5; 7; 5; 2; 11.
18. $\frac{2}{3}$; $\frac{36}{25}$; $\frac{12}{27}$; $\frac{2}{18}$; $\frac{5}{20}$.
19. $\frac{144}{289}$; $\frac{1024}{1225}$; $\frac{961}{1849}$.
20. $1\frac{60}{196}$; $2\frac{82}{121}$; $65\frac{16}{125}$.

## CUBE ROOT.

**Problem.** — Find the cube root of 74088.

### PROCESS AND ILLUSTRATION.

$$
\begin{array}{lllr}
t^3 + 3\,t^2 u + 3\,t\,u^2 + u^3 & = & & 74088(42 \\
t^3 & & & \underline{64} \\
\hline
\quad 3\,t^2 u + 3\,t\,u^2 + u^3 & & & \\
\text{or} \quad (3\,t^2 + 3\,t\,u + u^2)u = & & & 10088 \\
\quad 3\,t^2 & = 4800 & & \\
\qquad + 3\,t\,u & = \phantom{0}240 & & \\
\qquad\qquad + u^2 & = \phantom{000}\underline{4} & & \\
(3\,t^2 + 3\,t\,u + u^2)\,u & = 5044 \times 2 & = & \underline{10088}
\end{array}
$$

**Analysis.** — (1.) As the number has two periods, the root has two figures.

(2.) The cube of the tens is part of the left-hand period. The largest cube in 74 is 64; its root is 4, the tens' figure of the required root.

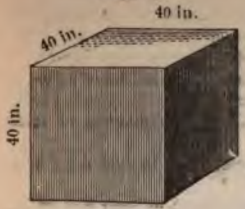
(3.) Subtract the cube of the tens, which leaves a remainder of 10088. This contains 3 times $t^2$ multiplied by $u$ plus 3 times $t$ multiplied by $u^2$ plus $u^3$, or 3 times $t^2$ plus 3 times $t$ multiplied by $u$ plus $u^2$, and this sum multiplied by $u$. Three times $t^2$, or 4800, is contained in 10088 twice, and 2 is probably the units' figure of the root.

(4.) Three times $t^2 = 4800$; 3 times $t$ multiplied by $u = 240$; $u^2 = 4$; the sum, $4800 + 240 + 4 = 5044$; and 5044 multiplied by $u = 5044 \times 2$, or 10088, the exact remainder of the power: hence the root is 42.

### GEOMETRICAL ILLUSTRATIONS.

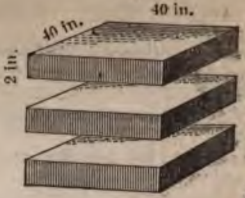
What is the length of each edge of a cubical block containing 74088 cu. in.?

Fig. 1.



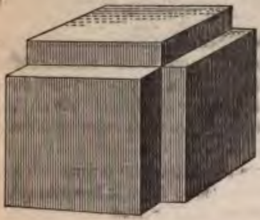
(a.)
40
40
1600
40
64000

Fig. 2.



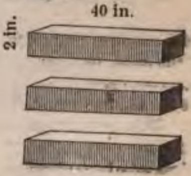
(b.)
40
40
1600, area.
3
4800
2
9600, cont.

Fig. 3.



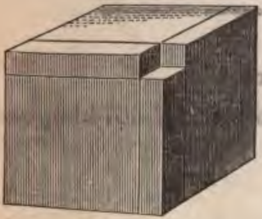
64000
9600
73600

Fig. 4.



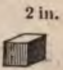
(c.)
40
2
80
2
160
3
480

Fig. 5.



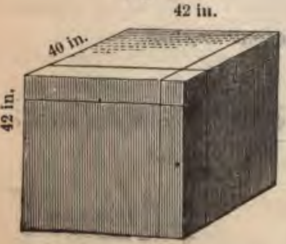
73600
480
74080

Fig. 6.



(d.)
2
2
4
2
8

Fig. 7.



(a.) PROCESS.

74088 74088(40 + 2 = 42, root.
64000 (a.) 40 × 40 × 40 = 64000
10088 40 × 40 × 3 = 4800)10088
9600 (b.) 4800 × 2 = 9600
488 488
480 (c.) 40 × 2 × 2 × 3 = 480
8 8
8 (d.) 2 × 2 × 2 = 8
0 0

ANALYSIS.—(1.) Since the product of the length, breadth, and thickness, gives the solid contents of a cube, the cube root of 74088 cu. in. will give the length of each side of the cubical block.

(2.) Since the number contains two periods, the root will contain two figures.

(3.) By trial the length of the edge of the largest cube in the left-hand period is 40 inches, and the solid contents of this cube is 64000 cu. in. ($40 \times 40 \times 40 = 64000$) (Fig. 1, *a*). This number subtracted from 74088 cu. in. leaves a remainder of 10088 cu. in. (Fig. 7, *a*).

In order to preserve the form of a cube, the remainder must be added to three of the sides (Fig. 3).

(4.) Since the addition to each side of the cube must be 40 in. long and 40 in. wide, the addition to each side at 1 in thick will contain 40 times 40 cu. in., or 1600 cu. in.; and the three sides will contain 3 times 1600 cu. in., or 4800 cu. in. (Fig 2, *b*).

(5.) If it requires 4800 cu. in. to make an addition 1 inch in thickness to the three sides of the cube, 10088 cu. in. will make an addition as many inches thick as 4800 cu. in. are contained times in 10088 cu. in., or 2. If the addition 1 inch thick contain 4800 cu. in., one 2 inches thick will contain 2 times 4800 cu. in., or 9600 cu. in. (Fig 2, *b*). This number subtracted from 10088 cu. in. leaves a remainder of 488 cu. in. (Fig. 7, *b*).

(6.) More nearly to complete the cube, 3 blocks are required, each 40 inches long, 2 in. wide, and 2 in. thick (Fig. 4), containing 480 cu. in. ($40 \times 2 \times 2 \times 3 = 480$) (Fig. 4, *c*). 480 cu. in. subtracted from 488 cu. in. leaves 8 cu. in. (Fig. 7, *c*).

(7.) To complete the cube, a block is required 2 in. long, 2 in. wide, and 2 in. thick, containing 8 cu. in. ($2 \times 2 \times 2 = 8$) (Fig. 6, *d*). 8 cu. in. is the exact remainder of the power (Fig. 7, *d*): therefore the block is 42 in. on each edge.

### CONTRACTED PROCESS.

PROBLEM.—What is the cube root of 3874.23, correct to three decimal places?

#### PROCESS.

$\dot{3}87\dot{4}.23\dot{0}$(15.705 + root.

$1^3 = 1$

Trial divisor, $1^2 \times 300 = 300$ | 2874 dividend.

$1 \times 5 \times 30 = 150$

$5 \times 5 = 25$ | [and 5.

Complete divisor, 475 | 2375. product of the complete divisor

Trial divisor, $(15)^2 \times 300 = 67500$ | 499.230, new dividend.

$15 \times 7 \times 30 = 3150$

$7 \times 7 = 49$ | [visor and 7.

Complete divisor, 70699 | 494.893, product of the complete di-

Trial divisor, $(157)^2 \times 300 = 7394700$ | 4.337000000, new dividend.

Trial div. $(1570)^2 \times 300 = 739470000$

$1570 \times 5 \times 30 = 235500$

$5 \times 5 = 25$

Complete divisor, 739705525 | 3.698527625, product of the complete divisor and 5.

.638472375, remainder.

RULE I. — *Separate the number into periods of three places each, beginning at unit's place.*

II. — *Find the root of the left-hand period; subtract its cube from the period, and to the remainder annex the next period for a new dividend.*

III. — *Multiply the square of the root found by* 300, *for a trial divisor; divide, and take the quotient for the next figure in the root.*

IV. — *Multiply the product of the root found and this last figure by* 30; *and the product plus the square of this last figure, added to the trial divisor, will form the complete divisor.*

V. — *Multiply and subtract as in simple division, and to the remainder annex the next period as a new dividend. Proceed as before until all the periods are exhausted.*

NOTES. — (1.) If the dividend does not contain the complete divisor, write a cipher in the root, and annex the next period as a new dividend, with which proceed as before.

(2.) If the given number contains a number of decimal places not divisible by *three*, make it divisible by annexing ciphers; thus 32.4 = 32.400.

(3.) When an integral number is not a perfect cube, annex as many periods of ciphers as there are decimal places in the required root.

---

REVIEW. — Define proportion. (94.) What are the antecedents? The consequents? The extremes? The means? What is a mean proportional? Recite Principle I. Principle II. Principle III. Define simple proportion. (95.) Compound proportion. (96.) What is a partnership or firm? (100.) A general partnership? A limited partnership? Who are the partners? (101.)

### WRITTEN EXERCISES.

21. $\sqrt[3]{185193}$
22. $\sqrt[3]{80621568}$
23. $\sqrt[3]{17984728}$
24. $\sqrt[3]{257259456}$
25. $\sqrt[3]{1860867}$
26. $\sqrt[3]{1879080904}$
27. $\sqrt[3]{41673648.563}$
28. $\sqrt[3]{75686967}$
29. $\sqrt[3]{8.144865728}$
30. $\sqrt[3]{4173.281}$
31. $\sqrt[3]{46}$
32. $\sqrt[3]{9}$
33. $\sqrt[3]{81\frac{10}{27}}$
34. $\sqrt[3]{\frac{729}{4096}}$
35. $\sqrt[3]{\frac{3}{5}}$

## *118.* EXTRACTION OF ROOTS BY INSPECTION.

Students will find the following method of great value in the computations of algebra.

TABLE.

| Root. | Power. | Square Root. | | | | | | Root. | Power. | Cube Root. | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **0** | **1** | **4** | **9** | **6** | **5** | | | **1** | **8** | **7** | **4** | **5** | **6** | **3** | **2** | **9** | **0** |
| 1 | 1 | | 1 | | | | | 1 | 1 | 1 | | | | | | | | | |
| 2 | 4 | | | 2 | | | | 2 | 8 | | 2 | | | | | | | | |
| 3 | 9 | | | | 3 | | | 3 | 27 | | | 3 | | | | | | | |
| 4 | 16 | | | | | 4 | | 4 | 64 | | | | 4 | | | | | | |
| 5 | 25 | | | | | | 5 | 5 | 125 | | | | | 5 | | | | | |
| 6 | 36 | | | | | 6 | | 6 | 216 | | | | | | 6 | | | | |
| 7 | 49 | | | | 7 | | | 7 | 343 | | | | | | | 7 | | | |
| 8 | 64 | | | 8 | | | | 8 | 512 | | | | | | | | 8 | | |
| 9 | 81 | | 9 | | | | | 9 | 729 | | | | | | | | | 9 | |
| 10 | 100 | 0 | | | | | | 10 | 1000 | | | | | | | | | | 0 |

N.B.—*The final figure of the power is at the head of the column, the final figure of the corresponding root is in the same column below it.*

It will be seen by the above table that the *square root* of any perfect power ending in zero must end in zero; and that the square root of any perfect power ending in 4 must either end in 2, or its complement, 8.

It also will be seen that the *cube root* of any perfect power ending in 7 must end in its complement, 3; and that the cube root of a perfect power ending in 9 must end in 9: thus the square root of 7569 may be found by the following

PROCESS.

8 7
75 69
64 49
— —
11 2

EXPLANATION.—The root of the largest square of the left-hand period is 8, which write above the corresponding period. As the power ends in 9, the unit figure of the root must end in 7, or its complement, 3. Try 7.

By trial the square of the 8 subtracted from its corresponding period 75, and the square of 7 subtracted from its corresponding period 69 (omitting the cipher), leaves 112 tens; which number of tens is equal to twice the product of the root figures; or $(8 \times 7) \times 2 = 112$ tens.

## METRIC SYSTEM OF WEIGHTS AND MEASURES.

**119. *The Metric System*** of weights and measures is formed upon the decimal scale, and has for its base an invariable unit derived from nature, and called a METRE.

**120. *The Metre*** is the *ten-millionth* part of the distance from the equator to the pole, and is the unit of linear measure.

**121. *The Are*** (*air*) is a square whose side is ten *metres*. It is the unit of land measure, and contains one hundred *square metres*.

**122. *The Stere*** (*stare*) is a cube whose edge is a *metre*. It is the unit of solid or cubic measure.

**123. *The Litre*** (*leeter*) is a cube whose edge is the *tenth* of a *metre*. It is the unit of all *measures of capacity*.

**124. *The Gramme*** is the weight of a cube of pure water at its greatest density, whose edge is the *hundredth* part of a *metre*. It is the unit of weight. A litre of water weighs 1,000 grammes.

**125. *The Names*** of the derivative denominations are formed by joining a Latin or a Greek prefix to the principal units. There are seven of these prefixes, derived as follows: —

*Latin.*
- MILLI, from *Millesimus*, *a thousandth.*
- CENTI, from *Centesimus*, *a hundredth.*
- DECI, from *Decimus*, *a tenth.*

| | | | |
|---|---|---|---|
| *Greek.* | DECA, | | *ten.* |
| | HECTO, | from *Hecaton,* | *one hundred.* |
| | KILO, | from *Kilioi,* | *one thousand.* |
| | MYRIA, | | *ten thousand.* |

The formation of the tables can be seen at a glance by the following:—

| | | | | | |
|---|---|---|---|---|---|
| *Milli*<br>*Centi*<br>*Deci*<br>—<br>*Deca*<br>*Hecto*<br>*Kilo*<br>*Myria* | METRE. | ARE.* | STERE. | LITRE. | GRAMME. |

### LINEAR MEASURE.

***126. The Centimetre*** is the unit generally used for measurements less than a metre.

***127. The Kilometre*** is the unit commonly used by surveyors in measuring distances.

TABLE.

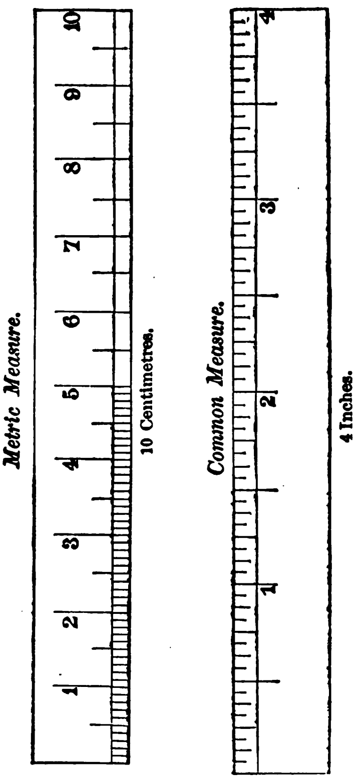
10 *Millimetres* (*mm.*) = 1 *Centimetre* (*cm.*)
10 *Centimetres* = 1 *Decimetre* (*dm.*)
10 *Decimetres* = 1 METRE (*M.*)
10 METRES = 1 Dekametre (*Dm.*)
10 Dekametres = 1 Hectometre (*Hm.*)
10 Hectometres = 1 Kilometre (*Km.*)
10 Kilometres = 1 Myriametre (*Mm.*)
1 Metre = 39.37 in.
1 Kilometre = 198 rods 13 ft. 10 in.

SCALE.

| Read as Kilometres. | | Read as Metres. | | | Read as Centimetres. | | Read as Millimetres. |
|---|---|---|---|---|---|---|---|
| *Mm.* | *Km.* | *Hm.* | *Dm.* | *Metre.* | *dm.* | *cm.* | *mm.* |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

---

* The *a* in *deca* and *myria*, and the o in hecto and kilo, are omitted before *a* in *are*.



NOTE.—By the accompanying illustration it will be seen that *one-tenth* of a metre, or ten centimetres, equals about $3\frac{15}{16}$ in., or a trifle short of 4 in.

This measure, as well as the other measures and weights, are written as whole numbers and decimals. The decimal point is placed at the right of the unit; thus, 4.167 *M.* may be written 416.7 *cm.* The diameter of the nickel five-cent piece is 2 *centimetres*, and its weight is 5 *grammes*.

## SQUARE MEASURE.

***128. The Square Metre*** is the unit commonly used in measuring sufaces of small extent. It contains about 1.196 + sq. yds. 100 square metres make one Are.

***129. The Are*** is the unit of land measure.

***130. The Hectare*** is much used by surveyors in estimating the contents of land. It contains 2.471 *acres. 100* Ares make 1 hectare.

TABLE.

1 Sq. Metre (*Sq. M.*) = 1 Centiare (*ca.*).
100 Sq. Metres
100 Centiares (*ca.*) } = 1 ARE (*A.*).
100 ARES = 1 Hectare (*Ha.*).

NOTE. — A square decimetre equals about 15.5. sq. in.

### CUBIC MEASURE.

**131.** ***The Cubic Metre or Stere*** is the unit used in measuring solids. It contains 35.316 + cu. ft.

TABLE.

1000 Cu. Decimetres (*cu. dm.*)
10 Decistere (*dst.*) } = 1 Stere (*St.*).
10 STERES = 1 Dekastere (*D.St.*).

NOTE. — A cubic decimetre is equal to 61.026 + cu. in.; and a decistere equals 3.531 + cu. ft.

### LIQUID AND DRY MEASURE.

**132.** ***The Litre*** is the unit commonly used by grocers. It equals 1.0567 qts. liquid measure, or .908 qts. dry measure, or a trifle more than a liquid quart.

**133.** ***The Hectolitre*** is the unit commonly used in the measurement of grain, roots, and liquids by the barrel. It equals 26.417 gallons liquid measure, or 2.837 bu. dry measure.

TABLE.

10 Decilitres (*dl.*) = 1 LITRE (*L.*).
100 Litres = 1 Hectolitre (*Hl.*).
10 Hectolitres = 1 { Kilolitre, or (*Kl.*). STERE. (*St.*).

### WEIGHT.

**134.** ***The Gramme*** is the unit commonly used in philosophical experiments by jewellers and druggists. *Its weight* is 15.432 Troy grains.

**135. *The Kilogramme*** is the unit commonly used by grocers, and is usually contracted to kilo. It is the weight of a litre of pure water, and equals 2.2046 lbs. or about $2\frac{1}{5}$ lbs. avoirdupois.

**136. *The Tonneau*** is the unit commonly used for weighing heavy bodies; as, coal, iron, marble, railroad freight, &c. It is the weight of a cubic metre of pure water, and equals 2204.6 lbs. avoirdupois.

### TABLE.

10 Decigrammes = 1 GRAMME (*G.*).
10 Grammes = 1 Dekagramme (*Dg.*).
10 Dekagrammes = 1 Hectogramme (*Hg.*).
10 Hectogrammes = 1 Kilogramme (*Kg.*).
10 Kilogrammes = 1 Myriagramme (*Mg.*).
10 Myriagrammes = 1 Quintal (*Q.*).
10 Quintals = 1 Tonneau (*T.*).

### EXERCISE IN READING.

The decimal point is usually placed at the right of the unit of the scale; thus:—

43.67 *M.* = 43 *M.* 67 *cm.*
25.41 *A.* = 25 *A.* 14 *ca.*
21.04 *S.* = 21 *S.* 4 *cs.*
134.41 *G.* = 1 *Hg.* 34 *G.* 41 *cg.*

### ORAL EXERCISES.

(1)
43.04 *M.* = ?
2.6 *M.* = ?
4134.2 *M.* = ?
6.7 *A.* = ?
12.14 *L.* = ?

(2)
34.4 *A.* = ?
50.3 *M.* = ?
.73 *L.* = ?
25.004 *M.* = ?
24.1 *S.* = ?

(3)
34.06 *S.* = ?
5.4 *L.* = ?
41.43 *G.* = ?
8964.11 *G.* = ?
8345.2 *L.* = ?

## TABLES OF EQUIVALENTS.

### LINEAR MEASURE.

1 in. = 2.54 cm.
1 ft. = 3.048 dm.
1 yd. = .9144 M.

$$1\text{ M.} = \left\{\begin{array}{l}39.37\text{ in.}\\ 1.093\text{ yds.}\end{array}\right.$$

### SQUARE MEASURE.

1 sq. ft. = .0929 sq. M.
1 sq. yd. = .8361 sq. M.
1 acre = 40.47 A.

$$\left.\begin{array}{l}1\text{ sq. M.}\\ 1\text{ ca.}\end{array}\right\} = 1.196\text{ sq. yds.}$$

1 are = 119.6 sq. yds.
1 Ha. = 2.471 acres.

### CUBIC MEASURE.

1 cu. in. = 16.39 cu. cm.

$$1\text{ cu. ft.} = \left\{\begin{array}{l}28.32\text{ L.}\\ .0283\text{ St.}\end{array}\right.$$

1 cu. yd. = .7645 St.
1 cord = 3.624 St.
1 gal. = 3.785 L.
1 bu. = .3524 Hl.

$$1\text{ L.} = \left\{\begin{array}{l}61.02\text{ cu. in.}\\ 1.056\text{ qts. liq. m.} =\\ .908\text{ qts. dry m.}\end{array}\right.$$

$$1\text{ Hl.} = \left\{\begin{array}{l}26.417\text{ gal.}\\ 2.837\text{ bu.}\end{array}\right.$$

$$\left.\begin{array}{l}1\text{ Kl.}\\ 1\text{ cu. M.}\\ 1\text{ Stere}\end{array}\right\} = \left\{\begin{array}{l}264.17\text{ gal.} =\\ 28.372\text{ bu.}\\ 35.316\text{ cu. ft.}\\ 1.308\text{ cu. yds.}\\ .2759\text{ cds.}\end{array}\right.$$

### WEIGHT.

1 gr. Troy = .648 dg.
1 oz. Troy = 31.1 G.
1 lb. Troy = 373.2 G.
1 lb. avoir. = 453.6 G.
1 T. avoir. = .907 Tonneau.

1 cg. = .154 gr. Troy.
1 G. = 15.43 gr. Troy.
1 Kg. = 2.2046 lbs. avoir.
1 Quintal = 220.46 lbs. avoir.
1 Tonneau = 2204.6 lbs. avoir.

REVIEW. — Define capital or stock. (102.) What is a dividend? (103.) An assessment? (104.) Define alligation. (107.) Alligation medial. (108.) Alligation alternate. (109.) What is a power of a number? (113.) What is the first power of a number? The second power, or square? The third power, or cube? Define involution. (114.) What is the root of a number? (115.) The first root? The square root? The cube root? What is a perfect power? An imperfect power? Make the radical sign. Define evolution. (116.) What is the base of the metric system? (119.) What is a metre? (120.) An are? (121.) A stere? (122.) A litre? (123.) A gramme? (124.) How is the centimetre used? (126.) The kilometre? (127.) The square metre? (128.) What is the unit of land measure? (129.) For what is the hectare used? (130.) The cubic metre? (131.) The litre? (132.) The hectolitre? (133.) The gramme? (134.) The kilogramme? (135.) The tonneau? (136.)

## REDUCTION.

**Problems.** — (*a.*) Reduce 30 bu. 3 pks. to hectolitres.

(*b.*) Reduce 25 M. to yards.

### PROCESS.

(*a.*)

$$\begin{array}{rcr} & \text{bu.} & \text{hl.} \\ 4\overline{).3524} \times 30 & = & 10.5720 \\ & \text{pk.} & \\ .0881 \times 3 & = & .2643 \\ \hline & & 10.8363 \end{array}$$

(*b.*)

$$\begin{array}{l} \text{yd.} \\ 1.093 = 1 \text{ M.} \\ \quad 25 \text{ M.} \\ \hline 5.465 \\ 21.86 \\ \hline 27.325 \text{ yds.} = 27 \text{ yds. } 11 \text{ in.} + \end{array}$$

**Anal. Steps.** (*a.*) — (1.) *Find number hl. in 30 bu.*
(2.) *Find number hl. in 1 pk., then 3 pks.*
(3.) *Find total number.*

**Anal. Steps.** (*b.*) — (1.) *Find number yds. in 25 M.*
(2.) *Find number inches in decimal of yds.*

### WRITTEN EXERCISES.

1. How many metres in 5 miles? In 3 miles, 2 rds.?

2. How many metres in 63½ yards of calico; and what is it worth, at 12 cts. per metre?

3. What are the dimensions of the box in feet and inches of a box 3.4 M. long, 1.52 M. wide, and 2.1 M. deep?

4. How many pounds and ounces in a sack of coffee weighing 25 Kg.

5. A merchant bought 130 pounds of sugar at 15 cts. per pound: what did it cost per kilogramme?

6. He sold the sugar at 25 % profit: what was the selling-price per kilo.?

## MENSURATION.

, ***137. A Solid*** is that which has length, breadth, and thickness.

***138. A Straight Line*** is one which does not change its direction; as, AB, Fig. 1.

1.

Fig. 1

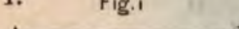


Fig. 2

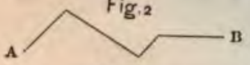
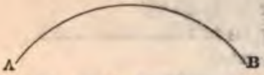


Fig. 3



1. A straight line measures the shortest distance between two points.

2. A **Broken Line** is one which occasionally changes its direction, and is composed of straight lines; as, AB, Fig. 2.

3. A **Curved Line** is one which continually changes its direction; as, AB, Fig. 3.

4. **Parallel Lines** are those everywhere equally distant from each other; as, AB, CD, Fig. 4.

***139. An Angle*** is the difference between two lines which meet; as, ABC, Fig. 5.

Fig. 4

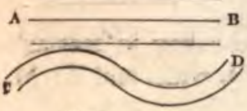
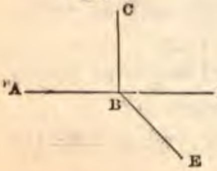


Fig. 5



1. The **Vertex** of the angle is the point at which the lines meet; and, when the angle is named, the letter at the vertex is placed second; as, ABC, Fig. 5.

2. A **Right Angle** is formed when a straight line meets another straight line, making the two adjacent angles equal; as, ABC = CBD, Fig. 5.

3. An **Acute Angle** is an angle less than a right angle; as, EBD, Fig. 5.

4. An **Obtuse Angle** is an angle greater than a right angle; as, ABE, Fig. 5.

*Note.* — *As* a right angle contains 90 degrees, an acute must contain less, and *an obtuse angle* more, than 90 degrees.

***140. A Surface*** has two dimensions, length and breadth.

1. A **Plane Surface** is one which does not change its direction, as the surface of a table, or of a plate of glass.

2. A **Curved Surface** is one that continually changes its direction, as the surface of a ball or cylinder.

***141. A Triangle*** is a plane surface having three sides bounded by straight lines; as, ABC, Fig. 6.

1. The **Base** of a triangle is the side on which it seems to rest. Either side may be the base; as, the base may be AB, or AC, or BC, Fig. 6.

2. The **Altitude** of a triangle is the perpendicular distance from the vertex to either base; thus, AC, or CB, or Ce, is the altitude of the triangle ABC, Fig. 6.

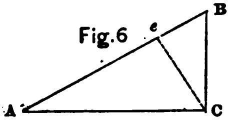


Fig.6

3. A **Right-angled Triangle** is a triangle having a right angle; as, ACB, Fig. 6.

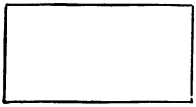
4. The **Hypothenuse** of a right-angled triangle is the side opposite the right angle; as, AB is the hypothenuse of the triangle ACB, Fig. 6.

***142. A Quadrilateral*** is a plane figure having four sides.

1. A **Parallelogram** has its opposite sides parallel; as, Fig. 7.

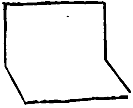
Fig.7

Fig.8



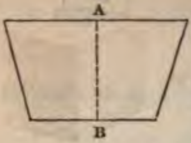
2. A **Rectangle** has its four angles right angles; as, Fig 8.

Fig.9



3. A **Square** has four equal sides and four right angles; as, Fig. 9.

Fig. 10



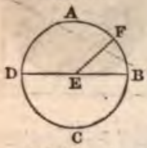
4. A **Trapezoid** has two of its four sides parallel; as, Fig. 10.

5. The **Altitude** of a parallelogram, a rectangle, a square, or a trapezoid, is the perpendicular distance between two of its parallel sides; as, A, B, Fig. 10.

***143. A Circle*** is a plane surface bounded by a line, every point of which is equally distant from a point called the centre; as, Fig. 11.

1. The **Circumference** of a circle is the line by which it is bounded; as, ABCD, Fig. 11.

Fig. 11.

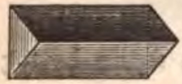


2. The **Diameter** of a circle is a straight line passing through the centre, and terminating in the circumference; as, DB, Fig. 11.

3. The **Radius** of a circle is the distance from the centre to the circumference. It is half the length of the diameter; as, EF, or EB, or ED, Fig. 11.

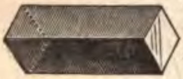
***144. A Prism*** is a solid whose sides are parallelograms, and whose ends are equal and similar; as, Fig. 13.

Fig. 12.



NOTE. — Prisms take their names from the forms of their bases, or ends.

Fig. 13.

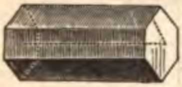


1. A **Triangular Prism** is one whose ends are triangles; as, Fig. 12.

2. A **Square Prism** is one whose ends are squares; as, Fig. 13.

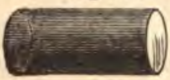
NOTE. — The square prism is also called a *parallelopipedon*.

Fig. 14.



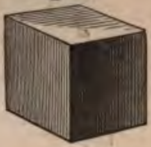
3. A **Hexagonal Prism** is one whose ends are hexagons; as, Fig. 14.

Fig. 15.



4. A **Cylinder** is a prism whose ends are circles; as, Fig. 15.

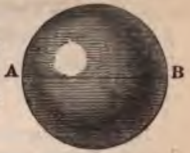
Fig. 16.



5. A **Cube.** — When all the sides of a rectangular prism are square, it is called a *cube*; as, Fig. 16.

***145. A Sphere*** is a solid bounded by a convex surface, every point of which is equally distant from a point within, called the centre; as, Fig. 17.

Fig. 17.



1. The **Circumference** of a sphere is the greatest distance around it.

2. The **Diameter** of a sphere is the greatest distance through it; as, AB, Fig. 17.

## PROBLEMS IN MENSURATION.

***146.*** PROBLEM I. — TO FIND THE AREA OF A RECTANGLE.

EXAMPLE. — How many square inches in a board 4 inches wide and 7 inches long?

PROCESS.
7 (length)
4 (width)
—
28 area.

ANALYSIS. — Since in 1 row of square inches there are 7 sq. inches, in 4 such rows there are 4 times 7 sq. in., or 28 square inches.

RULE. — *Multiply the length by the breadth.*

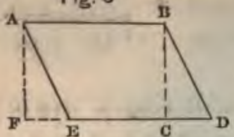
1. How many acres in a rectangular field 123 rods long and 42 rods wide?

2. How many square yds. in a pavement 80 ft. long and 14 ft. wide?

3. How many square yds. in the surface of one side of a wall 360 ft. long and $9\frac{1}{2}$ ft. high?

PROBLEM II. — TO FIND THE AREA OF A PARALLELOGRAM.

Fig. 18



EXAMPLE. — How many square inches in a board 6 inches wide and 8 inches long, cut in the shape of a parallelogram?

ILLUSTRATION. — Cut from a piece of cardboard a parallelogram; as, ABDE, Fig. 18. Cut the right-angled triangle, BCD, from one end, through the dotted line BC, and place it on the other end at AFE. The figure then becomes a rectangle 8 inches long and 6 inches wide.

PROCESS. $8 \times 6 = 48$ sq. in.

NOTE TO TEACHER. — Each pupil should be required to illustrate these problems *for himself*.

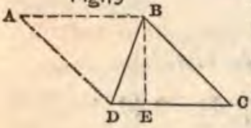
RULE. — *Multiply the length by the breadth.*

4. How many square feet in a board in the form of a parallelogram 90 inches long and 14 in. wide?

5. How many acres in a piece of land of diamond shape, 48 rods long and 18 rods wide?

PROBLEM III. — TO FIND THE AREA OF A TRIANGLE.

Fig. 19



EXAMPLE. — How many square inches in a triangular piece of board 8 inches on the base, the perpendicular distance from the vertex of the angle to the base being 4 inches?

PROCESS. $\frac{8}{2} \times 4 = 16$ sq. in.

ILLUSTRATION. — Cut from a piece of cardboard two triangles of the same size, as ABD and BCD; then place them so as to form the figure ABCD. The figure then becomes a parallelogram 8 inches long and 4 inches wide, one-half of which is the given triangle.

RULE. — *Multiply one-half of its base by its altitude.*

6. At $60 per acre, what will be the cost of a triangular piece of land, the base of which is 28 rods, and the perpendicular distance from the vertex to the base 14 rods?

7. Find the number of square feet of boards it will take to cover the gable-ends of a barn 28 feet wide, the ridge-pole being 18 feet higher than the foot of the rafters.

### PROBLEM IV. — THE SIDES OF A TRIANGLE BEING GIVEN TO FIND THE AREA.

EXAMPLE. — What is the area of a triangular piece of land, the sides of which are respectively 12, 18, and 24 rods?

PROCESS.

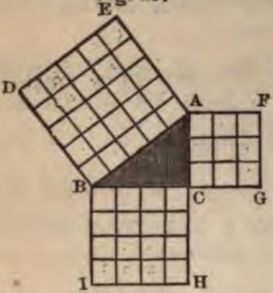
$12 + 18 + 24 = 54$, sum of sides.
$54 \div 2 = 27$, half sum of sides.
$27 - 12 = 15$, 1st remainder.
$27 - 18 = 9$, 2d remainder.
$27 - 24 = 3$, 3d remainder.
$27 \times 15 \times 9 \times 3 = 10935$.
$\sqrt{10935} = 104.57$ sq. rds. area.

RULE. — *From half the sum of the three sides subtract each side separately; then multiply the continued product of these remainders by half the sum of the sides, and extract the square root of the product. The result will be the area.*

8. The sides of a triangular field are respectively 30, 40, and 50 rods; how much is it worth at $50 per acre?

### PROBLEM V. — TO FIND THE HYPOTHENUSE OF A RIGHT-ANGLED TRIANGLE WHEN THE OTHER TWO SIDES ARE KNOWN.

Fig. 20.



EXAMPLE. — A piece of board in the shape of a right-angled triangle has a base of 4 inches, and a perpendicular of 3 inches: what is the length of the other side?

PROCESS.

$4^2 = 16$

$3^2 = \ \ 9$

$\sqrt{25} = 5$ Ans.

ILLUSTRATION. — (1.) Cut from colored cardboard the right-angled triangle ABC, also the squares BDEA, AFGC, and BCHI, so that the edges will coincide with the triangle ABC.

(2.) The square BCHI may be placed on the square BDEA, and the square AFGC cut so that the two will exactly coincide with BDEA, or, in other words, —

FORMULA. — *The square of the hypothenuse is equal to the sum of the squares of the other two sides.* Hence —

To find the hypothenuse of a right-angled triangle,

RULE. — I. *Add the squares of the base and perpendicular, and extract the square root of the sum.*

To find either of the other two sides,

II. *From the square of the hypothenuse subtract the square of the given side, and extract the square root of the remainder.*

9. The base of a right-angled triangle is 8 ft., the perpendicular is 6 ft.: what is the hypothenuse?

10. How high will a ladder reach against a perpendicular wall, if the ladder is 40 feet long, and its foot stands 10 ft. from the wall?

11. What is the distance between the opposite corners of the floor of a room 25 feet square?

PROBLEM VI. — TO FIND THE CIRCUMFERENCE OF A CIRCLE, THE DIAMETER BEING GIVEN.

RULE. — *Multiply the diameter by* 3.1416.

EXAMPLE. — What is the circumference of a circle, the diameter of which is 8 inches?

PROCESS. 8 in. × 3.1416 = 25.1328 in.

When the circumference is given, the diameter may be found by dividing the circumference by 3.1416.

12. The diameter of a circle is 18 inches: what is the circumference?

13. What is the diameter of a circle whose circumference is 25 feet?

14. What must be the inside measurement of a wagon-tire to fit a wheel 4 feet in diameter?

PROBLEM VII. — TO FIND THE AREA OF A CIRCLE.

RULE. — I. *Multiply the circumference by half the radius*, or

II. *Multiply the square of the radius by* 3.1416, or

III. *Multiply the square of the diameter by* .7854.

EXAMPLE. — What is the area of a circle, the diameter of which is 8 inches?

PROCESS. $4^2 \times 3.1416 = 50.2656$ sq. in.

15. How many acres in a circular pond one mile in diameter?

16. What is the area of the end of a log 22 inches in diameter?

17. What is the area of a circular piece of land, the circumference of which is 320 rods?

# ANSWERS.

## ADDITION.

**Page 18.**

*2.* 334166.
*3.* 2326369.
*4.* 3021197.
*5.* 2870561.

**Page 19.**

*6.* 2550245.
*7.* 1988881.
*8.* 52596760.
*9.* 107533540.
*10.* 133300644.
*11.* 175304305.
*12.* 3202103538.
*13.* 4601005537
*14.* 4100384269.
*15.* 4901478277.

**Page 20.**

*16.* 1031875.
*17.* 504268991.
*18.* 1307048626.
*19.* 311803918.
*20.* 901041406.
*21.* 1306088136.
*22.* 804125325.
*23.* 619927113.

**Page 21.**

*25.* $25.481.
*26.* $212.752.
*27.* $54.902.
*28.* $44.943.
*29.* $222.134.
*30.* $146.816.
*31.* $60.545.
*32.* $35.485.

**Page 23.**

*34.* $2.98.

**Page 24.**

*35.* $4.06.
*36.* $2695.
*37.* $109.
*38.* 394 barrels.
*39.* $2.22.
*40.* 1682 sq. miles.

**Page 25.**

*41.* $2684.
*42.* $836.
*43.* 868 sheep.
*44.* 659 peach-trees.
*45.* $777.
*46.* $987.
*47.* 320 sheep.
*48.* $6125.

**Page 26.**

*49.* $40772.
*50.* 986 miles.
*51.* 1045 pupils.
*52.* 7732 pounds.
*53.* $1085.
*54.* 4888 sheep.
*55.* 5408 sheep.
*56.* 4585 sheep.
*57.* 1821 sheep.
*58.* 348 melons.
*59.* $2295.

**Page 27.**

*60.* $94.34.

**Page 28.**

*61.* 100 acres.
*62.* 108 acres.
*63.* 62½ acres.
25 acres.
62½ acres.
64. 37 acres.

65. 124½ acres W.
113½ acres E.
66. 62 acres.
67. 7½ acres.
68. 300 rods.
200 rods.
69. 350 rods.
450 rods.
70. 300 rods.
360 rods.
71. 200 rods.
300 rods.

### Page 29.

72. 180 rods.
300 rods.
73. $559.02.
74. $126.90.
217 bushels.
75. 188 bushels.
$293.43.
76. 115 tons.
$250.50.
77. 749952 feet.
78. 160 acres.
79. 298 acres.
80. 3526 eggs.
81. 632 barrels.

### Page 30.

82. $3.15.
83. 908 trees.
84. $746.
85. 1483 pounds.
86. 107 yards.
87. $85.
88. 256 marbles.
89. $8348.
90. $2360.
91. 11379 bl'kbirds.

### Page 31.

92. $1708.
93. $1401.
94. 48501.
95. $4791.
96. 1796.
97. 2071.
98. $742.
$740.
$1482.
99. $745.
$1355.
100. 278.

### Page 32.

101. $9213.
102. $1273.
103. $5.66.
104. 201 rods.
105. 52 rods north.
47 rods south.
99 rods apart.
106. 40 rods.
107. 78 rods.

### Page 33.

108. $2538.
109. 96 years.
110. $923.
111. 60 dahlias.
112. 423 cents.
113. $1.31.
114. $16.52.
115. $5.87.
116. $122.815.
117. $86.315.
118. $21.253.
119. $70.506.
120. $2.342.
121. $53.66.

### Page 34.

122. $6.322.
123. $0.548.
124. $6.048.
125. $90.607.
126. $2007.02.
127. $5.555.
128. $60.603.
129. $50.361.
130. $11.111.
131. $1339.35.
132. $31.11.
133. $219.31.
134. $64.865.

### Page 35.

135. $43.99.
136. $24.10.
137. $4370.
138. $30.74.
139. $4.774.
140. $448.416.
141. $27.28.
142. $251.443.
520 bushels.
143. 817 cents.
$8.17.
67 pounds.
144. 170 cents.
56 in all.
37 oranges & lemons.

### Page 36.

145. $891.74.
146. 2931.
147. $71.636.
148. $43.799.
149. $3946.433.
150. $8015.002.

# SUBTRACTION.

**Page 44.**

1. 437 acres.
2. 92 children.
3. 111 barrels.
4. 268 men.
5. $1596.
6. $285.
7. $45.
8. 3 marbles.
9. 8.
10. 63.
11. 6263 lambs.

**Page 45.**

12. 300 cents.
13. 9279.
14. 1282.
15. 10269.
16. 39 years.
17. 71 years.
18. 128 years.
19. 1302.
20. 1769.
21. 85 years.
22. 111 years.
23. 1182.

**Page 46.**

24. 44.
25. 1774.
26. 797.
27. 3322.
28. 18 years.
29. 848.
30. 1060.
31. 286.
32. 3504.
33. 4580.
34. 34984.
35. 2464.
36. 42593.
37. 21997.
38. 116787.
39. 302122.
40. 434288.

**Page 47.**

41. $85470.
42. $99.43.
43. $388.357.
44. $51.366.
45. $305.66.
46. $282.87.
47. $94.05.
48. $99.
49. $9.90.
50. $0.09.
51. $0.009.
52. $3.189.
53. $2.211.
54. $2.764.
55. $56.666.
56. $75.787.
57. $8.287.
58. $51.50.
59. $2.50.
60. $944.961.
61. $61.213.
62. $24.625 lost.

**Page 48.**

63. $12.125.
64. $59.827.
65. $63.
66. $15 gained.
67. $5.
68. $53.
69. 82 miles.
70. 93 miles.

**Page 49.**

71. 53 years.
72. $0.03.
73. 72 nuts.
74. $0.14.
75. 135 sheep.
76. 20 oranges.
77. $46.
78. $11.
79. $0.14.
80. $195.

**Page 50.**

81. Nothing due.
82. $0.87.
83. 29 marbles.
84. $20.
85. $3.
86. $30.
87. $7.
88. $81.
89. $128.

**Page 51.**

90. 211 cents.
91. $5968.
92. $0.07.
93. $5.11.
94. 136 yards.

**Page 52.**

*95.* 88 nuts.
*96.* 1374 sheep.
*97.* $1227.
*98.* $4.17.
*99.* $3159.
*100.* Given.
*101.* $574.
*102.* 1282 sheep.

**Page 53.**

*103.* 2275 men.
*104.* 94 bushels.
*105.* 250 miles.
*106.* 120 minutes.
*107.* 131 minutes.
*108.* 252 minutes.
*109.* 58 minutes.
*110.* 259 minutes.
*111.* 639 minutes.
*112.* 366 minutes.

**Page 54.**

*113.* $87.37.
*114.* $19.81.
*115.* $5.36 lost.
*116.* 280 barrels.
$1880.
*117.* $4078.29.
*118.* $184.54 gain.
*119.* $4227.10.
*120.* $32.48.
*121.* $63.38.

**Page 55.**

*122.* $0.06.
*123.* $16.
*124.* 250 barrels.
*125.* 16 days.
*126.* 19 days.
*127.* $2379.74.
*128.* $0.86 lost.
*129.* 25 miles.
*130.* $18.352, amt. of bill.
$9.599.
*131.* $24.15, amt. of bill.
$15.85 change.

**Page 56.**

*132.* $54.23 amt. of bill.
$45.77 lacking.

# MULTIPLICATION.

**Page 65.**

*1.* Given.
*2.* $22.75.
*3.* $69.60.
*4.* $2.91.
*5.* $29.70.

**Page 66.**

*6.* $2.48.
$7.28.
*7.* 94 days.
*8.* 1341 days.
*9.* 1500 days.
*10.* 407 days.
*11.* 7398 horses.
*12.* $39.
*13.* $2.97.
*14.* $1.71.
*15.* $3.56.
*16.* $6.45.

**Page 67.**

*17.* $8.89.
*18.* $10.00.
*19.* $21.98.
*20.* 7616.
*21.* 1584 men.
*22.* 560 cows.
*23.* Given.
*24.* $61.

**Page 68.**

*25.* $100.
*26.* $48.
*27.* 127 miles.
*28.* 427 shillings.
*29.* 5 yards.
*30.* 4 yards.
*31.* $1.94.
*32.* 264 cents.
*33.* $97.
*34.* $129.
*35.* $105.

**Page 69.**

*36.* 188 pounds.
*37.* 23 gallons.
*38.* $3.12.
*39.* $4.08.
*40.* 1443 shillings.
*41.* $71.40.
*42.* 960 pounds.
*43.* $9.
*44.* 13 cents.

*45.* 182 trees.
*46.* 1000 trees.
*47.* 64 bushels.

**Page 70.**

*49.* 4200 dimes.
350 dimes.
*50.* 20000 mills.
75000 mills.

**Page 71.**

*51.* 2500 mills.
1830 mills.
*52.* Given.
*53.* $2.61.
$338.43.
*54.* $3.60 for hose.
$17.50 for boots
$6.00 for shoes.
$1.52 " gloves.
$7.00 " kid "
Bill $35.62.
*55.* $1.20 1st r'ders.
$2.50 2d "
$2.10 3d "
$7.20 4th "
$8.00 5th "
Bill $21.00.
*56.* $0.55 for sugar.
$2.70 " tea.
$0.88 for coffee.
$2.48 " raisins.
$0.64 " rice.
Bill $7.25.
*57.* $0.02 change.

**Page 72.**

*58.* $0.54 fr. $2 bill.
$3.54 " $5 "
$8.54 " $10 "
*59.* $0.65 cost.
$0.35 change.
*60.* $0.77.
*61.* $5.10.
*62.* $2.28.
*63.* $43.80.
*64.* $105.
$1277.50.
*65.* $31.25.
$2975.

**Page 73.**

*66.* 956 pounds.
2390 pounds.
183552 pounds.
*67.* $239.
$18355.20.
*68.* 376 gallons.
31576 gallons.
*69.* 736 pounds.
55648 pounds.
*70.* $4.41.
$59.19.
*71.* $13.75.
$459.25.
*72.* $1.44.
$5.04.
*73.* $4.80.
*74.* $1.78.
$8.22.
*75.* $1.20.
$3.84.
*76.* $9.63.
$11.50.
$8.46.
*77.* $6.23.
$100.90.

**Page 74.**

*78.* $97.25.
*79.* $294.90.
*80.* $6.20.
*81.* $15.375.
*82.* $69.12.
*83.* $129.235.
*84.* $28.233.
*85.* $297.30.
*86.* $0.453.
*87.* $302.945.
*88.* $651.03.

## DIVISION.

**Page 85.**

*9.* $148.
*10.* $3776.
*11.* $2401.
*12.* 3924 miles.

**Page 86.**

*13.* 3420 cents.
*14.* 680 trees.
*15.* 288 miles.
*16.* 450 bushels.
*17.* 2520 cents.
*18.* $2702.
*19.* 1351 boxes.
*20.* 72 years.
*21.* 347 pencils.

**Page 87.**

*22.* 89 pounds.
*23.* 974 barrels.
*24.* 30 men.
*25.* 14 men.
*26.* 50 persons.
*27.* 128 coats.
*28.* 123 hours.

29. 49 pounds.
30. $3082\frac{2}{3}$ oranges.
31. 30 pounds.
32. 140 rods.

### Page 88.

33. 12 rods.
34. 9 days.
35. Given.
36. 63360 inches.
37. 2016 gills.
38. 960 farthings.
39. 3000 families.
40. 1120 feet.
41. 1150 men.
42. 192 hours.
43. $\$774\frac{4}{5}$.

### Page 89.

44. $449.
45. $36835\frac{2}{4}$.
46. $8264\frac{4}{5}$.
47. Given.
48. $476\frac{8}{16}$ pounds.

### Page 91.

49. $144.
50. 60 pounds.
300 pounds.
51. 84 bushels.
52. 1000 gallons.
53. 25 barrels.
54. 40 barrels.

### Page 92.

55. 640 rods.
13440 rods.
56. 2 miles.
$22\frac{184}{320}$ miles.
57. 2.
2.
$52\frac{5}{9}$.
$198\frac{26}{46}$.
58. $10\frac{1}{46}$.
59. 5.
60. 36 sum.
61. 52 rem.
62. 1918 prod.
63. $6\frac{10}{92}$ quo.
64. 195 prod.
65. 378 sum.
66. $43\frac{10}{21}$ quo.
67. 399 rem.
68. 69.
69. 3.
70. 40.
71. 363.
72. 35.
73. $8\frac{2}{4}$.
74. $8\frac{2}{4}$.
75. $19\frac{2}{4}$.

### Page 93.

76. 11.
77. 11.
78. 132.
79. 38.
80. 6.
81. $2\frac{9}{11}$.
82. 455.
83. $(408 \div 12) - 3 = 31$.
84. $\$.20 + \$1.30 + \$.875 + \$2.30 = \$4.675$.
85. $\$.80 + \$.40 - \$1.00 = \$.20$.
86. $\$0.22 \times 8 = \$1.76$.
87. $\$72 \div 12 \times 5 = \$30$.
88. $(\$54 \div 18) - (\$25 \div 12) = \$0.91\frac{2}{3}$.

### Page 96.

89. $89.
90. $257.
$243.
91. $1.16.
$8.84.
92. 230 sheep.
193 sheep.
93. Given.
94. 18 sheep.
210 sheep.
95. 20 pounds.

### Page 97.

96. $107.
97. Nothing.
98. 82 cents.
$4.18.
99. $23.47.
100. $2.06.
$24\frac{6}{12}$ pounds.
101. 38 yards.
102. 11 miles.
103. 9 miles.
48 miles.

### Page 98.

104. 72 miles.
80 miles.
105. 95 miles.
245 miles.
106. Given.
107. 1143 cents.
1341 cents.
108. 360 cents.
1038 cents.

*109.* \$2568.
\$59896.
*110.* 760 problems.
2000 "
*111.* \$1372.
65289.
*112.* 972 miles.
6696 miles.
*113.* 648 miles.
64152 miles.

**Page 99.**

*114.* \$37544.
*115.* \$25311.
*116.* \$228.
\$60.
*117.* \$137.30.
\$54.92.
*118.* \$1368.
*119.* Given.
*120.* 32 men.
*121.* 8 days.
30 days.
*122.* 14 hours.
742 hours.
*123.* 110 lemons.
*124.* 28 barrels.

**Page 100.**

*125.* 18 dozen.
72 dozen.
*126.* 26 eggs.
3704 eggs.
*127.* 60 gallons.
57668$\frac{30}{35}$ galls.
*128.* \$4.60.
*129.* \$20.30 gain.
*130.* \$3.
\$4.
*131.* \$2 loss for ea.
*132.* 2 cwt.
4 cwt.
*133.* \$20 ea. for 2.
\$60 each for others.

**Page 101.**

*134.* \$6 each for 3.
\$30 for the other.

## U. S. CURRENCY.

**Page 103.**

*1.* 100 times.
111 times.
*2.* 100 times.
3700 times.

**Page 104.**

*3.* 100 times.
10 times.
26 times.
*4.* 9 times.
11 times.
*5.* 110 times.
11 times.
80 times.
*6.* 27 times.
135 times.
75 times.
*7.* \$1.625.
*8.* \$2.125.
*9.* \$1.625.
*10.* \$1.05.
*11.* \$8.875.
*12.* \$1.25.
*13.* \$2.80.
*14.* \$2.40.
\$4.20.
\$3.00.

**Page 105.**

*15.* \$20.82.
\$16.56.
*16.* \$2970.08.
*17.* \$5809.125.
*18.* \$6989.375.
\$8387.25.
*19.* \$4382.15.
\$7887.87.
*20.* Given.
*21.* \$0.472 +.
\$0.638 +.
\$1.208 +.
*22.* \$2.75.
\$37.125.
\$1377.75.
*23.* \$0.495.
\$50.655.
\$660.495.
*24.* \$0.167 +.
\$0.171 +.
\$0.178 +.
*25.* \$0.093 +.
\$0.156 +.
*26.* 20 oranges.
40 oranges.
*27.* 733 pounds.
*28.* 296 oranges
592 oranges.
*29.* 108 pounds.
54 pounds.

**Page 106.**

*30.* 24 yards.
1632 yards.
18437 yards.
*31.* $121.875.
$140.625.
$67.875.
*32.* $19.305.
$14.625.
$18.72.
*33.* $1.071 +.
$1.964 +.
$1.410 +.
*34.* $64.17.
$85.002 +.
$93.
*35.* $3.40.
$3.23.
$2.55.
$2.992.
*36.* $0.793 +.
$0.555 +.
*37.* $3.875.
*38.* $1.10.
$4.125.
$20.625.
*39.* $0.225.
$2.25.
*40.* $0.225.
$150.225.
*41.* $29.435.
*42.* $6355.045.
*43.* $32943.725.

**Page 107.**

*44.* $191.17.

**Page 108.**

*45.* $399.32.
*46.* $10933.32.
*47.* $93.25.
*48.* $412.74.

**Page 109.**

*49.* $6938.71.
*50.* $88204.10.

## PRIME FACTORS.

**Page 114.**

*1.* 2, 2, 2, 3.
*2.* 2, 2, 3, 3.
*3.* 2, 2, 2, 2, 3.
*4.* 2, 2, 2, 2, 3, 3.
*5.* 2, 2, 2, 3, 5.
*6.* 2, 2, 2, 17.
*7.* 5, 29.
*8.* 3, 3, 5, 5.
*9.* 2, 2, 199.
*10.* 2, 2, 2, 2, 2, 2, 3, 3.
*11.* 2, 2, 2, 2, 2, 2, 2, 2.
*12.* 2, 2, 2, 43.
*13.* 2, 2, 2, 2, 2, 3, 3, 3.
*14.* 3, 3, 3, 5, 7.
*15.* 2, 2, 2, 2, 2, 2, 5.
*16.* 2, 2, 5, 5, 5.
*17.* 5, 193.
*18.* 7, 11, 91.
*19.* 2, 2, 3, 7, 97.
*20.* 719.
*21.* 11, 83.
*22.* 2, 2, 2, 3, 11, 91.
*23.* 3, 5, 43.
*24.* 2, 2, 2, 3, 5, 7.

## GREATEST COMMON DIVISOR.

**Page 115.**

*25.* 6, 4, 6.
*26.* 3, 4, 2.
*27.* 9, 13, 24.
*28.* 78, 4.

**Page 116.**

*29.* 7.
*30.* 22.
*31.* 17.
*32.* 17.

## LEAST COMMON MULTIPLE.

**Page 117.**

*33.* 90.
*34.* 240.
*35.* 12600.
*36.* 504.
*37.* 1134.
*38.* 144.
*39.* 2520.
*40.* 194040.
*41.* 156240.
*42.* 144.
*43.* 19530.
*44.* 4095.
*45.* 6384.
*46.* 336.

## CANCELLATION.

**Page 118.**

*47.* $2\frac{2}{35}$.
*48.* 20.
*49.* $6\frac{82}{95}$.
*50.* 63.
*51.* $34\frac{2}{3}$.
*52.* 36.
*53.* $6\frac{3}{4}$.

**Page 119.**

*54.* $18\frac{2}{3}$.
*55.* 6.
*56.* 3.
*57.* 16.
*58.* 90 cents.
*59.* 6 tons.
*60.* 75 cents.

## FRACTIONS.

**Page 126.**

*1.* $\frac{1}{2}$.
*3.* $\frac{1}{3}$.

**Page 127.**

*5.* $\frac{2}{5}$, $\frac{2}{3}$.
*6.* $\frac{1}{7}$, $\frac{2}{7}$.

**Page 128.**

*7.* $\frac{8}{9}$, $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{9}$.
*8.* $\frac{3}{5}$, $\frac{11}{12}$, $\frac{5}{6}$.
*9.* $\frac{7}{11}$, $\frac{11}{20}$, $\frac{75}{88}$.
*10.* $\frac{3}{10}$, $\frac{7}{20}$, $\frac{11}{20}$, $\frac{22}{123}$.
*11.* $\frac{9}{50}$, $\frac{22}{25}$, $\frac{11}{40}$, $\frac{4}{7}$, $\frac{3}{4}$, $\frac{226}{107}$.
*12.* $\frac{3}{4}$, $\frac{22}{25}$, $\frac{19}{20}$, $\frac{3}{4}$, $\frac{134}{263}$. $\frac{278}{175}$, $\frac{3}{5}$.

**Page 129.**

*13.* $\frac{171}{4}$.
*14.* $\frac{201}{7}$.
*15.* $\frac{275}{3}$.
*16.* $\frac{607}{9}$.
*17.* $\frac{538}{9}$.
*18.* $\frac{187}{4}$.
*19.* $\frac{4384}{5}$.
*20.* $\frac{777}{9}$.
*21.* $\frac{827}{2}$.
*22.* $\frac{5461}{8}$.
*23.* $\frac{752}{9}$.
*24.* $\frac{5373}{13}$.
*25.* $\frac{10092}{11}$.
*26.* $\frac{12980}{17}$,
*27.* $\frac{32567}{79}$.

**Page 130.**

*28.* $170\frac{3}{4}$, $53\frac{1}{4}$.
*29.* $169\frac{1}{2}$, $224\frac{1}{4}$.
*30.* $179\frac{1}{5}$, $74\frac{1}{5}$.
*31.* 136, $82\frac{3}{5}$.
*32.* $1118\frac{5}{6}$, 78.
*33.* $701\frac{6}{7}$, $63\frac{2}{5}$.
*34.* $1229\frac{1}{2}$.
*35.* $520\frac{3}{8}$.
*36.* $717\frac{1}{11}$.
*37.* $8376\frac{2}{5}$.
*38.* $304\frac{9}{16}$.
*39.* $129\frac{1}{22}$.
*40.* $1658\frac{34}{37}$.
*41.* $31\frac{57}{67}$.
*42.* $10\frac{187}{191}$.
*43.* $159\frac{5}{6}$.
*44.* $1\frac{158}{631}$.
*45.* 1.
*46.* $1\frac{309}{397}$.
*47.* 1.
*48.* $35\frac{208}{241}$.
*49.* $7\frac{665}{874}$.
*50.* $7\frac{56}{83}$.
*51.* $75\frac{13}{92}$.

**Page 134.**

*52.* $\frac{608}{5}$.
*53.* $\frac{82}{3}$.
*54.* $\frac{303}{7}$.
*55.* 28.
*56.* $159\frac{1}{4}$, $117\frac{7}{8}$.
*57.* $82\frac{3}{5}$.
*58.* $8\frac{8}{13}$.
*59.* $\frac{36}{3}$, 12 pecks.
*60.* $\$\frac{68}{4}$, $17.
*61.* 65 boys, 156 boys.
*62.* 420.
*63.* $21.75, $28.

## COMMON DENOMINATOR.

**Page 137.**

*64.* $\frac{72}{168}$, $\frac{105}{168}$, $\frac{84}{168}$, $\frac{60}{168}$, $\frac{126}{168}$.
*65.* $\frac{288}{336}$, $\frac{72}{336}$, $\frac{112}{336}$, $\frac{308}{336}$, $\frac{231}{336}$.
*66.* $\frac{672}{756}$, $\frac{84}{756}$, $\frac{648}{756}$, $\frac{135}{756}$, $\frac{108}{756}$.
*67.* $\frac{616}{3388}$, $\frac{168}{3388}$, $\frac{2662}{3388}$, $\frac{363}{3388}$.
*68.* $4\frac{24}{56}$, $\frac{42}{56}$, $\frac{36}{56}$, $11\frac{48}{56}$.
*69.* $3\frac{35}{56}$, $3\frac{32}{56}$, $5\frac{21}{56}$, $9\frac{16}{56}$.
*70.* $4\frac{180}{420}$, $5\frac{60}{420}$, $\frac{315}{420}$, $\frac{112}{420}$.
*71.* $4\frac{189}{252}$, $3\frac{231}{252}$, $\frac{18}{252}$, $\frac{84}{252}$.
*72.* $9\frac{224}{280}$, $3\frac{140}{280}$, $4\frac{56}{280}$, $\frac{105}{280}$.
*73.* $4\frac{21}{70}$, $8\frac{25}{70}$, $\frac{60}{70}$, $9\frac{28}{70}$.
*74.* $3\frac{648}{756}$, $5\frac{672}{756}$, $\frac{84}{756}$, $\frac{162}{756}$.
*75.* $3\frac{96}{168}$, $5\frac{126}{168}$, $\frac{7}{168}$, $3\frac{6}{168}$.
*76.* $\frac{594}{693}$, $5\frac{99}{693}$, $6\frac{378}{693}$, $3\frac{308}{693}$.
*77.* $\frac{112}{126}$, $6\frac{72}{126}$, $5\frac{54}{126}$, $6\frac{72}{126}$.

## ADDITION OF FRACTIONS.

**Page 140.**

*78.* $144\frac{367}{504}$ yards.
*79.* $\$1.56\frac{1}{3}$.
*80.* $\$0.33\frac{19}{24}$.
*81.* $581\frac{23}{24}$ bushels.
*82.* $\$37\frac{11}{12}$, $\$62\frac{7}{12}$.
*83.* $\$1250\frac{1}{4}$.
*84.* $313\frac{5}{21}$ acres.
*85.* $\$475\frac{5}{12}$.
*86.* Lost $\$137\frac{1}{4}$.
*87.* Lost $\$0.59\frac{5}{14}$.

**Page 141.**

*88.* $\$303\frac{14}{15}$.
*89.* $4\frac{5}{8}$ shares.
*90.* $1\frac{17}{60}$ times.
*91.* $10\frac{51}{70}$.
*92.* $3\frac{17}{18}$ apples.
*93.* $6\frac{2}{5}$ times.
*94.* $22\frac{167}{168}$ pears.
*95.* $9\frac{473}{630}$.
*96.* $8\frac{40}{63}$ times.
*97.* $37\frac{31}{44}$ times.

## SUBTRACTION OF FRACTIONS.

**Page 144.**

*98.* $\frac{1}{2}$.
*99.* $\frac{1}{2}$.
*100.* $\frac{26}{99}$.
*101.* $\frac{9}{77}$.
*102.* $\frac{1}{9}$.
*103.* $2\frac{224}{323}$.
*104.* $\frac{5}{132}$.
*105.* $2\frac{49}{52}$.
*106.* $22\frac{7}{24}$.
*107.* $24\frac{89}{92}$.
*108.* $17\frac{33}{56}$.
*109.* $8\frac{52}{77}$.
*110.* $15\frac{6}{7}$.
*111.* $\frac{1}{2}$.
*112.* $8\frac{11}{12}$.
*113.* $10\frac{7}{10}$.
*114.* $\$\frac{10}{13}$.

**Page 145.**

*115.* $223\frac{53}{56}$ pounds
*116.* $126\frac{11}{30}$ barrels.
*117.* $\$3.72\frac{1}{2}$.
*118.* $\$8\frac{8}{21}$.
*119.* $226\frac{1}{24}$ lbs. tea.
204 " coffee.
$1\frac{45}{56}$ " rice.
*120.* $198\frac{6}{35}$.

*121.* $353\frac{4}{5}$.
*122.* $115\frac{1}{8}$.
*123.* $\$39\frac{29}{60}$.
*124.* $\$5\frac{13}{35}$.
*125.* $\$209\frac{1}{21}$.
*126.* $\$6.44\frac{5}{6}$.
*127.* $\$237\frac{1}{14}$.

**Page 146.**

*128.* $\$1759\frac{23}{24}$.
*129.* $\$22\frac{4}{35}$.
*130.* $\$160\frac{53}{56}$.
*131.* Gain $\$0.29\frac{7}{13}$.

# MULTIPLICATION OF FRACTIONS.

**Page 148.**

*1.* 2050.
$2733\frac{1}{3}$.
3075.
$3758\frac{1}{3}$.
*2.* $1511\frac{1}{4}$.
$3324\frac{3}{4}$.
1209.
2418.
*3.* $1618\frac{2}{5}$.
$4855\frac{1}{5}$.
$3236\frac{4}{5}$.
4046.
*4.* 1361.
$2268\frac{1}{3}$.
$3629\frac{1}{3}$.
$5897\frac{2}{3}$.
$4990\frac{1}{3}$.
$13156\frac{1}{3}$.
*5.* $298\frac{3}{8}$.
$215\frac{1}{4}$.
*6.* $\$2.62\frac{1}{2}$.
$\$9.37\frac{1}{2}$.
$\$32.62\frac{1}{2}$.

**Page 149.**

*7.* $\$103\frac{5}{7}$.
$\$2102\frac{4}{7}$.
*8.* $181\frac{1}{3}$ loaves.
$333\frac{1}{3}$ loaves.

**Page 150.**

*9.* $\$0.62\frac{1}{2}$.
$\$0.78\frac{1}{8}$.
$\$3.54\frac{1}{6}$.
*10.* $0.84.
$11.40.
*11.* $628.54.
$\$235.70\frac{1}{4}$.
$\$1846.33\frac{5}{8}$.
*12.* $13.11.
$\$20.10\frac{1}{5}$.
$\$33.01\frac{7}{9}$.
$\$2.38\frac{4}{11}$.
*13.* $\$1.88\frac{8}{9}$.
$\$4.97\frac{7}{9}$.
*14.* $\$13\frac{1}{3}$.
$\$16.16\frac{2}{3}$.
$\$18.71\frac{2}{3}$.

**Page 152.**

*15.* $\$\frac{13}{18}$.
$\$1\frac{2}{27}$.
*16.* $15\frac{15}{16}$ bushels.
$17\frac{1}{7}$ bushels.
*17.* $\$0.37\frac{11}{12}$.
$\$0.42\frac{7}{15}$.
$\$0.50\frac{3}{8}$.
*18.* $\$45\frac{3}{5}$.
$\$102\frac{3}{5}$.
$\$148\frac{1}{5}$.
*19.* $228.
$342.
$256.50.

**Page 153.**

*21.* 42 lbs.
*22.* 86 lbs.
$19\frac{1}{9}$ lbs.
$391\frac{7}{9}$ lbs.
*23.* $105\frac{4}{5}$ yds.
$196\frac{20}{63}$ yds.
$63\frac{2}{9}$ yds.
*25.* $96\frac{3}{16}$ sq. inches.
*26.* $211\frac{13}{21}$ lbs.
*27.* $20\frac{1}{18}$ qts.

**Page 155.**

*28.* $190\frac{6}{7}$.
$71\frac{4}{7}$.
$47\frac{5}{7}$.
$40\frac{44}{49}$.
*29.* $70\frac{7}{27}$.
$42\frac{7}{45}$.
$21\frac{7}{90}$.
$10\frac{1}{27}$.
*30.* $303\frac{13}{16}$.
$86\frac{45}{56}$.
$14\frac{269}{328}$.
$30\frac{61}{160}$.
*31.* $\frac{28}{55}$ dimes.
*32.* $\$\frac{13}{40}$.
$\$\frac{7}{30}$.

**Page 156.**

*33.* $\$\frac{1}{99}$.
$\$0.02\frac{7}{99}$.
$\$0.04\frac{14}{45}$.
*35.* $\$0.40\frac{8}{15}$.
$\$4.50\frac{2}{15}$.

36. $\$3.75$.
$\$14.87\frac{1}{2}$.
37. $\$1.44\frac{9}{10}$.
$\$4.34\frac{7}{10}$.
38. $9\frac{13}{24}$ shillings.
$26\frac{7}{8}$ shillings.
39. $\$0.40\frac{8}{27}$.
$\$0.83\frac{13}{63}$.
$\$8.59\frac{1}{45}$.
40. $\$0.65\frac{25}{64}$.
$\$2.11\frac{13}{21}$.
41. $12\frac{13}{36}$ yds.
42. $\$0.15\frac{5}{18}$.
$\$0.76\frac{7}{18}$.
$\$1.01\frac{23}{27}$.
43. $73\frac{13}{14}$ lbs.
44. $\$4.31\frac{117}{128}$.
$\$4.35\frac{75}{112}$.

**Page 157.**

45. $114\frac{1}{3}$.
$89\frac{5}{6}$.
$134\frac{3}{4}$.
$70\frac{7}{9}$.
46. $106\frac{2}{7}$.
$70\frac{13}{20}$.
$21\frac{9}{35}$.
$19\frac{3}{8}$.
47. $24\frac{7}{11}$.
$15\frac{13}{63}$.
$6\frac{241}{276}$.
$18\frac{97}{100}$.
48. $130\frac{27}{338}$.
$50\frac{417}{871}$.
$40\frac{807}{1075}$.
$179\frac{13}{35}$.

**Page 159.**

50. $12\frac{7}{9}$ persons.
$13\frac{4}{5}$ persons.
$12\frac{1}{21}$ persons.
51. $2\frac{39}{203}$ persons.
$1\frac{47}{76}$ persons.
$1\frac{11}{40}$ persons.
52. $11\frac{1}{2}$ times.
$20\frac{4}{7}$ times.
$185\frac{1}{22}$ times.
53. $656\frac{1}{2}$ oranges.
1023 oranges.
$680\frac{7}{16}$ oranges.
54. $\$0.45\frac{5}{11}$.
$\$0.77\frac{7}{9}$.
$\$0.47\frac{1}{17}$.
$\$0.07\frac{9}{13}$.
55. $\$0.80$.
$\$0.857\frac{1}{7}$.
$\$0.923\frac{1}{13}$.
$\$0.285\frac{5}{7}$.

**Page 161.**

57. $388\frac{1}{3}$ tons.
58. $12\frac{5}{14}$ bbls.
59. $150\frac{1}{16}$ lbs.
$166\frac{2}{35}$ lbs.
$140\frac{1}{30}$ lbs.
60. $\$44\frac{211}{435}$.

**Page 162.**

62. $\$0.64\frac{1}{6}$.
$\$0.825$.
63. $\$15\frac{61}{110}$.
$\$12\frac{103}{183}$.
64. $72\frac{15}{16}$ lbs.
$\$22.36\frac{3}{4}$.
65. $\$1.375$.
66. $\$5.13\frac{21}{58}$.
67. $\$1.37\frac{2}{9}$.
$\$0.19\frac{38}{63}$.
$\$0.37\frac{14}{33}$.
$\$3.88\frac{43}{64}$.

**Page 163.**

69 $\$0.21\frac{1}{3}$.
70. $\$2.66\frac{497}{1113}$.
71. $\$0.25\frac{365}{782}$.
72. $\$\frac{49}{340}$.
73. $100\frac{49}{110}$ bushels.
74. $\$5.25$.
75. $\$34.28\frac{4}{35}$.
76. $\$14.50$.
77. $\$15.55\frac{5}{8}$.
78. $\$11.50\frac{1}{2}$.
79. $\$5.95$.
80. $\$1.89\frac{7}{8}$.
81. 60 cords.

**Page 164.**

82. $80\frac{5}{8}$ miles.
83. $\$24.87\frac{3}{8}$.
84. $77\frac{38}{81}$ feet.
86. $6\frac{10}{13}$ yds.
$1\frac{7}{48}$ yds.
87. $1\frac{7}{293}$ yds.
$5\frac{110}{293}$ yds.
$6\frac{267}{293}$ yds.
88. $162\frac{8}{11}$ qts.
$276\frac{1}{4}$ qts.

**Page 165.**

89. $7\frac{25}{33}$ yds.
$83\frac{5}{81}$.
90. $\$0.14$.
91. $\$0.26\frac{1}{3}$.
$\$0.22\frac{74}{189}$.
$\$0.38\frac{18}{47}$.
92. $\$1.92\frac{31}{45}$.
$\$2.75\frac{25}{48}$.
$\$2.21\frac{1037}{1146}$.
93. $\$0.50\frac{2}{5}$.
$\$0.72\frac{9}{20}$.
$\$7.16\frac{2}{5}$.
94. $7\frac{17}{24}$ shillings.
$80\frac{5}{8}$ shillings.
95. $\$0.04\frac{5}{28}$.
$\$0.06$.

*96.* $\$0.05\frac{1}{48}$.
$\$0.04\frac{5}{17}$.
$\$0.18\frac{146}{261}$.

**Page 166.**

*98.* $\frac{21}{22}$.
*99.* $2\frac{4}{9}$.
*100.* $\frac{31}{40}$.
*101.* $8\frac{3}{11}$.
*104.* $1\frac{5}{9}$ shillings.
$18\frac{11}{20}$ shillings.
$402\frac{2}{3}$ shillings.
*105.* $\$1.57\frac{1}{2}$.
$24.30.
*106.* $\$1\frac{1}{8}$.
$\$3\frac{1}{2}$.
$\$34\frac{11}{18}$.

**Page 167.**

*107.* $8.
$\$40\frac{4}{11}$.
$\$504\frac{6}{11}$.
*108.* $\$12\frac{1}{2}$.
$\$62\frac{1}{2}$.
$\$91\frac{2}{3}$.
*109.* 75 lbs.
$55\frac{5}{9}$ lbs.
$85\frac{5}{7}$ lbs.
*110.* $122\frac{1}{2}$ lbs.
3136 lbs.
$4573\frac{1}{3}$ lbs.
*111.* 40 lbs.
2796 lbs.
*112.* $87.71.
$313.60.
$627.20.
$19913.60.
*113.* $\$20\frac{4}{9}$.
$\$116\frac{8}{15}$.
$\$19626\frac{2}{3}$.
$\$107946\frac{2}{3}$.

*114.* $\$0.41\frac{2}{3}$.
$\$0.61\frac{7}{8}$.
$3.20.
$\$11.73\frac{1}{3}$.
*115.* $29\frac{11}{16}$ years.
*116.* $180.15 in safe.
$480.40 in both.

**Page 168.**

*117.* 18.
$70\frac{7}{8}$.
$3526\frac{1}{4}$.
*118.* 50.
$13\frac{5}{7}$.
*119.* $\frac{5}{32}$ bushel.
$\frac{9}{40}$ bushel.
*120.* $\$\frac{1}{12}$ more.
$\$1\frac{7}{12}$ both.
$\$8\frac{5}{12}$ left.
*121.* $\$3\frac{5}{8}$.
$\$27\frac{47}{56}$.
*122.* $\$14\frac{1}{4}$ both.
$\$5\frac{3}{4}$ lacking.

**Page 169.**

*126.* $\frac{11}{12}$.
$\frac{1}{12}$.
$1503.
*127.* $\$30\frac{10}{147}$.
*128.* $\$43\frac{4}{5}$.
*129.* $\$6\frac{1}{24}$.
*130.* $\$1\frac{14}{25}$.
$\$8\frac{1}{25}$.
*132.* $\$9\frac{29}{60}$.
*133.* $\frac{15}{56}$ too large.
$\frac{12}{35}$ too large.
1 too small.
$1\frac{4}{77}$ too large.
*134.* $\$\frac{1}{90}$ gained.
*135.* $\$\frac{41}{110}$.

**Page 170.**

*136.* $\$5\frac{31}{40}$.
$\$6\frac{3}{10}$.
$\$12\frac{3}{8}$.
$\$36\frac{3}{4}$.
*137.* $\$72\frac{2}{9}$.
$91\frac{13}{27}$.
*138.* $\$\frac{2}{5}$.
$\$\frac{3}{44}$.
$\$\frac{42}{715}$.
$\$27\frac{3}{11}$.
*140.* $1\frac{31}{54}$.
$\$38\frac{6}{7}$.

**Page 171.**

*141.* $\$0.05\frac{5}{14}$.
$\$1.33\frac{13}{14}$.
$\$4.30\frac{45}{56}$.
$\$21.54\frac{1}{56}$.
*142.* $\$15.91\frac{2}{3}$.
$79.58\frac{1}{3}$.
*143.* $\$5.68\frac{3}{4}$.
$250.
*145.* 46 bushels.
$8\frac{3}{5}$ bushels.
86 bushels.
$64\frac{1}{2}$ bushels.
$67\frac{4}{7}$ bushels.
*146.* $\$94\frac{2}{7}$.
*147.* $10\frac{18}{55}$.
$\frac{8}{27}$.
$\frac{33}{58}$.
*148.* $\$51.428\frac{4}{7}$.
$\$83.636\frac{4}{11}$.

**Page 172.**

*149.* $128\frac{2}{25}$ acres.
*150.* $\$24.47\frac{1}{7}$.
*151.* $750.
*152.* $\$1166.66\frac{2}{3}$.
*153.* $\$10\frac{20}{23}$.
154. $8\frac{33}{35}$ bushels.

## ADDITION OF DECIMALS.

**Page 177.**

1. 24.7.
2. 39.31.
3. 57.99.
4. 97.17.
5. 88.41.
6. 93.492.
7. 81.807.
8. 43.762.

**Page 178.**

9. .6367.
10. 622.6363.
11. 6.00186.
12. 108.33033.
13. 11.88237.
14. 18.27005.

## SUBTRACTION OF DECIMALS.

15. 58.74.
16. 19.97.
17. .05023.
18. .494541.
19. .995299.
20. 19.997.
21. .297.
22. 2999.997.
23. 100999999.999899.
24. 78999.9921.

**Page 179.**

25. 58.41.
3.33.
36.963.
26. 59.34 years.
2.54 years.
27. $311.50.
236.64.
28. 12.03 gal.
97.5 gal.
29. 54.50 rods.
9 rods more.
30. $108.93.
$22.13.
31. 1.60.
32. 1.81.
33. 13.64.
34. 4.91.
35. 4.859.
36. 11.834.

**Page 181.**

37. 4.492.
38. 5.738.
39. 6.158.
40. 7.474.
41. 6.567.
42. 12.872.
43. 5.502.
44. .91021.
45. 3.54476.

## MULTIPLICATION OF DECIMALS

**Page 184.**

46. 12.9.
75.09.
47. 2.952.
60.66.
48. .003784.
.01624.
49. *.0162.*
*.000618.*
50. .4824.
.032272.
51. 240.024.
2.40024.
3.60036.
52. .06934.
.0080006.
.00014086.
53. $197.75.
$226.59.
54. $2.46.
$2.55.
$2.314.
55. $84.
$0.75.
56. $5.925.
$2.025.

**Page 185.**

*57.* $14.90 lost.
*58.* \$1.841$\frac{2}{3}$ gained.
*59.* $0.50 gained.
*60.* $1.50 lost.
*61.* \$8.944$\frac{4}{9}$.
*62.* 6.077$\frac{1}{2}$.
*63.* 68.38.
*64.* 3.51225.
*65.* 13.
*66.* 0.
*67.* 0.

## DIVISION OF DECIMALS.

**Page 188.**

*68.* .8525.
*69.* 33.5.
*70.* 13.05714 +.
*71.* 167.5.
*72.* 223.333 +.
*73.* .25.
*74.* 87.5.
*75.* 16.6666 +.
*76.* 571.42857.
*77.* 4000.
*78.* 5.
*79.* 10.
*80.* .01.
*81.* 1.
*82.* 10.
*83.* 1.6666 +.
*84.* .6666 +.
*85.* 2.
*86.* .1.
*87.* 1000.

**Page 189.**

*88.* 108.1 cords.
282.9 cords.
*89.* $69.60.
164.40
*90.* $62.10.
$6630.90.
*91.* $13.35.
$3.3642.
$228.1782.
*92.* 114.5 gal.
123.675 gal.

**Page 190.**

*93.* $2.60.
$3.785.
*94.* 38.1904 + lbs.
286.428 + lbs.
*95.* 562.5 lbs.
2338.75 lbs.

## COMMON FRACTIONS TO DECIMALS.

**Page 191.**

*97.* .625.
.54545 +.
.571428 +.
.81818 +.
(*1.*) $.1666 +.
$.1428 +.
$.2857 +.
$.375.
(*2.*) .5714 +.
.3333 +.
.3636 +.
.8571 +.
(*3.*) .8888 +.
.0945 +.
.2162 +.
.0947 +.
(*4.*) .0740 +.
.1052 +.
.0061 +.
.0683 +.
(*5.*) .0364 +.
.0133 +.
.0146 +.
.0019 +.

**Page 192.**

*98.* $\frac{1}{2}$.
$\frac{1}{20}$.
*99.* $\frac{1}{4}$.
$\frac{3}{4}$.
$\frac{11}{20}$.
$\frac{1}{8}$.
$\frac{37}{200}$.

**Page 193.**

*100.* (*1.*) $\frac{3}{11}$.
$\frac{8}{9}$.
$\frac{5}{9}$.
$\frac{5}{11}$.
(*2.*) $\frac{7}{9}$.
$\frac{7}{11}$.
$\frac{7}{30}$.
$\frac{4}{11}$.
(*3.*) $\frac{13}{30}$.
$\frac{29}{30}$.
$\frac{1}{9}$.
$\frac{7}{9}$.
(*4.*) $\frac{1}{7}$.
$\frac{3}{7}$.
$\frac{4549}{11000}$.
$\frac{4}{5}$.

*102.* (*1.*) $\frac{7}{8}$.
$\frac{21}{50}$.
$\frac{11}{20}$.
$\frac{3}{4}$.
$\frac{9}{20}$.
$\frac{151}{200}$.
$\frac{1}{8}$.
(*2.*) $\frac{3}{20}$.
$\frac{3}{200}$.
$\frac{1}{40}$.
$\frac{1}{5}$.
$\frac{4}{5}$.
$\frac{7}{25}$.
$\frac{3}{8}$.
(*3.*) $\frac{1}{3}$.
$\frac{2}{3}$.
$\frac{7}{9}$.
$\frac{8}{9}$.
$\frac{6}{25}$.
$\frac{63}{100}$.
$\frac{4}{9}$.
(*4.*) $\frac{6}{25}$.
$\frac{7}{20}$.
$\frac{1}{250}$.
$\frac{5}{11}$.
$\frac{19}{75}$.
$\frac{37}{50}$.
$\frac{7}{25}$.

**Page 194.**

*103.* 8.0714.
*104.* 1.15625 lbs.
*105.* \$0.921.
*106.* \$0.0557.
*107.* 5.94 bbls.
*108.* 57.8687.
*109.* .675.
*110.* 5.6438 +.
*111.* 15.7527 +.
*112.* 18.5279 +.

**Page 195.**

*113.* \$33.87.
*114.* \$457.76.
*115.* \$76.45.
\$23.62.
\$2.58.
*116.* \$17.85.
\$2.47.
\$0.47.
*117.* \$1.46.
\$0.16.
\$190.78.
*118.* \$11.47.
\$7.98.
*119.* \$4.41.
*120.* \$2.07.

## EXAMPLES FOR REVIEW.

*121.* \$2.493.
*122.* \$13.482.
*123.* \$6.842.
*124.* \$6.593.
*125.* \$160.714.

**Page 196.**

*126.* \$274.288.
*127.* \$11.342.
*128.* \$1.086.
*129.* \$17.192.
*130.* \$2.925.
*131.* \$78.187.
*132.* \$58.65.
*133.* \$21.511.
*134.* \$11.226.
*135.* \$63.685.
*136.* \$929.88.
*137.* \$16.056.
*138.* \$8.875.
*139.* \$2084.197.
*140.* \$183.174.
*141.* \$114.858.
*142.* \$1131.212.
*143.* \$65.38 +.
*144.* \$13.375.
*145.* \$68.45.
*146.* \$183.19.
*147.* \$1001.10 received.
\$58.97 paid out.
\$942.13 less.
*148.* \$29.725 bo't.
\$9.632 sold.
\$20.093 bo't more.

## DENOMINATE NUMBERS.

**Page 206.**

*2.* $412\frac{1}{2}$ yards.
$1237\frac{1}{2}$ feet.
*3.* $11104\frac{1}{2}$ feet.
2349 feet.
*4.* 7716 inches.
1872 inches.
*5.* 231 feet.
$397\frac{1}{2}$ feet.

*6.* 5280 feet.
36960 feet.
$15856\frac{1}{2}$ feet.

**Page 207.**

*7.* 42240 ties.
*8.* 156 yards.
$273.
*9.* 378400 ties.
$30272.
*10.* 132 feet.
198 feet.
660 feet.
*11.* 396 feet.
96 boards.
$11.52.
*12.* 78 feet.
26 yards.
$0.78.
*13.* 310 panels.
$579.70.

**Page 208.**

*14.* 517 steps.

**Page 210.**

*15.* 30 ft. 7 in.
72 ft. 8 in.
*16.* 782 yds. 1 ft.
268 yds. 2 ft. 1 in.
*17.* 4 rds. 5 yds.
149 rds. 4 yds. 2 ft. 6 in.
25 rds. 1 ft. 6 in.
*18.* $12.30.
$16.40.
$46.33.
*19.* $284.24.
$222.64.
$167.20.
*20.* $119.
$153.

**Page 211.**

*22.* $3.18.
*23.* $2.58.
$5.34.
$4.92.
*24.* $13.25.
$8.51.
*25.* $260.66.

**Page 216.**

*27.* 234 sq. in.
520 sq. in.
*28.* 7500 sq. ft.
6460 sq. ft.
5762 sq. ft.
*29.* 1296 sq. in.
9 sq. ft.
*30.* 1248 sq. ft.
$138\frac{2}{3}$ sq. yds.
*31.* 120 sq. ft.
$13\frac{1}{3}$ sq. yds.
*32.* 648 sq. ft.
72 sq. yds.
*33.* 1458 sq. ft.
162 sq. yds.
*34.* 400 sq. ft.
$44\frac{4}{9}$ sq. yds.
*35.* $151\frac{5}{9}$ sq. yds.
*36.* $169\frac{1}{3}$ sq. yds.
*37.* $28.
*38.* 180 sq. yds.
$32.40.

**Page 217.**

*39.* $63.84.
*40.* 640 acres.
$21333.33.
*41.* 40 chains.
$11264.
*42.* 40 chains.
80 acres.
400 acres.
$6300.

**Page 218.**

*43.* 7 acres.
1280 acres.
1273 acres.
$12730.
*44.* $14836.80.
*45.* 160 acres.
$3573.33.
*46.* $556.80.
*47.* $393.60.
$18653.25.
*48.* 64 rods.
64 acres.
$1088.
$2707.20.
$896.
$1248.
$5939.20.
$3520.
$6459.20 more
$6459.20 gain.

**Page 219.**

*49.* $1008 each.
*50.* 7680 acres.
*51.* $53096.33.
*52.* $6.91.
*54.* $106.17.
$61.10.

**Page 220.**

*55.* $28.
$48.89.
*56.* $120.
*57.* $206.08.
*58.* 221 sq. ft.
425 sq. ft.

*59.* 168 sq. ft.
154 sq. ft.
*60.* 240 sq. ft.
198 sq. ft.
*61.* 306 sq. ft.
196 sq. ft.
*62.* $94.40.
*63.* $24.
*64.* $389.33.

### Page 224.

*66.* 320 c. ft.
*67.* 48 c. ft.
68 c. ft.
*68.* 688 c. ft.
*69.* 1280 c. ft.
*70.* 24 c. ft.
*71.* 128 c. ft.
221184 c. in.
*72.* 2688 bricks.
$ 3 44.
*73.* 4448 bricks.
$ 05.47
*74.* 26880 bricks.
$301.06.

### Page 225.

*75.* 51 pchs. $17\frac{3}{4}$ c. ft.
*76.* 62 pchs. $1\frac{1}{2}$ c. ft.
*77.* 1280 c. ft.
*78.* 2211840 blocks.
*79.* $9575\frac{15}{231}$ gals.

### Page 226.

*80.* 9 cords.
*81.* $3\frac{3}{8}$ cords.
$12.66.
*82.* $11.81.
*83.* $22.22.
*84.* 2400 feet.
*85.* $60.
*86.* 1600 feet.
*87.* $30.80.
*88.* 2100 feet.
$48.99.
*89.* 330 feet.
$8.25.

### Page 227.

*90.* $67.20.
*91.* $215.24.
*93.* 115 gals. 3 qts.
14 gals. 1 qt. 1 pt. 3 gills.
*95.* 172 qts.
$15.48.

### Page 228.

*96.* 84 gals.
$26.88.

### Page 229.

*97.* 1200 pints.
3008 pints.
*99.* $3.84.
$529.92.
*100.* $6.40.
*101.* $6.60.
*102.* Gained $2.25.
Lost $0.42.
*103.* $26\frac{2}{3}$ bottles.
80 pt. bottles.
64 cups each.
*104.* $4.77.
$5.36.
*105.* $8.70.
$18.90.
*106.* 1 bu. 1 pk. 7 qts.
*107.* $2.50.
$2.97.

### Page 230.

*109.* $24.38.
$33.44.
*110.* $41.21.
$53.97.
*111.* $25.52.
35.56.

### Page 231.

*113.* $36\frac{36}{215}$ bush.
$77\frac{169}{1075}$ bush.
$1\frac{1}{215}$ bush.
*114.* $1671\frac{159}{215}$ bush.
*115.* $97\frac{269}{1075}$ bush.

### Page 232.

*117.* 151984 c. in.
$657\frac{217}{231}$ gals.
*118.* $375\frac{223}{231}$ gals.
$2631\frac{175}{231}$ gals.
*119.* $11913\frac{93}{231}$ gals.

### Page 233.

*120.* 160 pills.
120 pills.
*121.* $4.20.
*122.* $9.60.
$8.04.
*123.* $20.25.

### Page 234.

*124.* 1200.
10352.
*125.* 1346 lbs.
1237 lbs.
*126.* 107 cwt.
293 cwt.
*127.* 444 oz.
*128.* 3 cwt. 47 lbs.
4 cwt. 76 lbs.
1 cwt. 54 lbs 3 oz.
820 cwt.

### Page 235.

*130.* $49.28.
$55.09.
*131.* $9.73.
$7.94.
$10.38.
*132.* $118.97.
$79.38.
$97.16.
*133.* $8\frac{11}{28}$ bushels.
$15.36.
*134.* $108\frac{22}{32}$ bushels oats.
$62\frac{6}{56}$ bushels corn.
$57\frac{28}{60}$ bushels wheat.
$72\frac{22}{48}$ bushels barley.
$57\frac{28}{60}$ bushels potatoes.
$61\frac{1}{57}$ bushels onions.
$69\frac{28}{50}$ bushels salt.
*135.* $7\frac{47}{56}$ bushels rye.
$7\frac{12}{60}$ bushels beans.
$7\frac{12}{60}$ bushels clover.
$6\frac{12}{70}$ bushels ears of corn.
$9\frac{43}{44}$ bushels hempseed.
*136.* $0.59.
*137.* $0.94.
*138.* $1.62.
*139.* $5.12.
*140.* $4.88.

### Page 236.

*141.* $17.46.
*142.* $15.
*143.* $135.06.
*144.* $33.36.
*145.* $29.73.

### Page 238.

*146.* 9 d. 1 hr. 31 m. 4 sec.
*147.* 58 d. 10 hr. 40 min.
*148.* 19 hr. 57 min. 14 sec.
*149.* 94608000 seconds.
*150.* 7776000 seconds.
*151.* 7862400 seconds.
*152.* 4 hr. 5 min. 20 sec.

### Page 239.

*154.* 5 hr. 8 min.

### Page 240.

*155.* 18 min. 28 sec.
*156.* 2 hr. 26 min. 48 sec.
*157.* 6 hr. 30 min. 32 sec.
*158.* 4 hr. 49 min. 28 sec.
*159.* 60° 49′.
*160.* 41° 35′ 15″.
*161.* 15° 34′ 30″.
*162.* 60° 15′ 30″.
*163.* $3.60.
$3.24.
*164.* Gained $2.28.

### Page 241.

*166.* 60 shillings.
40 shillings.
6540 shillings.
*167.* 680 shillings.
8160 pence.
32640 farthings.

### Page 242.

*168.* 62 bu. 3 pks. 7 qts.
*169.* 52 lb. 5 oz.
*170.* 97 A. 55 p.
*171.* 85 rds. 14 ft.
*172.* £74 5 s. 2 d.
*173.* 378 A. 80 p.

### Page 243.

*174.* £36 18 s. 1 d. 3 far.
*175.* 5 lb. 3 oz. 14 pwt. 17 grs.
*176.* 3 lb. 9 ℥ 2 ʒ 16 grs.
*177.* 1 m. 20 rds. 4 yds.
*179.* 3 yrs. 1 mo. 20 days.

*180.* 16 yrs. 1 mo. 8 days.
*181.* 2 yrs. 3 mo. 2 days.
*182.* 9 yrs. 5 mo. 10 days.
*183.* 1 mo. 26 days.
*184.* 1 yr. 11 mo. 26 days.
*185.* 1 yr. 7 mo. 27 days.
*186.* 8 days.
*187.* 64° 48′ 24″.
*188.* 25° 51′ 50″.

### Page 244.

*189.* 56° 36′.
*190.* 157°.
*191.* 3 h. 25 min. $12\frac{4}{15}$ sec.
*192.* 25 gallons, 1 pint.
*193.* 97 pounds, 9 ounces.
*194.* 221 A. 126 perches.

### Page 245.

*195.* £16 3 s. 1 d. 2 far.
£37 13 s. 11 d. 2 far.
*196.* 17 rds. 1 yd. 1 ft. 6 in.
51 rds. 4 yds. 1 ft. 6 in.
*197.* 3 d. 2 hr. 14 min. 1 sec.
9 d. 23 hr. 49 min. 54 s.
*198.* 43 bushels.
*199.* 55 pounds, 8 ounces.
83 pounds, 4 ounces.
*200.* 2 lb. 6 oz. 12 pwt.
$42.84.
*201.* 4 lb. 5 oz. 10 pwt. 12 gr.
7 lb. 1 oz. 12 pwt. $19\frac{1}{5}$ gr.
*202.* 30 bu. 1 lb. $1\frac{1}{15}$ oz. rem.
*203.* 1 oz. 13 pwt. 20 gr.
*204.* 67 gal. 2 qt. $1\frac{1}{7}$ gills.

### Page 246.

*205.* 85 A. 46 p.
*206.* 12 m. 254 rds. 4 yds. 2 ft. $9\frac{3}{19}$ in.
*207.* 14 bu. 1 pk. $2\frac{1}{4}$ qt.
*208.* £2 10 s. 2 d. $3\frac{3}{7}$ far.
*209.* $27\frac{1}{2}$ spoons.
*210.* 88 days.
*211.* 96 bottles.
*212.* $725\frac{1}{3}$ packages.

### Page 248.

*213.* 192 rods.
96 rods.
*214.* 2 yards.
2 feet 9.66 inches.
*215.* 18 inches.
32.4 inches.
514.8 inches.
*216.* 178 rds. 1 yd. $4\frac{6}{7}$ in.
*217.* 19 cwt. 35 lbs.
*218.* 7 oz. 15 pwt. $6\frac{6}{7}$ grs.
123 rds. 3 yds. 5.85 in.
*219.* $704.
*220.* $7267.20.
$52.36$\frac{4}{11}$.

### Page 249.

*221.* $\frac{17}{18}$ yards.
.94 yards.
*222.* $\frac{49}{2112}$ miles.
.0232 + miles.
*223.* $1\frac{3}{32}$ gallons.
1.09375 gallons.
*224.* $0.90.
*225.* $0.309.
*226.* .0015 rods.
$\frac{53}{132}$ rods.
*227.* 1.43 rods.
$1\frac{43}{99}$ rods.

### Page 250.

*228.* $34.71.
229. $14.30.

### Page 251.

*231.* £20 4 s.
$98.29 +.
*232.* $144.03.
*233.* $79.32.
*234.* $14.36.
*235.* $21.13.
*236.* $2.80.
*237.* 9 spoons: 5 pwt. 9 grs. rem.
*238.* $30.88.
*239.* $1.25.
$1.62.

### Page 252.

*240.* $2.62.
*241.* $6.19.
$7.43.
*242.* $0.17.
$0.92.
$1.64.
*243.* $83.55.
*244.* $0.38.
*245.* 188 doses.
*246.* 384 pills.
*247.* $8.31.
$6.34.
$4.27.
*248.* $5062.50.
*249.* 60 yards.
$133\frac{1}{3}$ yards.
*250.* 4856 square feet.
*251.* $201.67.
*252.* $175.

### Page 253.

*253.* 9 square feet.
$0.18.
*255.* $100.79.
*256.* $72.
*257.* $7560.30.
*258.* $54.11.
*259.* 240 loads.
*260.* $450.
*261.* 99 cubic feet.

### Page 254.

*262.* 424 square feet.
$10.60.
*263.* 80 perches.
*264.* $66.69.
*265.* $15.45.
*266.* $0.25.
*267.* 56 bushels.
*268.* 19.285 + bushels.
*269.* $209\frac{5}{12}$ pounds.
*270.* $12.80
*271.* $6.60.
$0.26.

### Page 255.

*272.* 15 loads: $17\frac{223}{1075}$ bush. rem.
*273.* $363\frac{7}{11}$ barrels.
*274.* $43.33.
*275.* $426\frac{2}{3}$ rods.

# ANSWERS.

## SECOND PART.

### PERCENTAGE.

**Page 266.**
*1.* 2.73.
18.63.
*3.* 73.52 gals.
*4.* 182.07 gals.
*5.* 5.25 gals.
*6.* $6.50.
*7.* $74.40.
$855.60.

**Page 267.**
*8.* $68.60.
*10.* $25.27.
*11.* $6.45.
*12.* $21.81.
*13.* $174.40.
*14.* $39.
*15.* $85.
*16.* $26.33.
*17.* $129.60.

**Page 268.**
*18.* .68¾.
*19.* 3.57 yds.
3.54¾ gals.
*20.* 1.12 bar.
62 trees.
*21.* $60.
*22.* $7.70.
*23.* $45.
*24.* $1575.
*25.* $16.67.
*26.* $16.32.
*27.* 9⅜.

**Page 269.**
*28.* $23.44.
*29.* $221.25.
$1548.75.
$4.13.
*30.* $2365.
$1419.
*31.* $98.15.
*32.* 150 men.
155 sheep.
*34.* 2 cwt. 37 lbs.

**Page 270.**
*35.* 5 gals. 1.28 pts.
*36.* 2 tons, 8 cwt. 91 lbs.
$6.33.

**Page 271.**
*37.* $1875.
$3125.
*38.* $253.82.
$442.47.

**Page 272.**
*39.* 147.
*40.* Son, $6890.
Dau'ter, $4240
Wife, $11130.
*41.* 1125 sheep.
425 sheep.
700 sheep.
*42.* 78.
*43.* $192.
*44.* $1.65.
*45.* $3.75.
*46.* 2350 bbls.

**Page 273.**
*47.* $3656.25.
*48.* 46 cents.
*49.* $8.32.
$7.20.
$8.
*50.* $2362.50.
*51.* $210.
$35.
*52.* $7.70.
*53.* $273.
*54.* $3900.
*55.* $6683.40.
*56.* 336 A.
*57.* $31943.

### Page 274.

58. $40.
59. $426.87.
60. 396.
61. $2.70.
62. (1.) $.12.
(2.) $.144.
(3.) $1.44.
(4.) $.096.
(5.) $.42.
(6.) $4.50.
(7.) $.24.
(8.) $1.056.
(9.) $.384.
(10.) $.216.
(11.) $2.64.
(12.) $.45.

### Page 276.

63. 6%.
64. 25%.
65. $16\frac{2}{3}$ %.
66. 5 %.
67. $7\frac{1}{2}$ %.
68. 70 %.
69. $\frac{1}{2}$ %.
70. $4\frac{1}{6}$ %.
2400 %.

### Page 277.

71. 50 %.
72. 3750 %.
73. 50 %.
100 %.
125 %.
160 %.
190 %.
125 %.
$9\frac{1}{11}$ %.
75. $6\frac{1}{4}$ %.
$\frac{25}{48}$ %.
76. $13\frac{8}{9}$ %.
5 %.
77. $12\frac{4}{33}$ %.
78. $27\frac{7}{9}$ %.
79. $7\frac{1}{7}$ %.
80. $21\frac{17}{23}$ %.

### Page 278.

81. 2 %.
82. 75 %.
$18\frac{2}{11}$ %.
83. 2 %.
$6\frac{1}{2}$ %.
84. 5 %.
85. 25 %.
86. $6\frac{2}{33}$ %.
87. $31\frac{1}{4}$ %.
$9\frac{1}{11}$ %.
$33\frac{1}{3}$ %.
50 %.
88. $3\frac{1}{3}$ %.
89. $22\frac{94}{103}$ %.
90. 70 %.
91. 25 %.

### Page 279.

92. $56\frac{14}{31}$ % sold.
$43\frac{17}{31}$ % unsold.
93. 15 % @ $.90.
35 % @ $1.
50 % @ $1.10.
$972.90.
95. $12\frac{1}{2}$ %.
96. $7\frac{23}{31}$ %.
97. $2\frac{1}{2}$ %.
$50.
98. 100 %.
20 %.
200 %.
99. 20 %.
50 %.
75 %.
100 %.
100. $1\frac{1}{3}$ %.
$98\frac{2}{3}$ %.

### Page 280.

101. 32 %.
102. $2\frac{1}{2}$ %.

### Page 281.

103. $1680.
104. $333.33.

### Page 282.

106. 100 bu.
50 bu.
25 bu.
20 bu.
107. 9600.
3600.
108. 40.
200.
109. $100.
$200.
$126.90.
110. $920.
$20.
$200.

### Page 283.

111. $165.
112. $3000.
113. $11600.
114. $1843.80.
115. $4116.
116. 40 cts.
117. 4 lbs.
118. 63 gals.
119. $3440.
120. $570.
121. $6054.40.

**Page 285.**

*122.* 70 cts.
*123.* $312.50.
*124.* $1320.
*125.* $3.

**Page 286.**

*126.* $6736.
*127.* $22500.
*128.* $3950.
*129.* $9.37½.
*130.* (1.) $4.
(2.) $5.
(3.) $2.50.
(4.) $17.50.
(5.) $3.
(6.) $.90.
*131.* 150.

**Page 287.**

*134.* 390 bbls. and $5.20 remainder.
*135.* $3840.
*136.* $1555.
1244 bushels.
*137.* (1.) $20.62½.
(2.) $13.03½.
(3.) $6.22½.
(4.) $3.75.
(5.) $.37½.
(6.) $.30.
(7.) $.30.
(8.) $2.62½.
*138.* $124.26.
$165.68.
$41.42.
*139.* 75.
100.
120.
$148\frac{52}{101}$.
$149\frac{281}{401}$.
$149\frac{353}{803}$.
*140.* $3349.51.
*141.* $420.

**Page 288.**

*142.* $400.
*143.* 50 cts.
*144.* 60 bushels.
*145.* $1187.50.
*146.* $1.25.
$1.33⅓.
*147.* $46.
*148.* 200.
*149.* $20000.
*150.* $35500.
*151.* $1.28.
*152.* $1.28.
*153.* 16 cts.
*154.* $48.
*155.* $1000.
*156.* $2.50.

**Page 289.**

*157.* 2 %.
*158.* 37½ %.
*159.* $122\frac{2}{9}$ %.
*160.* $1\frac{9}{11}$ %.
*161.* ⅜ %.
*162.* $14.40.
*163.* $4.
*164.* 5947½ bu.

**Page 290.**

*165.* $2472.
*166.* $4.25.
*167.* $130.50.
*168.* $6.71.
*169.* $42.
*170.* 1⅓ %.
98⅔ %.
*171.* 27 %.
*172.* $423.
$70.50.
*173.* $4.60.
$1.15.
*174.* $303.98.
*175.* $3380.
*176.* $21823.75.
*177.* $288.75, horse.
$218.50, car'ge
$2.25, gain.

**Page 291.**

*178.* $2.56.
*179.* $219.64.
*180.* 25 %.
$600.
*181.* $1.60.
$90.
*182.* 23 bbls.
*183.* 80 %.
$1668.25.
*184.* $1620.
62½ %.
37½ %.
*185.* $147.50.
$4\frac{1}{12}$ %.
$31\frac{5}{24}$ %.
*186.* $4.65, Albert.
$5.35, Arthur.
70 cts.

**Page 292.**

*187.* $2693.78.
*188.* (1.) $12½ cts.
(2.) 15 cts.
(3.) $1.50.
(4.) 10 cts.
(5.) 43¾ cts.
(6.) $2.18¾.
(7.) 25 cts.

(8.) $1.10.
(9.) 40 cts.
(10.) $22\frac{1}{2}$ cts.
(11.) $2.75.
(12.) $46\frac{7}{8}$ cts.
*190*. 32.
*191*. 60 %.
*192*. 4, average.
80 %.
40 %.

### Page 293.

*193*. $91\frac{2}{3}$ %.
*194*. $87\frac{1}{2}$ %.
*195*. $18.75.
*196*. $9.03.
*197*. 27 %.
$58.10, Henry.
$74.70, John.
$91.30, Mary.
$605.90, left.
*198*. $1.35, gain.
$1\frac{52}{76}$ %.
*199*. 1800 ft.
*200*. 3000 bu.
*201*. 960.
*202*. 9600.
*203*. 960.
*204*. 2500.
*205*. .25.
*206*. 25.
*207*. 1200, B.
$653\frac{1}{8}$, A.
$28\frac{1}{8}$, P.
$596\frac{1}{8}$, D.

### Page 294.

*208*. 1060, B.
15 %.
126, P.
*209*. 72 A. $121\frac{1}{5}$ sq. rds. wood.
36 A. $60\frac{3}{5}$ sq. rds. past.
254 A. $104\frac{1}{5}$ sq. rds. cultivated.
*210*. $14250.
*211*. 15 %.
*212*. $4\frac{1}{2}$ %.
*213*. $14\frac{2}{7}$ %.
*214*. $2.
*215*. 4760 bu.
*216*. 205 bbls.
*217*. $104242\frac{14}{33}$ lbs.

### Page 295.

*218*. $680.
*219*. $5526.60.
*220*. $2892.50.
*221*. 9 %.
*222*. $8750.
$12500.
*223*. $304.94.
*224*. $11686.19.
*225*. $6124.33.
*226*. $6.
*227*. $44090.

### Page 296.

*228*. 779 bbls. and $7.85, remainder.
*229*. $1.20.
20 %.
*230*. $1.04.
*231*. $6\frac{1}{4}$ %, loss.
*232*. $520.
*233*. 44 cts.
*234*. 1 %, loss.
*235*. 4 %.
*236*. 17 %.
*237*. 130 %.
*238*. $2.08, loss.
*239*. $134.

### Page 297.

*240*. $18.56.
*241*. $28\frac{4}{7}$ %.
*242*. $18000.
$12000.
*243*. $45.30.
*244*. $4.62.
*245*. $30.75.
*246*. $2880.
$50000.
*247*. $22425.56.

### Page 298.

*248*. $48.
*249*. $14\frac{2}{7}$ %.
*250*. $$4.66\frac{2}{3}$.
*251*. $565.19.

## STOCKS.

### Page 299.

*252*. $1270.
$4699.
$17399.
*253*. $7312.50.

### Page 300.

*254*. $8.75.
$3788.75.
*255*. $4326.
$13390.
*256*. $1260.

$5250.
*257.* $1030.
$5150.
*258.* $870.
*259.* $3587.50.
*260.* $892.50.
*261.* $3123.75.
*262.* 24 shares.
*263.* 10 %.
*264.* $50.
*265.* $183.75.

**Page 301.**
*266.* $5540, sur.
$125, div.
*267.* $36765.
*268.* $145700.
*269.* 10 %.
$2613.50, surplus.
*270.* 24 % and $9600, sur.
*271.* $2\frac{102}{169}$ %.
*272.* $250.

**Page 302.**
*273.* $1925.
*274.* $42.25.
*275.* 70 shares.
*276.* 10772 lbs., 1 ct., rem.
*277.* $143.75.
*278.* $90\frac{10}{11}$ cts.
$45.45.
*279.* 37 bonds, $15.38, rem.
*280.* $413.60, gain.
$22\frac{44}{45}$ %.

## INSURANCE.

**Page 304.**
*2.* $20.80.
*3.* $5.25.
*4.* $3750.
*5.* $50000.
*6.* $11\frac{1}{4}$ %.
*7.* $1\frac{1}{2}$ %.

**Page 305.**
*8.* $143.75.
*9.* $196.88.
*10.* $11.89.
*11.* $1395.
*12.* 4 %.
$1\frac{1}{2}$ %.

**Page 306.**
*14.* $56.25.
*15.* $4556.25.
*16.* $7443.75.
*17.* $425.
$1\frac{7}{10}$ %.
*18.* $3840.
*19.* $550.
$2450.
*21.* $400.
$600.

**Page 307.**
*22.* $12000.
$16000.
*23.* $7410.

## LIFE INSURANCE.

*1.* $51.

**Page 308.**
*2.* $1638.
*3.* $237.50.
*4.* $2474.24.
*5.* $2880.

## TAXES.

**Page 310.**
*2.* $98.54.
*3.* $85.05, B's tax.
$49, C's tax.
$203, McF's tax.

**Page 311.**
*4.* $1\frac{1}{4}$ %.
*5.* $994152.
*6.* $1\frac{1}{2}$ %.
*7.* $149.46.
*8.* $268.75.

## CUSTOMS.

**Page 313.**

*2.* $1129.57.
*3.* $1159.54.
*4.* $668.
*5.* $1535.
*6.* $26400.
*7.* $245.70.

## INTEREST.

**Page 320.**

*1.* $45.
*2.* $496.
*3.* $201.10.
*4.* $38.04.
$50.72.
*5.* $76.88.
$104.79.
*6.* $403.
$604.50.
*7.* $483.35.

**Page 321.**

*8.* $1072.19.
*9.* $163.08.
$1036.73.
*10.* $561.32.
*11.* $124.94.
*12.* $735.01.
*13.* $1.02.
$2.02.

**Page 322.**

*14.* $68.17.
*15.* $20.12.
*16.* $523.65.
*17.* $66.43.
*18.* $203.90.

**Page 323.**

*19.* $326.17.
*20.* $940.93.
*21.* $4419.87.
*22.* $432.31.
*23.* $1460.70.
*24.* $850.47.
*25.* $1259.41.
*26.* $638.19.
*27.* $58.33.

**Page 324.**

*28.* $178.29.
*29.* $946.66.
*30.* $597.63.

**Page 326.**

*31.* $94.87.
*32.* $75.23.
*33.* $366.84.
*34.* $452.81.
*35.* $895.75.
*36.* $196.57.
*37.* $99.43.
*38.* $41.35.
*39.* $79.90.
*40.* $383.14.
*41.* $8.24.

**Page 327.**

*42.* $8.52.
*43.* $3780.60.
*44.* $110.57.
*45.* $1329.02.
*46.* $1097.06.
*47.* $422.09.
*48.* $911.92.
*49.* $1.29.
*50.* $4387.89.

**Page 328.**

*51.* $72.64.
*52.* $33.91.
*53.* $251.67.
*54.* $159.96.
*55.* $232.60.
*56.* $70.64.
*57.* $3.04.

**Page 329.**

*58.* $1.18.
*59.* $1.46.
*60.* $1.40.
*61.* $0.35.

**Page 331.**

*62.* 280.
*63.* 233.
*64.* 184.
*65.* 267.
*66.* 427.
*67.* 363.
*68.* 241.
*69.* $2.20.
*70.* $2.31.

**Page 332.**

*71.* $87.08.
*72.* $97.79.
*73.* $10.37.
*74.* $81.39.
*75.* $3.03.
$3.53.
*76.* $880.54.
*77.* $8.17.
$9.53.
$6.81.
*78.* $357.22.
$356.23.
$359.21.

**Page 334.**

*79.* $209.38.
*80.* $1415.81.
*81.* $1350.
*82.* $647.32.
*83.* $4218.75.
*84.* $3164.06.

**Page 335.**

*85.* $56.80.
*86.* $530.

**Page 336.**

*87.* $77.
*88.* $125.
*89.* $3600.

**Page 337.**

*90.* 8 %.
*91.* 4½ %.
*92.* 3 %.
*93.* 4 %.
*94.* 5 %.
*95.* 10 %.
*96.* 26⅔ %.

**Page 339.**

*97.* 3 yrs. 7 mos. 6 da.
*98.* 1 yr. 9 mos. 6 da.
*99.* 1 yr. 1 mo. 27 da.
*100.* 4 yrs.
*101.* 14 yrs. 3 mos. 13 da.
*102.* 33 yrs. 4 mos.
*103.* 20 yrs.
12 yrs. 6 mos.
10 yrs. 6 mos. 9 da.
6 yrs. 8 mos.
*104.* 14 yrs. 3 mos. 13 da.
11 yrs. 1 mo. 10 da.
5 yrs.
11 yrs.

## COMPOUND INTEREST.

**Page 341.**

*105.* $367.51.
$67.51.
*106.* $2551.81.
*107.* $403.75.
*108.* $50.54.
*109.* $25.02.
*110.* $85.83.

## ANNUAL INTEREST.

**Page 343.**

*111.* $1744.
*112.* $7025.63.
*113.* $60.39.
*114.* $1024.91.
*115.* $93.18.
*116.* $180.
$190.80.
$191.02.

## PARTIAL PAYMENTS.

**Page 346.**

*117.* $375 55.

**Page 347.**

*118.* $441.28.
*119.* $375.18.
*120.* $36.93.
*121.* $30.38.
*122.* $1031.99.

**Page 348.**

*123.* $561.67.
*124.* $255.48.
*125.* $276.10.

**Page 349.**

*126.* $287.06.

## MERCANTILE RULE.

**Page 350.**
*127.* $134.51.
*128.* $108.53.
*129.* $71.95.
*130.* $228.12.
*131.* $73.75.

## BANKING.

**Page 352.**
*132.* $61.11.
*133.* $29.93.
*134.* $4.94.
*135.* $9.48.
*136.* $139.31.

**Page 353.**
*137.* (1.) $348.08.
(2.) $197.55.
(3.) $441.34.
(4.) $493.88.
(5.) $772.40.
(6.) $355.50.
(7.) $394.85.
(8.) $984.50.
*138.* $9.21.

*139.* $564.06.
*140.* $100.75.
*141.* $162.05.
*142.* $152.45.
*143.* $234.34.
*144.* $20.41.
*145.* $214.43.
*146.* $71.33.
*147.* 30 bbls.

**Page 354.**
*148.* $499.73.
*149.* $698.65.
*150.* $624.63.
*151.* $81.27.
*152.* 25 bbls.

**Page 355.**
*153.* $620.38.
*154.* $377.13.
*155.* $700.25.
*156.* $328.63.
*157.* $38.39.
*158.* $458.02.
*159.* $97.77.
*160.* $510.55.
*161.* $563.95.
*162.* $353.71.
*163.* $636.51.
*164.* $285.91.
*165.* $1508.30.
*166.* $3682.07.
*167.* $132.96.

## PRESENT WORTH.

**Page 357.**
*168.* $181.37.
*169.* $350.
*170.* $14.91.
*171.* $14.45.

*172.* $325.69.
$40.71.
*173.* $68.84.
*174.* $131.76.
*175.* $151.85.
*176.* $346.

**Page 358.**
*177.* $291.69.
$0.36.
*178.* $0.24.
$0.24.
*179.* Lose,$412.77

## BONDS.

**Page 364.**
$3375.
$1792.
*4.* $9722.25.
*5.* $3845.63.
*6.* *$23000.*

*7.* 27.
*8.* 48.
*9.* 10.
*10.* 4.
*11.* 80.

**Page 365.**
*12.* 80.
*14.* $720.
$3\frac{91}{103}$ %.
15. $5\frac{5}{11}$ %.

*16.* $5\frac{55}{109}$ %.
$4\frac{64}{109}$ %.
*17.* $3\frac{5}{9}$ %.
*18.* $4\frac{1}{6}$ %.
*19.* $4\frac{6}{11}$ %.
*20.* $2025.
$3\frac{69}{77}$ %.
*21.* $4\frac{46}{51}$ %.
*22.* $4\frac{4}{19}$ %.
*23.* $5\frac{15}{97}$ %.
$4\frac{4}{9}$ %.
$5\frac{5}{39}$ %.

## EXCHANGE.

**Page 368.**

*13.* $780.
*14.* $945.
$942.
$875.25.
*15.* $468.
*16.* $740.95.

**Page 369.**

*17.* $714.
*18.* $351.75.
*19.* $611.70.
*20.* $1245.30.
*21.* 770.20.
*22.* $292.88.

**Page 370.**

*24.* $1197.70.
*25.* $1632.65.
*26.* $2223.30.
*27.* $603.02.
*28.* $631.25.
$1485.15.
*29.* $445.96.
*30.* 2 % premium.
*31.* Gain, $2.28.
*32.* $10.75.

**Page 372.**

*33.* $779.56.
*34.* $4152.25.
*35.* $1548.50.

**Page 373.**

*36.* $281.37.
*38.* $478.93.
*39.* $216.60.
*40.* £1377 3*s.* 8*d.*
*41.* £1690 4*s.* 5*d.*

**Page 374.**

*42.* 10540 fr.
*43.* 1204.81 florins
*44.* £622 8*s.* $1\frac{11}{25}$*d.*

## EQUATION OF PAYMENTS.

**Page 376.**

*45.* 6 mos. 9 da.
*46.* 7 mos. 2 da.
*47.* 4 mos.
*48.* 9 mos. 1 da.
Nov. 2.
*49.* Nov. 27.

**Page 377.**

*50.* $3\frac{1}{2}$ mos.
*51.* 1 mo. 16 da.
*52.* 29 da.
*53.* 7 mos. 11 da.

**Page 379.**

*54.* July 23.
*55.* June 27.
*56.* Sept. 24.
*57.* May 16.
*58.* July 8.

**Page 380.**

*60.* 6 mos. 10 da.
*61.* 18 mos. 21 d

## RATIO.

**Page 383.**

*1.* $2\frac{1}{2}$.
$\frac{2}{5}$.
2. $\frac{1}{8}$.
$\frac{3}{4}$.
$\frac{7}{40}$.
*3.* $\frac{1}{2}$.
$\frac{1}{4}$.
$\frac{13}{576}$.
*4.* $\frac{55}{72}$.
*5.* $\frac{1}{2}$.
6. $\frac{5}{24}$.
7. $\frac{26}{31}$.
$5\frac{1}{2}$.
*8.* $4\frac{4}{13}$.
*9.* $1\frac{1}{3}$.
*10.* 6.

## PROPORTION.

**Page 384.**

1. 5.
2. 9.
3. 20.
4 $6\frac{1}{4}$.
5. 5 : 6 :: 15 : 18.

**Page 385.**

6. 8.
7. 3.
8. 5.
9. 11.
10. 576.

**Page 386.**

11. $108.
12. $82.50.
13. $24640.
14. 72 feet.
15. 75.
16. $29.04.
17. $182.67.
18. 850 bbls.
19. 16 h. 48 m.
20. 10 da.

**Page 387.**

21. 6 da.
22. $1064.25.
23. $1680.
24. $54$\frac{3}{8}$
25. $6300.
26. $21.85.
27. 28.
28. 50.
29. $20.30.
30. 16 da.
31. 33 h. 20 m.
32. $1$\frac{7}{8}$.
33. 180.
34. $3\frac{1}{16}$ mos.

**Page 389.**

35. 10.
36. $12\frac{1}{2}$.
37. $259\frac{1}{5}$.
38. 200.
39. 3.
40. 120.

**Page 390.**

41. 13358400.
42. 194400.
43. $124.75.

**Page 391.**

44. $52.70.
45. $39.42.

**Page 392.**

46. 88.
47. $34.
48. $4860.
49. 5 h. 20 min.
50. 288.
51. 8.

**Page 394.**

52. $388$\frac{4}{5}$.
53. 4 mos.
54. $236.11.
55. 33 da.

## PARTNERSHIP.

**Page 397.**

1. A, $150.
B, $112.50.
C, $87.50.
2. A, $700.
B, $980.
3. A, $1510.
B, $2265.
C, $755.
4. J, $420.
T, $504.
5. A, $90.
B, $180.
C, $270.
6. A, $200.
B, $100.
C, $50.
7. B, 702 bu.
H, 624 bu.

**Page 398.**

8. A, $130.
B, $216.67.
C, $173.33.
9. H, $952.38.
C, $1777.78.
M, $1269.84.
10. $8500, profit.
$4250.

**Page 398.**

11. $841.25.
12. A, $900.

B, $600.
C, $2250.
*13.* A, $48.
B, $70.
*14.* J, $850.50.
F, $1102.50.
N, $1680.

## ALLIGATION.

**Page 400.**

*1* $.66.

**Page 401.**

*2* $1.
*3.* $.95½.
*4.* $.40.
*5.* $269.

**Page 403.**

[An indefinite number of correct answers may be given to the remaining examples of Alligation. The teacher should accept any which the pupil can show to be correct.]

*6.* 1 lb. at 28 cts.
2 lbs. at 32 cts.
1 lb. at 40 cts.
1 lb. at 48 cts.
*7.* 2 of the 1st to 1 each of the 2d & 3d.
*8.* 1 of the 1st to 2 of the 2d.
*9.* 1⅓ of the 2d to 1 each of the 1st & 3d.

**Page 404.**

*10.* 12 at 8 cts.
12 " 10 cts.
12 " 12 cts.
*11.* 24 " 40 cts.
8 " 60 cts.
8 " 70 cts.
*12.* 100 " 35 cts.
100 " 40 cts.
*13.* 125 " $1.15.
625 " $1.20.
125 " $1.35.
125 " $1.50.
*14.* 400 " 90 %.
800 " 85 %.

## POWERS AND ROOTS.

### POWERS.

**Page 406.**

*1.* 7569.
5776.
3249.
441.
*2.* 2401.
7225.
4096.
2916.
*3.* 1156.
8649.
*1024.*
*4489.*
*4.* 4624.
3025.
576.
1444.
*5.* $\frac{225}{324}$.
$\frac{196}{225}$.
$\frac{441}{625}$.
$\frac{729}{900}$.
*6.* .0289.
.000625.
.000961.
.00000625.

**Page 407.**

*7.* 12167.
91125.
13824.
148877.
*8.* 103823.
571787.
884736.
*9.* 19683.
592704.
250047.
157464.
10. 50653.

24389.
941192.
$\frac{1728}{4096}$.
$\frac{15625}{32768}$.

$\frac{13824}{2197}$.
$\frac{132651}{13824}$.
*12.* 91.125.

.015625.
.000005832.
.000000046656.

## SQUARE ROOT.

**Page 411.**
*2.* 25.
24.
38.
55.
65.

**Page 412.**
*4.* 36.
74.
633.

*15.* 719.
427.
4879.
*16.* 7.5.
17.5.
825.
*17.* 8.68 +.
2.54 +.
2.64 +.
2.23 +.
1.41 +.
3.31 +.

*18.* .81 +.
$\frac{6}{8}$.
$\frac{2}{3}$.
$\frac{1}{3}$.
$\frac{1}{2}$.
*19.* $\frac{12}{17}$.
$\frac{32}{35}$.
$\frac{31}{43}$.
*20.* $\frac{16}{14}$.
$\frac{18}{11}$.
8.07 +.

## CUBE ROOT.

**Page 416.**
*1.* 57.
*2.* 432.
*3.* 262.
*4.* 636.
*5.* 123.

*26.* 1234.
*27.* 346.7.
*28.* 423.
*29.* 2.012.
*30.* 16.1.

*31.* 3.58 +.
*32.* 2.08 +.
*33.* $4\frac{1}{3}$.
*34.* $\frac{9}{16}$.
*35.* .84 +.

## METRIC SYSTEM.

**Page 423.**
*1.* 8046.72 M.
4838.09 + M.
*2.* 58.06 + M.

$6.97.
*3.* 11 ft. 1.8 + in.
4 ft. 11.8 + in.
6 ft. 10.6 + in.

*4.* 55 lbs. 1.8 + oz.
*5.* 33 cts.
*6.* 41 cts.

# MENSURATION.

**Page 427.**

*1.* 32 A. 46 sq. rds.
*2.* 124 sq. yds. 4 sq. ft.
*3.* 380 sq. yds.

**Page 428.**

*4.* 8 sq. ft. 108 sq. in.
*5.* 5 A. 64 sq. rds.

**Page 429.**

*6.* $73.50.
*7.* 504 sq. ft.
*8.* $187.50.

**Page 430.**

*9.* 10ft.
*10.* 38.7 ft.

**Page 431.**

*11.* 35.3 ft.
*12.* 4 ft. 8.54 + in.
*13.* 7.95 + ft.
*14.* 12.566 + ft.
*15.* 502 A. 104.[illegible] + sq. rds.
*16.* 2 sq. ft. 92.[illegible] + sq. in.
*17.* 50 A. 148.7 sq. rds.